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THE

Posthumous Works

OF

Mr. JOHN WARD,

AUTHOR of the

Young Mathematician's GUIDE.

IN TWO PARTS.

PART I. Containing, His New Method of Navigation by Parallel Parts, by which all Questions in Sailing may be answered with great Expedition and Truth, in a different Manner from Plain Mercator, and Great Circle Sailing, by the Solution of a plain Triangle only.

ALSO,

Compendiums of Practicall and Speculative Geometry, and of Plain Trigonometry, with their Application to Plain Mercator, and Middle Latitude Sailing, with several curious Questions in Surveying.

PART II. Containing, The Doctrine of the Sphere, and the Demonstrations and Calculations of Spherical Trigonometry, in which the Construction of the Figures are New, and drawn so as to represent Solids, by which the Demonstrations are made easy to the meanest Capacity.

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M. DCC. XXX.





T H E
P R E F A C E.



THE Author was pleased to make me acquainted with the Manuscript of the following Treatise, which, falling more immediately under my Care after his Death, I apprehended myself under the strongest Obligations to print the same, that whatever was New, or Improved therein, might be published for general Use and Advantage: And the generous Reception which all real Improvements of Arts and Sciences meet with in this Nation, and the Glory Great-Britain

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has acquir'd by the Success of them, especially in Mathematical Learning (in which our Author might claim a Share) is a sufficient Inducement to discharge this Obligation without any further Apology, since thereby is discovered to the Publick, the ingenious Author's New Method of Sailing by Parallel Parts, and his Demonstration of the Truth thereof; together with a Table of Parallel Parts by him calculated for Practice. And, as this New Method is compared by various Examples with the several Methods of Plain, Mercator, and great Circle Sailing, the Spirit of this Invention; and the Usefulness, Conciseness, and Accuracy thereof (by which all Nautical Questions are answered by the Solution of a plain Triangle only) are submitted to the candid Censure of the Learned and Judicious.

The Author also, in Explaining this New Method of Sailing by Parallel Parts, has treated of the Theory and Practice of Sailing in general, so fully, that

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that this Book will evidently appear far to excel any other of its Bulk or Price extant, with respect to the several Parts of Sailing now known and practis'd; and to introduce the Learner into the Knowledge thereof, by instructing him plainly and easily to form a great many Figures in Practical Geometry, which not only inures him to the Use of Scale and Compass, but also from the necessary Parts given of a Geometrical Figure, to project the Whole, in such a Manner, that all the Angles and Sides of that Figure, may be measured by the Scale from which the Parts given were laid down: And as to the demonstrative Part, there is a Compendium of Speculative Geometry, which contains all those Propositions which are necessary for understanding the Whole, without any Reference to Euclid; also the plain Trigonometry, with the Application thereof, as well to this new Method, as to Plain and Mercator's Sailing, is in a most easy and familiar Method, and, that as well in Regard to the Projection of the Figures, as the
Cal-

Calculation of the Parts, or finding the Parts required by the Gunter's Scale: Together with Tables of Latitude and Departure, and of Meridional Parts to very great Exactness. These, with some very curious Problems in Surveying, are the Subject of the First Part.

The Second Part contains the Projection of the Sphere, with the several Cases of Spherical Trigonometry, and their Applications to those Problems of Use in Navigation, in which the Schemes are intirely New, being made to represent those Solids the several Problems relate to, which will be (it is presum'd) a vast Help to the Imagination, it being very difficult (except to those well acquainted therewith) to conceive the Demonstration of Solids from Lines drawn on a Plane.

Thus I have endeavoured to give a very brief Account of the Contents of this Book, and must beg leave now, to do Justice to my ingenious Friend Mr. George Gordon, to whom I am obliged

The PREFACE. vii

*obliged for the Second Part intirely,
and for his Assistance in the Whole.*

*The History of the Life and Works
of the Author, may be here expected,
as in Modern Prefaces of this kind; but
the several Treatises he has published,
so manifestly shew the Power and
Strength of his Genius, and the Great-
ness of his Industry and Application in
the several Inventions and Improvements
he has made in the abstruse and difficult
Parts of the Mathematicks, which are
made by him more useful, because easy
to the meanest Capacity, that they only
can give a just Idea of the Author, and
in them is his Monument. If this Or-
phan I have introduced to the World,
proves of any Advantage to the Publick
in General, Entertainment to the Ju-
dicious and Curious, Instruction to the
Lovers and Learners of the several
Branches therein; and any Increase of
Character to the Author, all the Views
I had are answered: Yet I entertain some
Hopes, from the Fineness of the Invention,
that it may be encouraged and cherished
into*

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into Practice, especially should it be received by so great a Patron as has favour'd the Mercator's Chart with his Improvements; to whose Perusal the Original Manuscript was submitted, and who was so well acquainted with the Author's Hand-writing, and Ability, as to express his Willingness to recommend the printing thereof to any Person I should intrust therewith.

G. W.





A
COMPENDIUM
 OF
PRACTICAL GEOMETRY.

S E C T. I.
 D E F I N I T I O N S.

I.

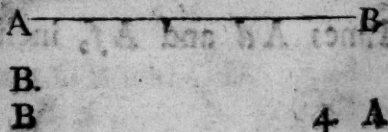


GEOMETRY is the Science of the Relations and Properties of extended Quantities, of which there are three Kinds, *viz.* Quantities extended only in Length, which are called Lines: Quanti-

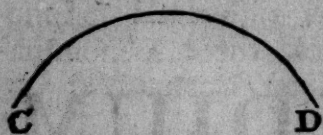
ties extended in Length and Breadth, called Surfaces: And Quantities extended in Length, Breadth, and Thickness, which are called Solids, or Bodies.

2. A Point Mathematical is incapable of being divided, and therefore hath no Parts.

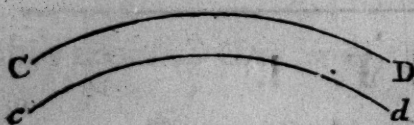
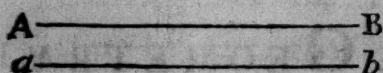
3. Lines are generated by the Motion of a Point, and are of several Kinds; first, A *strait* Line is that which lies equally betwixt (or is the nearest Way between) its Points A and B.



4. A circular, crooked, or oblique Line, is that which lies bending between those Points which limit its Length, as the Lines CD , or FG , &c.

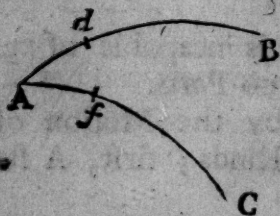
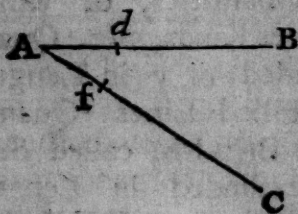


5. Parallel Lines are those that lie equally distant from one another in all their Parts,



viz. Such Lines as being infinitely extended (upon the same Plain) will never meet, as the Lines AB and ab , or CD and cd .

6. Lines not parallel, but inclining (*viz.* leaning) one towards another, whether they are right



Lines, or circular Lines, will (if they are extended) meet, and make an Angle; the Point where they meet is called the Angular Point, as at A : And according as such Lines stand nearer or farther off each other, the Angle is said to be lesser or greater, whether the Lines that include the Angle be long or short. That is, the

Lines Ad and Af , include the same Angle as AB

PRACTICAL GEOMETRY. 3

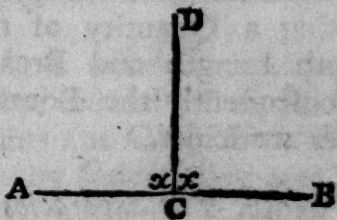
A B and A C doth, notwithstanding that A B is longer than A d, &c.

7. All Angles included between right Lines, are called Right-lin'd Angles; and those included between circular Lines, are called Spherical Angles; but all Angles, whether right-lin'd or spherical, fall under one of these three Denominations.

Viz. $\left\{ \begin{array}{l} \text{A Right Angle.} \\ \text{An Obtuse Angle.} \\ \text{An Acute Angle.} \end{array} \right.$

8. A Right Angle is that which is included betwixt two Lines that meet one another perpendicular.

That is, when a right Line, as D C, meets with another right Line as A B, so directly as that it neither inclines nor declines to one Side more than the other, but makes the Angles on both Sides of it equal, as at x, x ; then are those Angles called Right Angles, and the Lines so meeting, are said to be perpendicular to each other.

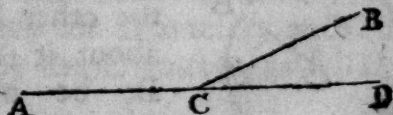


That is, A C and C B are perpendicular to D C, as well as D C is to either, or both of them.

9. An obtuse Angle is that which is greater than a right Angle.

Such is the Angle included between the Lines A C and C B.

10. An acute Angle is that which is less than a right Angle, as the Angle included between the Lines C B and C D.



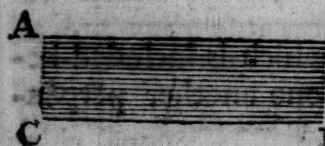
B 2

These

These two Angles are generally called Oblique Angles.

11. A Superficies or Surface, is the upper, or the very Out-side of any visible Thing: But by Superficies in Geometry, is meant only so much of the Out-side of any thing, as is inclosed within a Line or Lines, according to the Form or Figure of the Thing designed; and it is produced or formed by the Motion of a Line, as a Line is described by the Motion of a Point; thus,

Suppose the Line AB were equally moved (upon the same Plain)



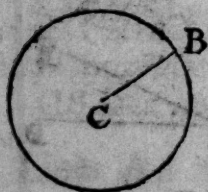
to CD ; then will the Points at A and B , describe the two Lines

AC and BD ; and by so doing, they will form (and inclose) the Superficies or Figure $ABCD$, being a Quantity of two Dimensions, *viz.* It hath Length and Breadth, but not Thickness: Consequently the Bounds or Limits of a Superficies are Lines.

Note, The Superficies of any Figure, is usually called its Area.

12. A Circle is a plain regular Figure, whose Area is bounded or limited by one continued Line, called the Circumference, or Periphery of the Circle, which may be thus described or drawn:

Suppose a right Line, as CB , to have one of its extreme Points, as C , so fix'd upon any Plain, as that the other Point at B may move about it; then if the Point at B , be moved round about (upon the same Plain) it will describe a Line equally distant in



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in all its Parts from the Point C, which will be the Circumference, or Periphery of that Circle, the Point C will be its Center, and the contained Space will be its Area; and the right Line CB, by which the Circle is thus described, is called the *Radius* of the *Circle*.

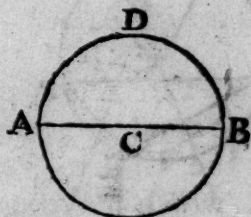
C O N S E C T A R Y.

From hence 'tis evident, that an infinite Number of right Lines may be drawn from the Center of any Circle, to touch its Periphery, which will be all equal to one another, because they are all Radius's.

And with a little Consideration, it will be easy to conceive, that no more than two equal right Lines can be drawn from any Point within a Circle, to touch its Periphery, but from the Center only.

13. Equal Circles are those which have equal Radius's; for it's plain, by the last Definition, that one and the same Radius (as CB) must describe equal Circles, how many soever they are.

14. The Diameter of a Circle is twice its Radius join'd into one right Line as AB, drawn through the Center C, and ending at the Periphery on each Side.

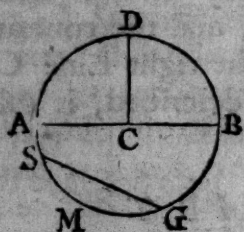


That is, the Diameter divides the Circle into two equal Parts.

15. A Semicircle (*viz.* Half a Circle) is a Figure included between the Diameter, and Half the Periphery cut off by the Diameter, as ABD.

16. A Quadrant is half a Semicircle, *viz.* one

Quarter of a Circle; and it's made by the Radius (as DC standing perpendicular upon the Diameter at the Center C, cutting the Periphery of the Semicircle in the Middle, as at D).



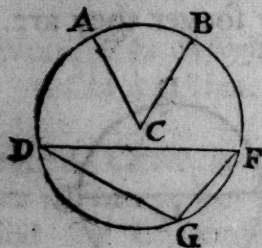
Therefore a Quadrant, or half the Semicircle, is the Measure of a right Angle.

17. A Chord Line, or the Subtense of an Arch, is any right Line that cuts the Circle into two unequal Parts, as the Line SG, and is always less than Diameter.

18. A Segment of a Circle, is a Figure included betwixt the Chord and that Arch of the Periphery which is cut off by the Chord; and it may either be greater or less than a Semicircle, as the Figure SMG, or SDG.

19. A Sector is a Figure included between two

Radius's of the Circle, and that Arch of its Periphery where they touch, as the Figure ACB; and the Arch AB is the Measure of the Angle at C, included betwixt the Radius's AC and BC.



Note, All Angles of Sectors are called Angles at the Center of a Circle.

20. An Angle in the Segment of a Circle, is that which is included between two Chords that flow from one and the same Point in the Periphery,

phery, as at D, and meet with the Ends of another Chord Line, as at F and G.

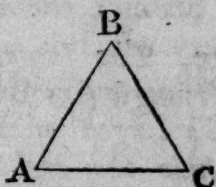
That is, the Angles at D, at F, and at G, are called Angles at the Periphery, or Angles standing on the Segment of a Circle.

There are two Kinds of Triangles, *viz.* Plain and Spherical; but we have referred the Definitions of spherical Triangles, to those Sections that treat of spherical Geometry.

21. A plain Triangle is a Figure whose Area is contained within the Limits of three right Lines called Sides, including three Angles, and it may be divided, and takes its Name either according to its Sides or Angles.

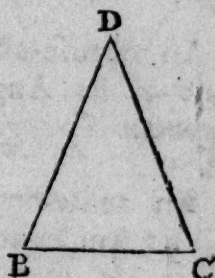
First by its SIDES.

22. An equilateral Triangle is that which hath all its three Sides equal, as the Figure ABC.



That is, $AB = BC = AC$.

23. An Isosceles Triangle, is that which hath only two of its Sides equal, as the Figure BDG.

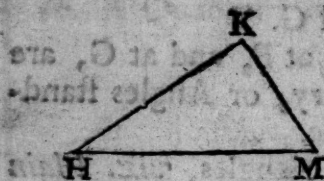


That is $BD = DC$; but the third Side BC may be either greater or less, as Occasion requires.

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24. Scalenous Triangle is that which hath all its three Sides unequal.

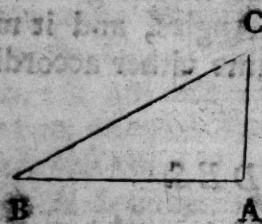
Such as the Figures HKM.



Second, By its ANGLES.

25. A right-angled Triangle is that which hath one right Angle; that

is, when two of its Sides are perpendicular to each other, as CA is suppos'd to be to BA. Therefore the Angle at A, is a right Angle, per Defi. 8.



Note, The longest Side of every right-angled Triangle (as BC) is called Hypothenufe, and the longest of the other two Sides, which include the right Angle (as BA) is called Base. The third Side (as CA) is called Cathetus, or Perpendicular.

26. An obtuse-angled Triangle is that which hath one of its Angles obtuse, and is called an *Amblygonium Triangle*, such is the third Triangle HKM.

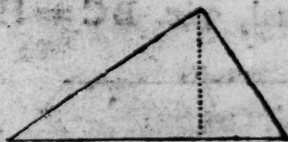
27. An acute-angled Triangle is that which hath all its Angles acute, and is called an *Oxygonium Triangle*, such are the first and second Triangles ABC, and BDC.

Note, All Triangles that have not a right Angle, whether they are acute or obtuse, are, in general Terms, called Oblique Triangles, without any other Distinction as before. And the

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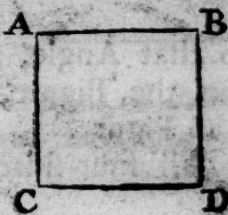
the longest Side of every Oblique Triangle, is usually called the Base; the other two are only called Sides, or Legs.

28. The Altitude or Height of any plain Triangle, is the Length of a right Line let fall perpendicular from any of its Angles, upon the Side opposite to that Angle from whence it falls, and may fall either within or without the Triangle, as Occasion requires, and is denoted by the two pricked Lines in the annexed Triangles.



29. A Square is a plain regular Figure, whose Area is limited by four equal Sides, all perpendicular one to another.

That is, when $AB = BC = CD = DA$; and the Angles A, B, C, D , are all equal, then it's usually called a Geometrical Square.

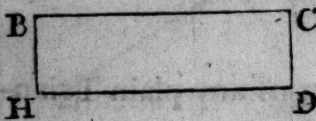


30. A Rhombus, or Diamond-like Figure, is that which hath four equal Sides, but no right Angle:

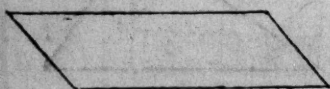


That is, A Rhombus is a Square moved out of its right Position, as the annexed Figure.

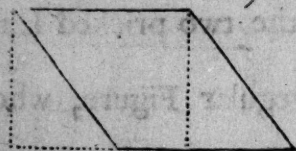
31. A Rectangle, or a right-angled Parallelogram (often called an Oblong, or long Square) is a Figure that hath four right Angles, and its two opposite Sides equal, *viz.* $BC = HD$ and $BH = CD$.



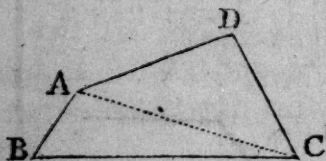
32. A Rhomboides is an oblique-angled Parallelogram; that is, It is a Parallelogram moved out of its right Position, like the annexed Figure.



33. The Altitude, or Height of any oblique-angled Parallelogram, *viz.* either of the Rhombus, or Rhomboides, is a right Line let fall perpendicular from any Angle upon the Side opposite to that Angle, and may either be within or without the Figure, as the prick'd Lines in the annexed Figure.



34. All four-sided Figures, which differ from those before mentioned, are called *Trapezia*.



That is, when they have neither opposite Sides, nor opposite Angles, equal; as the Figure $ABCD$.

35. A right Line drawn from any Angle in a four-sided Figure, to its opposite Angle, is called a *Diagonal Line*, and will divide the Area of the Figure into two Triangles, being denoted by the prick'd Line AC , in the last Figure.

36. All right-lined Figures that have more than four Sides, are called *Polygons*, whether they be regular or irregular.

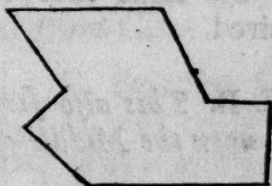
PRACTICAL GEOMETRY. 11

37. A regular Polygon is that which hath all its Sides equal, standing at equal Angles, and is named according to the Number of its Sides (or Angles):

That is, If it has five equal Sides, it is called a *Pentagon*; if it has six equal Sides, it is called a *Hexagon*; if seven, it is a *Heptagon*; if eight, it is an *Octagon*, &c.

Note, *All regular Polygons may be inscribed in a Circle; that is, Their angular Points, how many soever they have; will all just touch the Circle's Periphery.*

38. An irregular Polygon is that Figure which hath many unequal Sides, standing at unequal Angles (like unto the annexed Figure, or otherwise); and of such Kind of Polygons there are infinite Varieties; but they may be all reduced to regular Figures, by drawing diagonal Lines in them.



These are the most general and useful Definitions that concern *Plain* or *Superficial Geometry*.

PROB.

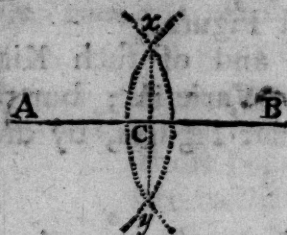
12 A COMPENDIUM of

P R O B. I.

To Biffect, or Divide a right Line given (as AB) into two equal Parts.

1. **F**ROM the Point A , with any Radius more than half the Line, describe an Arch.
2. From the Point B , with the same Radius, describe another Arch that intersects the former in two Points, as x and y .
3. A Ruler laid to the Intersections, will divide the Line into two equal Parts in c , as was required.

N. B. This also serves to raise a Perpendicular upon the Middle of any finite Line given.



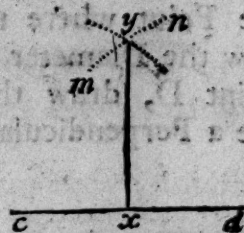
P R O B. II.

To raise a Perpendicular upon a given Point x , in a given right Line AB .

1. **F**ROM the given Point x , take xc , equal to xd , then from d , with a Radius greater than dx , describe the Arch mn .
2. From c , with the same Radius, describe an Arch to intersect the former, as at y .

3. A

3. A Ruler laid to the Point x , and the Intersection y , will give the Perpendicular requir'd.



P R O B. III.

To let fall a Perpendicular, as Cx , upon a given right Line AB , from any assigned Point that is not in it, as from C .

1. **F**ROM the given Point C , describe such an Arch of a Circle, as will cross the given Line AB , in two Points, as at d and y .
2. Then bisect the Distance between those two Points dy (by *Prob. I*).
3. Draw the right Line Cx , and it will be the Perpendicular required.



P R O B. IV.

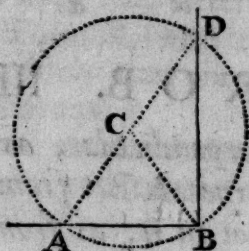
To raise a Perpendicular upon the End of any given right Line, as at B ; or upon a Point assigned in that Line.

1. **U**PON any Point out of the given Line, as at C , describe such a Circle as will pass through the Point from whence the Perpendicular

dicular is to be raised, as at B (*viz.* make CB Radius) and also let it cut the Line in any other Part, as at A.

2. And from the Point where the Circle cuts the given Line, draw the Diameter ACD.

3. From the Point D, draw the right Line DB, and it will be a Perpendicular, as was required.



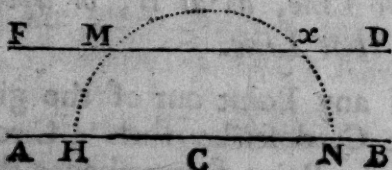
P R O B. V.

To draw a right Line as FD, parallel to a given right Line AB, that shall pass through any assigned Point, as at x.

1. TAKE any convenient Point in the given Line, as at C (the further off x the better) make Cx Radius, and with it, upon the Point C, describe a Semicircle, as HM x N.

2. Take the Arch HM, equal to the Arch x N.

3. Through the Points M and x , draw the right Line FD, and it will be the Parallel required.



P R O B.

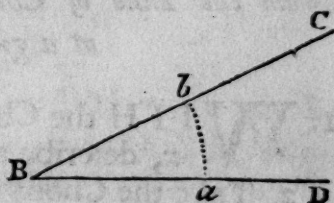
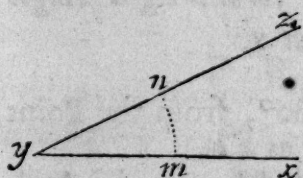
P R O B. VI.

To make an Angle DBC, from the Point B, equal to a given Angle x y z.

1. **F**ROM the Point *y*, with any Radius, describe an Arch as *m n*, and from the Point *B*, with the same Radius, describe the Arch *a b*.

2. In the Compasses take the Length of the Arch *m n*, and lay on the other Arch from *a* to *b*.

3. From the Point *B*, through the Points *a* and *b*, draw out the Lines *B D* and *B C*, and the Angle *DBC*, is equal to the given *x y z*.



P R O B. VII.

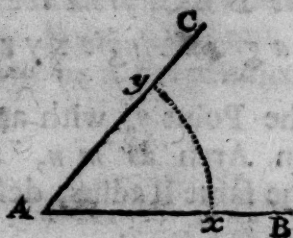
From a Point given A, to make an acute Angle viz. of 50°.

1. **F**ROM a Scale or Line of Chords, take the Chord of 60 Degrees, and with that Radius, and one Foot of the Compasses in the given Point *A*, describe an Arch, as *x y*.

2. From the Line of Chords take the given Number 50° and lay upon the Arch from *x* to *y*.

16 A COMPENDIUM of

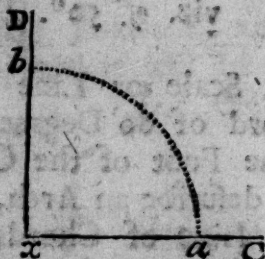
3. From the Point A, through x and y , draw out the Lines AB and AC, so shall the Angle BAC, be an Angle of 50° , as was required.



P R O B. VIII.

From the Line of Chords, to form a right Angle at a given Point x .

1. **W**ITH the Chord of 60° , from the Point x , describe an Arch as $a b$.
2. Take the Chord of 90° and lay on the Arch from a to b .
3. From the Center x , through a and b , draw out the Lines xC and xD , and the Angle DCx will be a right Angle.

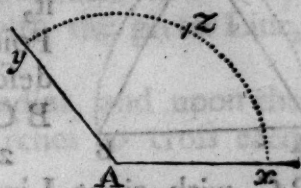


P R O B.

PROB. IX.

From a given Point A, to make an obtuse Angle, viz. of 130° .

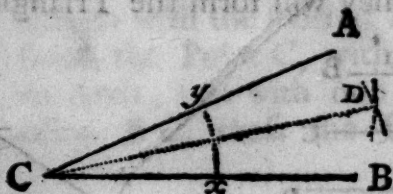
1. **W**ITH the Chord of 60° , from the Point A, describe an Arch, as $x y$.
2. From the Chords take 65° , Half the given Angle, and lay on the Arch twice from x to z , and from z to y .
3. Through x and y , draw out the Lines Ax and Ay , and the Angle yAx will be equal to 130° , as was required.



PROB. X.

To bisect a right-lined Angle given ACB, into two equal Angles.

1. **U**PON the angular Point C, with any convenient Radius, describe an Arch, as $x y$.
2. From those Points x and y , describe two equal Arches crossing each other, as at D.
3. Join the Points C and D with a right Line, and it will bisect the Arch yx , and consequently the Angle, as was required.

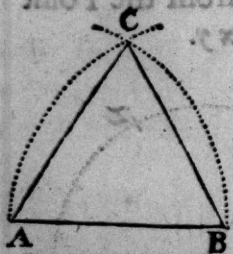


C

PROB.

P R O B. XI.

Upon a right Line given, as AB, to describe an Equilateral Triangle.



1. **M**AKE the given right Line Radius, and with it, upon each of its extreme Points or Ends, as at A and B, describe an Arch, viz. AC and BC.

2. Join the Points AC and BC with right Lines, and they will make the Triangle requir'd.

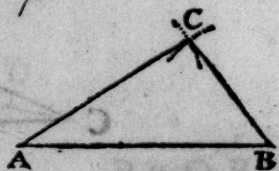
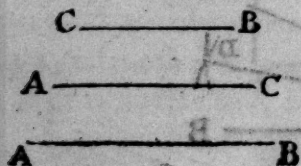
P R O B. XII.

Three right Lines being given to form them into a Triangle, provided any two of them, being taken together, be greater than the third.

1. **M**AKE either of the shorter Lines Radius, as AC, and upon either End of the longest Line, as at A, describe an Arch.

2. Make the other Line CB Radius, and upon the other End of the longest Line, as at B, describe another Arch, to cross the first Arch, as at C.

3. Join the Points CA and CB with right Lines, and they will form the Triangle requir'd.



P R O B.

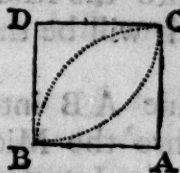
P R O B. XIII.

Upon a given right Line, as A B, to form a Square.

1. **U**PON one End of the given Line, as at B, erect the Perpendicular B D (by *Prob. IV.*) equal in Length with the given Line, *viz.* make B D equal A B.

2. Make the given Line Radius, and upon the Points A and D describe Arches to cross each other, as at C.

3. Join the Points C A and C D with right Lines, and they will form the Square requir'd.



P R O B. XIV.

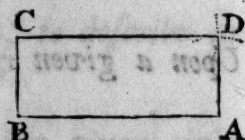
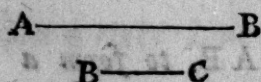
Two unequal right Lines, as A B and B C, being given, to form, or make of them, a right-angled Parallelogram.

1. **U**PON the End of the longest Line, as at B, erect a Perpendicular (by *Prob. IV.*) of the same Length with the shortest Line B C.

2. Then from the Point C, with the Radius A B, make an Arch; and with one Foot in A, with the Radius B C, cross the former Arch in D;

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3. Join D A and C D with right Lines, and the Parallelogram will be form'd as was requir'd.



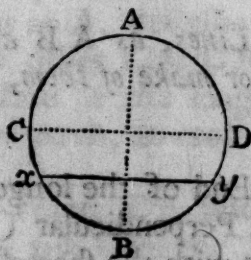
PROB. XV.

To find the Center of a given Circle.

1. **I**N the Circle draw any Line, as xy , to touch the Circumference.

2. Divide the Line xy into two equal Parts (by Prob. I.) and thro' the Middle thereof, draw the Line A B, which will be the Diameter of the given Circle.

3. Divide the Line A B into two equal Parts (by Prob. I.) and thro' the Middle thereof, draw the Line D C, and the Intersection of the Lines D C and A B, will be the Center of the given Circle.



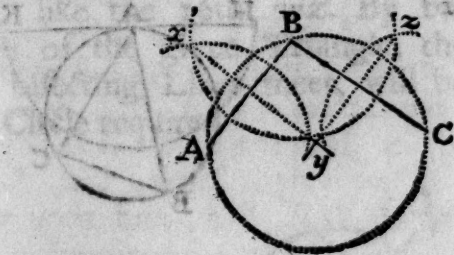
PROB.

P R O B. XVI.

To describe a Circle that shall pass (or cut) thro' any three given Points not lying in a right Line, as the Points A B C.

1. **J** O I N the Points B A and B C with right Lines.
2. Divide the Line A B into two equal Parts (by Prob. I.) and draw the Line $x y$.
3. After the same Manner divide the Line B C into two equal Parts, and draw the Line $z y$, and where the Line $z y$ intersects the Line $x y$, as at y , will be the Center of a Circle that will pass thro' the Points A B and C.

N. B. If a Segment of a Circle be given, the Whole may be compleated by taking any three Points in the given Segment's Arch, and then proceed as before, in the above Problem.

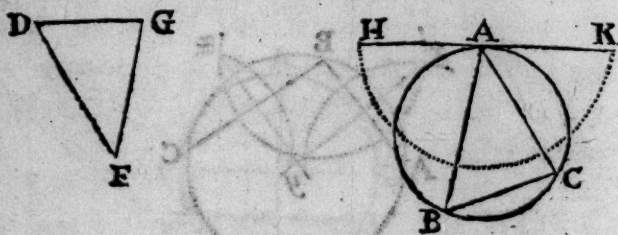


P R O B. XVII.

In any given Circle to inscribe or make a Triangle, whose Angles shall be equal to the Angles of a given Triangle, as the Triangle FGD.

Note, Any right-lined Figure is said to be inscribed in a Circle, when all the angular Points of that Figure do just touch the Circle's Periphery.

1. **D**RAW any right Line (as HK) so as just to touch the Circle, as at A, and then make the Angle KAC equal to any one Angle of the given Triangle, as DFG.
2. Make the Angle HAB equal to another Angle of the Triangle (as DGF) then will the Angle BAC be equal to the Angle FDG.
3. Join the Points B and C with a right Line, and it will form the Triangle required.

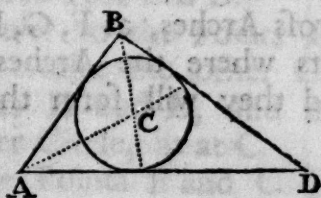


P R O B.

P R O B. XVIII.

In any given Triangle, as ABD, to describe a Circle that will touch all its Sides.

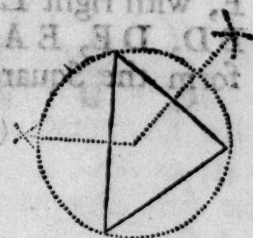
BISECT any two Angles of the Triangles, as A and B (by *Prob. X.*) and where the bisecting Lines cross one another, as at C, will be the Center of the Circle requir'd, and its Radius will be the nearest Distance to the Sides of the Triangle.



P R O B. XIX.

To describe a Circle about any given Triangle.

THIS Problem is perform'd in all respects like the 16th, *viz.* By bisecting any two Sides of the given Triangle, the Points where the bisecting Lines meet, will be the Center of the Circle required.



P R O B. XX.

To describe a Square about any given Circle.



1. **D**R A W a Diameter D A in the given Circle, divide it into two equal Parts (by *Prob. I.*) and draw E B, which will be another Diameter.
2. From the extrem Points of the Diameter A D and B E, with the Radius A C describe cross Arches, as F, G, H, K; then join those Points where the Arches cross, with right Lines, and they will form the Square requir'd.

P R O B. XXI.

In any given Circle, to describe the largest Square it can contain.



1. **D**R A W the Diameters, as D A and E B, bisecting each other at right Angles in the Center C, by *Prob. XX.*
2. Join the Points A B D and E, with right Lines, viz. A-B, B-D, D-E, E-A, and they will form the Square required.

PROB. B, OXXII.

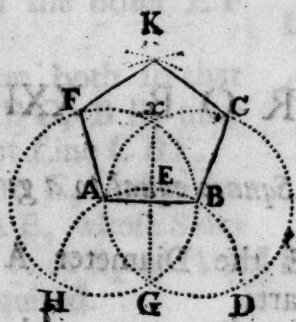
Upon any given right Line, as *AB*, to describe a regular Pentagon, or five-sided Polygon.

1. **M**AKE the given Line Radius, and upon each End of it describe a Circle, and thro' those Points where the Circles cross each other, as at *x* and *G*, draw the right Line *GEx*.

2. From the Point *G*, with the same Radius, describe the Arch *HAEED*.

3. Lay a Ruler to the Points *D* and *E*, and mark where it crosses the other Circle, as at *F*; also the Points *H* and *E*, and mark where it crosses the other Circle, as at *C*.

4. From the Points *F* and *C* (with the same Radius as before) describe cross Arches, as at *K*, join the Point *A* *F*, *FK*, *KC*, and *CB*, with right Lines, and they will form the Pentagon requir'd, *viz.* $AF = FK = KC = CB = AB$, and the Angles at *A, B, C, K, F*, will be equal.



PROB.

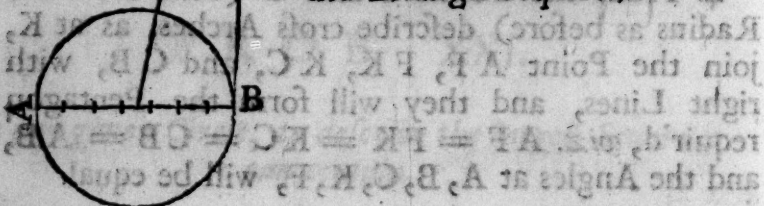
PROB. XXIII.

To make a right-lined Triangle equal to a Circle given.

1. **D**IVIDE the Diameter AB of the given Circle, into seven equal Parts.

2. Raise a Perpendicular upon B , three times as long as the Diameter AB , and a seventh Part over, as BC .

3. Draw a right Line from C to D , the Center of the given Circle, and that gives you the Triangle required.



PROB. XXIV.

To make a Square equal to a given Circle.

1. **D**IVIDE the Diameter AB into seven equal Parts.

2. Double this Diameter, and add a seventh Part of itself to it, as AC .

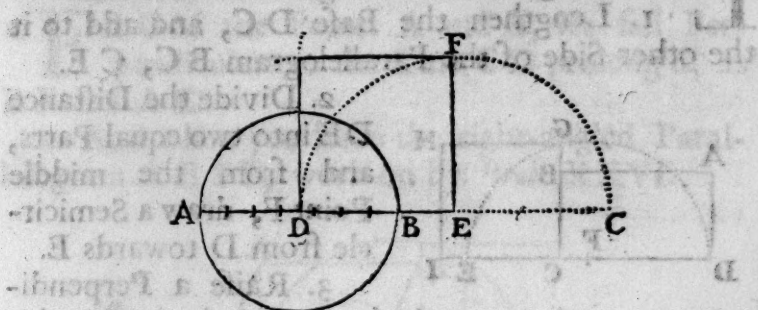
3. Divide the first Diameter AB , into two equal Parts, as AD .

4. Divide

PRACTICAL GEOMETRY. 27

4. Divide the Line DC into two equal Parts, and taking the middle Point E, draw from D to C an Arch, as DFC.

5. Raise a Perpendicular upon E, to touch the Circle in F; this Line EF shall be a Side of the required Square; the rest is wrought by Prob. X.



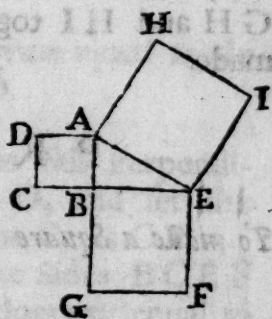
PROB. XXV.

To make one Square equal to two.

1. **L**ET one of the given Squares be equal to ABCD, and the other EFGB.

2. Join them both so that the Sides of BC, BE, may make one right Line CBE.

3. Join A to E, and make a Square on AE, whose Sides shall be equal to AE, as AEIH, which will be the Square required.

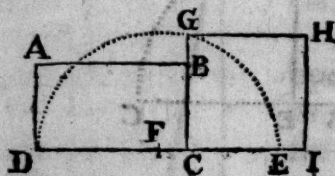


PROB.

PROB. XXVI.

To make a Square equal to a Parallelogram.

LET the given Parallelogram be ABCD.
1. Lengthen the Base DC, and add to it the other Side of the Parallelogram BC, CE.



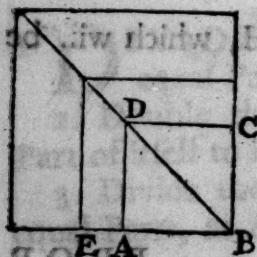
2. Divide the Distance DE into two equal Parts, and from the middle Point F, draw a Semicircle from D towards E.

3. Raise a Perpendicular upon C, to touch the Semicircle in G; this will be a Side of the required Square; set it upon the first Line, from C to I, and you will have another Side, as CI.

4. Keep the same Distance, and from the Points GI, make two Arches to intersect at H; join GH and HI together, and your Square will be made.

PROB. XXVII.

To make a Square two, three, or four times greater than it is.



LET the given Square be ABCD.

1. Lengthen the Side AB, and take the Distance BD, and set upon the lengthened Side AB, from B to E, and it will be a Side of a Square double to that given at first.

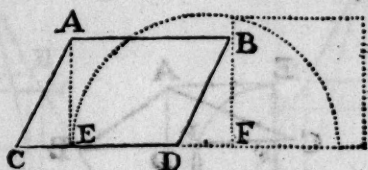
2. Observe what was said above in Prob. XI.

PROB.

P R O B. XXVIII.

To make an equilateral right-angled Square equal to an oblique-angled Parallelogram ABCD.

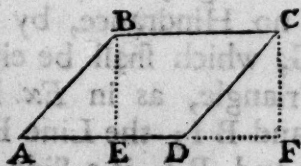
1. FROM the Points A and B, let fall Perpendiculars to the Line CD prolong'd; as BF and AE.
2. And when you have the right-angled Parallelogram AEBF, work on by Prob. XXVI.



P R O B. XXIX.

To make a right-angled Parallelogram equal to the Rhomb, ABCD.

1. FROM the Points BC, let fall Perpendiculars upon the Line AD, and set the Distance BE from the other Point C, as CF.
2. Join EF together, and the Sides BCEF will make a right-angled Parallelogram equal to a given Rhomb.

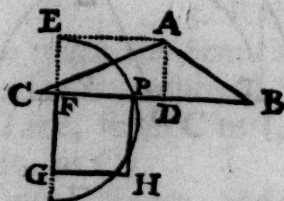


PROB.

P R O B. XXX.

To make an equilateral right-angled Square equal to the Triangle ABC.

1. FROM A let fall the Perpendicular AD, make FD equal $\frac{1}{2}$ CB, and draw EF parallel to AD, and equal to it, and join EA.
2. Then, by Prob. XXVI. make the Square PHGF equal Parallelogram EADF, which will likewise be equal to the Triangle ABC.



P R O B. XXXI.

In a given equilateral Triangle ABC, to inscribe a Square, and the greatest Hexagon that can be described in the said Triangle.

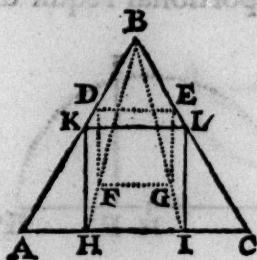
IN any Place of the Triangle, draw the Line DE parallel to the Base AC, upon DE make the Square DEFG; which Square, whether it fall within the Triangle, as in Ex. I. or without, as in Ex. II. is no Hindrance, by means of the Points F and G, which shall be either within or without the Triangle, as in Ex. I. & II. draw the Lines BH and BI; the Line HI, cut off by the Lines BH and BI, is a Side of the Square HILK to be inscribed.

For

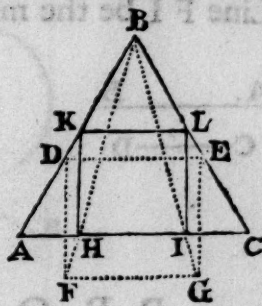
PRACTICAL GEOMETRY. 31

For the Solution of the second Part of the Proposition, to draw the greatest Hexagon in a given equilateral Triangle, divide each Side of the Triangle into three equal Parts, join the Extrems of the said Divisions by right Lines, and you will have the Hexagon requir'd, which is so plain by *Ex. III.* that there's no need of farther Explanation.

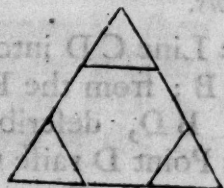
Example I.



Example II.



Example III.

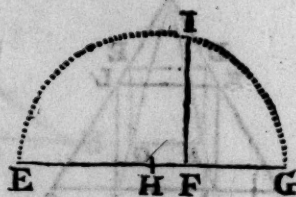
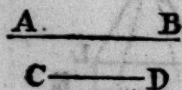


PROB.

P R O B. XXXII.

Between two given Lines, A B and C D, to find a mean Proportional.

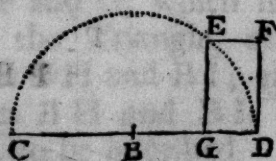
DR A W a Line and set A B from E to F, and C D from F to G, divide E G into two equal Parts in the Point H, upon which, as a Center, with the Distance H E, or H G, describe the Semicircle E I G; upon F raise the Perpendicular F I, to touch the Circle in I; then will the Line F I be the mean Proportional requir'd.



P R O B. XXXIII.

The Sum of the two Extrems being given in one Line C D, and their mean Proportional A, to find the Extrems.

DI V I D E the Line C D into two equal Parts in the Point B; from the Point B, with the Distance B C, or B D, describe the Semicircle C E D; upon the Point D raise the Perpendicular D F equal to the mean Proportional given A, draw F E parallel to C D, and E G parallel to F D, then C G and G D, will be the Extrems required.

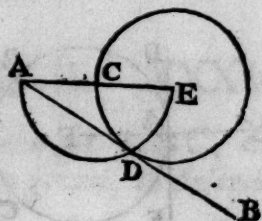


P R O B.

P R O B. XXXIV.

From a Point given A, to draw the Tangent A B.

FROM the Point E, draw the Line E A; so from the Point C, being the Middle of A E, as a Center, with the Distance C A, or C E, describe the Semicircle E D A; lastly, from A, by the Point D, which is the Touch-Point, draw the Line A B, which touches the Circle only in the Point D, and is the Tangent required.



P R O B. XXXV.

By a Point given in the Circumference of a Circle, as A, to draw a Tangent, as B C.

FROM the Center E, thro' the given Point A, draw the Line E D so that A D may be equal to A E; bisect E D in A with the Line B C, which Line will be the Tangent required.



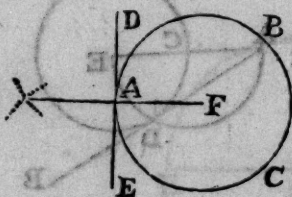
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P R O B.

P R O B. XXXVI.

Given the Circle ABC, and the Tangent DE, to find the Touch-Point.

FROM the Center F, let fall a Perpendicular upon the Line DE; the Point A, where the Perpendicular cuts the Tangent, is the Touch-Point required.



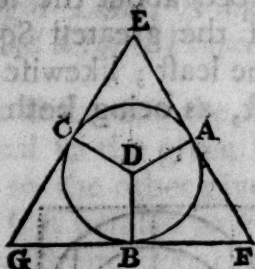
P R O B. XXXVII.

About a given Circle to circumscribe a regular Polygon of any Number of Sides.

DIVIDE your Circle into as many equal Parts as you would have your Polygon to contain Sides, and draw Lines from the Center to those Divisions which gives the Touch-Points, for as many Tangents as your Polygon contains Sides; for Example, Let the Circle be ABC, about which to draw an equilateral Triangle divide the Circle into three equal Parts in the Points A, B, and C, to which Points draw Lines from the Center D, as DA, DB, and DC, to the Points A B C draw

PRACTICAL GEOMETRY. 35

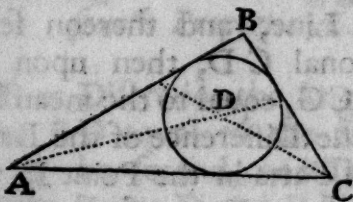
draw the Tangents EF, FG, and GE, *per* Prob. XXXV. which, by their Intersections, will form the equilateral Triangle EFG required.



P R O B. XXXVIII.

In a right-lined Triangle ABC, or in any other regular Polygon given, to describe a Circle.

DIVIDE two of the Angles, as A and C, into two equal Parts, and draw the Lines AD and CD, their Intersection D will be the Center of the Circle required.



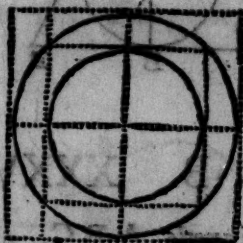
P R O B. XXXIX.

To describe a Circle whose Area shall be double to that of a given Circle.

ABOUT the given Circle circumscribe a Square by Prob. XX. and about that Square circumscribe another Circle by Prob. XXI. this last contains in its Superficies, double to that of the

D 2
given

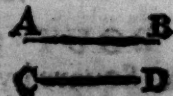
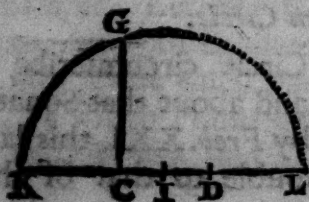
given Circle; for if you circumscribe another Square about this last Circle, this Square circumscribed about the greatest Circle, is double to the Square circumscribed about the least Circle, because the Side of the greatest Square is equal to the Diagonal of the least; likewise the great Circle is double the least, as being both inscrib'd in the said Squares.



P R O B. XL.

The mean Proportional AB, and the Difference of the Extreams CD being given, to find the Extreams.

DR A W a Line, and thereon set the mean Proportional CD , then upon C raise the Perpendicular CG , equal to the mean Proportional AB ; divide the Difference of the Extreams CD , into two equal Parts in the Point I , with the Distance IG , and one Foot of the Compasses in I , describe the Arch KGL , upon the Difference CD prolonged both Ways toward K and L , then will KC and CL be the Extreams required.

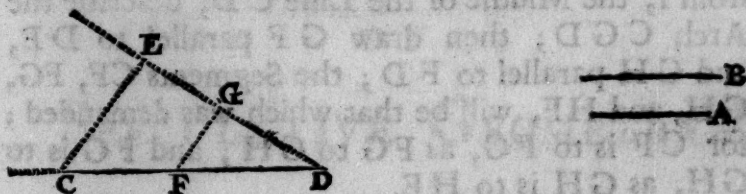


P R O B.

P R O B. XLI.

Two unequal right Lines, A and B, being given, to find a third Proportional.

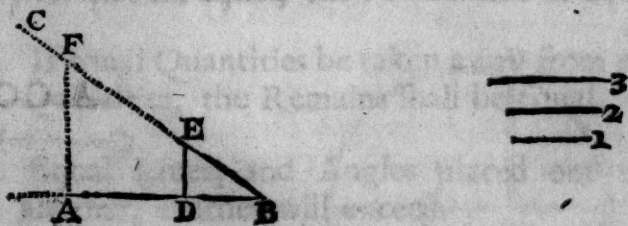
MAKE at Discretion the Angle CDE, make DF equal to the given Line B, and FC, or DG equal to the other given Line A; draw FG, and draw CE parallel to FG; the Line EG will be the third Proportional required.



P R O B. XLII.

Three different right Lines being given, to find a fourth Proportional.

MAKE at Discretion the Angle ABC, then make BD equal to the first Line given, BE equal to the second, and DA equal to the third; join the Points D and E by the right Line DE, and make AF parallel to DE, which gives you the fourth Proportional FE.



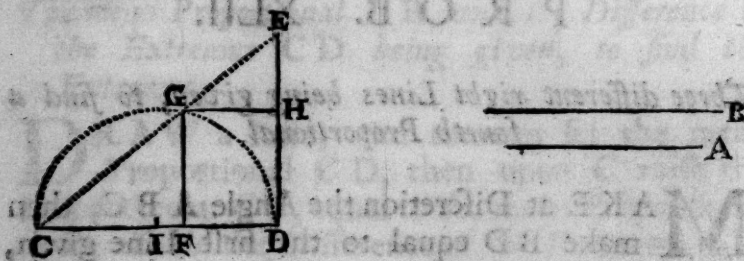
D 3

P R O B.

P R O B. XLIII.

Two right Lines A and B being given, to cut each of them into two Parts, so that the four Segments may be proportional.

DR A W the Line C D equal to the given Line B, and the Perpendicular D E, equal to the other given Line A; draw the Line C E, and from I, the Middle of the Line C D, describe the Arch C G D; then draw G F parallel to D E, and G H parallel to F D; the Segments C F, F G, G H, and H F, will be that which was demanded; for C F is to F G, as F G to G H; and F G is to G H, as G H is to H E.



A COM.



A
COMPENDIUM
O F
SPECULATIVE GEOMETRY.

S E C T. II.



IN order to demonstrate some of the following Propositions, I shall premise the following Axioms, or self-evident Truths.

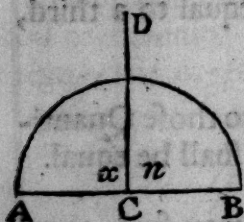
1. Those Quantities that are equal to a third, are equal among themselves.
2. If equal Quantities are added to those Quantities that are equal, their Sums shall be equal.
3. If equal Quantities be taken away from equal Quantities, the Remains shall be equal.
4. Equal Lines, and Angles placed one upon another, neither will exceed.

Significations of some Characters used in the following Sections.

- + Signifies that the two Quantities between which it is placed, should be added together.
- Signifies, that the Quantity to the right Hand of the Line, should be subtracted from that on the left.
- × Signifies the Multiplication of the two Quantities.
- ÷ Signifies the Division of the left Hand Quantity by the right.
- = Equality; and signifies, that the Numbers on each Side of this Sign are equal.
- ∠ Signifies an Angle in general.

P R O P. I.

If a right Line stand upon (or meet with) another Line, the Angles made by the Meeting of those two Lines, will be two right Angles, or two Angles equal to two right ones.



SUPPOSE the Lines to be AB and DC, meeting in the Point C; upon C describe a Circle at Pleasure, then will the Arch AD be the Measure of the Angle α , and the Arch DB, the Measure of the Angle n ; but the Arches AD and DB, are equal to a Semicircle, or 180 Degrees, consequently the Angles α and n are equal 180 Degrees, which was to be demonstrated.

CO-

C O R O L L A R Y.

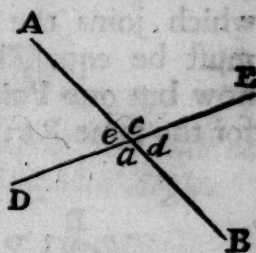
It's evident from the foregoing Proposition, that if the Angle x be equal to 90 Degrees, then the Angle n is also equal to 90 Degrees; but if the Angle x is obtuse, then the Angle n will be acute.

Also, That if several right Lines stand upon, or meet with any right Line at one and the same Point, all the Angles taken together will be equal to 180 Degrees, or two right Angles.

P R O P. II.

If two Lines intersect or cut each other, the two opposite Angles will be equal.

LET the two Lines be A B and D E, intersecting each other in the Center c , then the Angle c , and the Angle e , will be equal to 180 Degrees, by Prop. I. and the Angles e and a , equal to 180 Degrees, by the same Prop. I. consequently if $\angle e$ be taken away, which is common to both, the Angle at a will remain equal to the Angle at c , by Axiom III.



C O R O L.

From hence it's evident, that if two Lines intersect one another, they will make four Angles, which taken together, will always be equal to four right Angles.

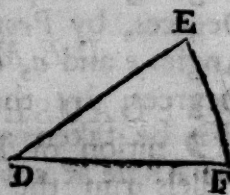
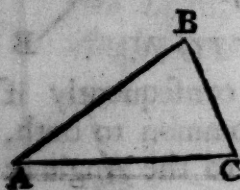
P R O P.

P R O P. III.

If two Triangles have two Sides of the one, equal to two Sides of the other, and the Angles between these Sides equal, these Triangles will be equal in all Respects; that is, if $AB = DE$, and $AC = DF$, and the Angle $A = \text{Angle } D$, then is $CB = EF$, and $\angle C = \angle F$.

D E M O N S T R A T I O N.

IT is evident by *Axiom IV.* that if DF is laid on the Line AB , they will not exceed each other, and the $\angle A = \angle D$ by Supposition, and the Lines AC and FD must have the same Direction, consequently the Triangles laid one upon another, FD must fall on AC ; if so, the Line which joins the Points C and B , and E and F , must be equal, because the Points C and F are now but one Point, as is likewise B and E ; and for the same Reason is $\angle C = \angle F$.



C O R O L.

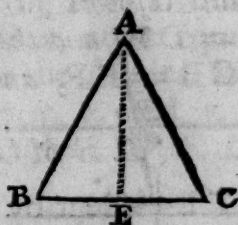
From hence it will follow, that if two Triangles have one Side of one, equal to one Side of the other, and the Angles at each End respectively equal, the Triangles are equal in all Respects,

P R O P.

P R O P. IV.

In an Iſoſceles Triangle ABC, the Angles B and C, oppoſite to the equal Sides AB and AC, are equal:

LET AE biſect BC, then AB will be equal to AC, and BE equal to EC; and AE being common to both, the Triangles AEB and AEC, it follows, that the Angles at B and C muſt be equal, by the laſt Part of *Prop. III.* It follows alſo, that AE is perpendicular to BC, and that it biſects the Angle ABC.



P R O P. V.

If a Line cut, or croſs two Parallels, the external Angle is equal to the internal oppoſite Angle.

DEMONSTRATION.

BECAUSE by the Definition of Parallels, Parallels run the ſame Way, they muſt therefore have the ſame Inclination to any ſtrait Line; for if it were not ſo, they could not be parallel; therefore the Angle *a* is equal to the Angle *b*, and the Angle *y* equal to the Angle *z*.

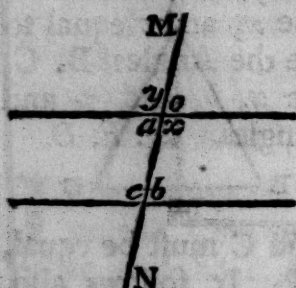


P R O P.

P R O P. VI.

If any strait Line MN, cross two Parallels, the alternate Angles a and b , also x and y , are equal.

DEMONSTRATION.

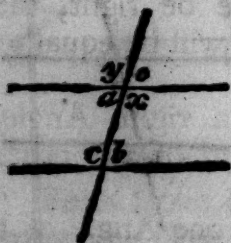


THE Angle a is equal to the Angle o , by Prop. II. Also the Angle o equal to b , by Prop. V. Therefore o being equal to a , as also to b , a must be equal to b , by Axiom I. Again, the Angle x is equal to the Angle y , by Prop. II. and the Angle c equal to the Angle y , by Prop. V. therefore the Angle x is equal to the Angle c . Q. E. D.

P R O P. VII.

The internal Angles x and b , are equal to two right Angles.

DEMONSTRATION.



THE Angles o and c are equal to two right Angles, by Prop. I. and x to c , by Prop. VI. and b equal to o , by Prop. V. Therefore x and b are equal to o and c , and therefore equal to two right Angles.

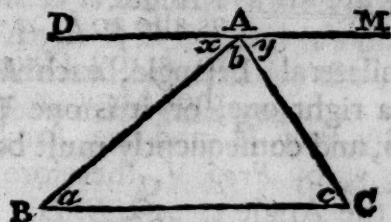
After the same Manner (a and y) may be demonstrated to be equal to two right Angles.

P R O P.

P R O P. VIII.

The three Angles of every Triangle, is equal to two right ones.

IN the Triangle ABC , draw DM parallel to the Base BC , then the Angles x , b , and y , are equal to two right Angles, by *Prop. I.* but the Angle B is equal to the Angle x , and c equal to y , because alternate; therefore the Angles B , C , and b , are equal to the Angles x , b , and y , and therefore equal to two right Angles. *Q. E. D.*



C O R O L.

1. The three Angles of any one Triangle taken together, are equal to the three Angles of any other Triangle taken together.

2. If in a Triangle one Angle be right (or obtuse) the rest are acute.

3. If in a Triangle one Angle be right, the other two Angles taken together, will be equal to one right one.

4. In every Triangle having a right Angle, this right Angle is equal to the Sum of the other two Angles taken together.

5. The Number of Degrees of one Angle of a Triangle being known, the Sum of the other two Angles is also known; and, on the contrary, the Sum of two Angles being known, the third is also known.

6. The

6. The two Angles of one Triangle, either separately or jointly, being equal to the two Angles of another Triangle, then the third Angle of the one Triangle, is equal to the third Angle of the other Triangle.

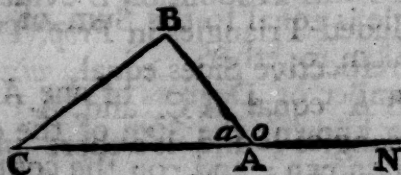
7. When two Triangles have two Angles of the one, equal to two Angles of the other, then the remaining Angle of the one, will be equal to that of the other.

8. When an Iſosceles Triangle, the Angle contained by the equal Sides, is a right one, the other two are each of them equal to half a right Angle; and the Angles of an Iſosceles Triangle, which are at the Base, are always Acute.

In an equilateral Triangle, each Angle is two Thirds of a right one, or it is one Third of two right Angles, and consequently must be two Thirds of one right.

P R O P. IX.

IF the Side of any Triangle (as AC) be produced to N, the external Angle o will be equal to the two opposite internal Angles B and C taken together; for o and C make together two right Angles; by *Prop. I.* but by *Prop. VI.* B and C make with A two right ones; therefore o is equal to B and C taken together.

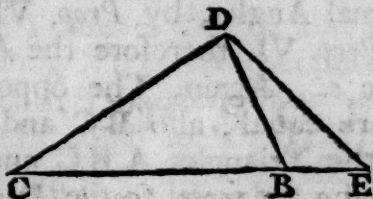


P R O P.

P R O P. X.

In every plain Triangle, the longest Side subtends the greatest Angle; consequently the greatest Angle of every Triangle, is subtended by the greatest Side.

THIS Proposition is evident by Inspection; for let any of the Sides of a plain Triangle, as CB, be produced, suppose to E, join DE with a right Line, then it's evident, that because CE is now made longer than the Side CB, therefore the Angle at D is become greater than it was before, by the Angle BDE; and it's plain, that the longer the Side had been made, the Angle at D would have been more enlarged.



P R O P. XI.

If the three Sides of two Triangles are equal, the Angles opposite to those Sides will be equal.

THE Truth of this Proposition is evident by the two included Triangles in Prop. IV. for they have their respective Sides equal, viz. BC equal EC, and BA equal AC, and AE common to both Triangles; and it's there proved, that the Angles opposite to those equal Sides, are equal. Q. E. D.

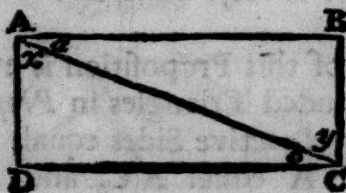
N. B. *The Converse of this Proposition holds not true; for the Angles of two Triangles may be equal, and their opposite or subtending Sides unequal, as it will hereafter appear.*

Hence it follows, that two Angles mutually equilateral, are also mutually equiangular.

P R O P. XII.

The opposite Sides and Angles of Parallelograms are equal, and the Diagonal AC divides it in the Middle.

IN the Parallelogram $ABCD$, the opposite Angles a and o are equal; for a is equal to o , because alternal Angles, by *Prop. VI.* but x is equal to y by *Prop. VI.* therefore the Angle a is equal to Angle o . Again, The opposite Sides BA and CD are equal; also BC and AD are equal; for in the Triangles ABC and ADC , the Angles x and o are equal to the Angles y and a , and the Side AC common to both Triangles; therefore ABC is equal to the Triangle ADC by *Corol. Prop. III.* consequently the Sides AD and BE , as also AB and DE , must be equal.



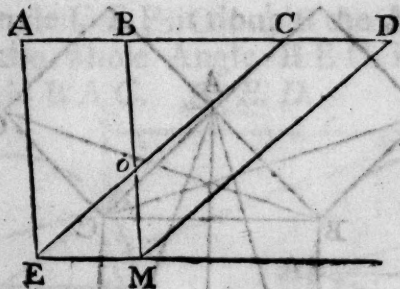
C O R O L

The Line A E drawn to the opposite Angles, which is called the Diagonal, divides the Parallelogram into two equal Parts.

P R O P. XIII.

Parallelograms and Triangles on the same Base, and between the same Parallels, are equal.

THUS the Parallelogram A E M B is equal to the Parallelogram C E M D; for since, by the last Proposition, A B is equal to E M, and E M equal to C D, therefore A C is equal to B D; also A E equal to B M, and E C equal to M D, therefore the Triangles C A E, and D M B, must be equal by *Prop. XI. & III.* therefore taking away B o C from both Triangles, there will remain the Space A E o B, equal to C o M D, by *Axiom III.* then, by adding E o M to both these Spaces, you will have the Parallelograms A E M B, equal to the Parallelograms C E M D, by *Ax. II.* Q.E.D.

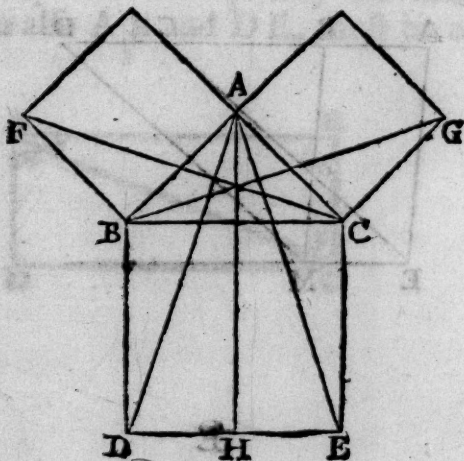


C O R O L.

Hence Triangles upon the same Base, and between the same Parallels, are equal, because they are Halfs of Parallelograms described on the same Base, and between the same Parallels.

P R O P. XIV.

IN a right-angled Triangle, as ABC , the Square of the Hypotenuse BC (or Side subtending the right Angle) is equal to the Sum of the Squares of the other two Sides BA and AC : Draw the Line AH parallel to BD , and join AD and AE : The Angle DBC is equal to the Angle FBA , and add the Angle ABC to both, then is the Angle ABD equal to the Angle FBC . Moreover, AB is equal to FB , and BD equal to BC , by the Definition of a Square; therefore is the Triangle ABD equal to the Triangle FBC , by *Prop. III.* but the Parallelogram BH is equal to twice the Triangle ABD , and



SPECULATIVE GEOMETRY. 51

and the Parallelogram CH is equal to twice the Triangle ACE, then the Square AG is equal to the Rectangle CH, and AF equal to BH, and by consequence the Squares AF and AG are equal to the Square BDEC. *Q. E. D.*

PROP. XV.

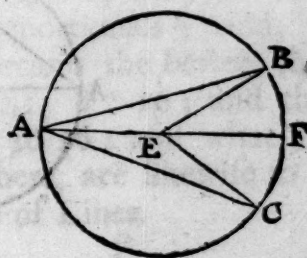
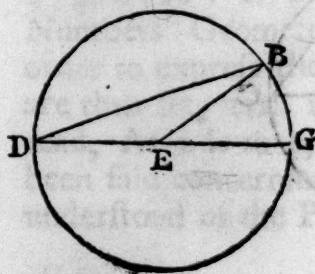
The Angle at the Center, is double the Angle at the Circumference, when both Angles have the same Arch for their Base; that is, the Angle GEB is double the Angle GDB.

DEMONSTRATION.

IN an Isosceles Triangle, the Angle GEB being external, is equal to the Angles D and B; but the Angle D is equal to the Angle B; therefore the Angle GEB is double of the Angle D. *Q. E. D.*

CASE II.

Having drawn AF through the Center E, the Angle BEF is double of the Angle BAF, and the Angle CEF is double the Angle CAF; therefore the whole Angle BEC is double the whole Angle BAC. *Q. E. D.*



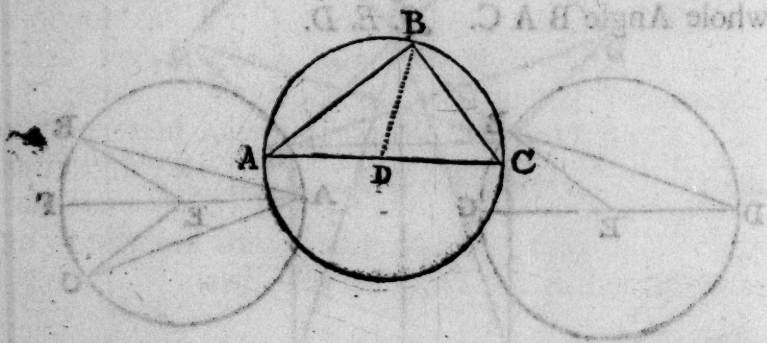
PROP. XVI.

1. *The Angle ABC in a Semicircle, is a right Angle.*
2. *In a Segment greater than a Semicircle, the Angle BAC is less than a right Angle.*

DEMONSTRATION.

I.

HAVING drawn the Line DB, there will be two Iſofceles Triangles, BDA and BDC, whose Angles at the Bases will be equal; therefore the Angle DBA will be equal to the Angle DAB, and the Angle DBC will be equal to the Angle DCB; but the whole Angle ABC is equal to BAC, together with BCA; but in the Triangle ABC, all the three Angles are equal to two right ones; therefore, on the one Part the two remaining Angles BAC, and BCA, are equal to one right Angle; consequently the Angle B is a right Angle.



II.

By the first Part the two Angles B A C and B C A together, make one right Angle, therefore B A C alone, is less than a right Angle.

In order to demonstrate some of the following Propositions, it is necessary to say something of Proportion, which I shall do in the following Lemmas.

LEMMA I.

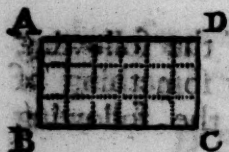
Three Numbers are said to be in Geometrical Proportionals continu'd, when the Product of the first and last Term is equal to the Square of the Mean; as suppose, 2, 4 and 8 the Numbers, they are called *Geometrical Proportionals*, if 2×8 is $= 4 \times 4$, both which I find to be equal 16, therefore are the three Numbers Geometrical Proportions continu'd.

LEM. II.

If four Numbers are continu'd, so that the Product of the First and Fourth, is equal to the Product of the Second and Third; as suppose 2. 4. 8. 16: If $2 \times 16 = 4 \times 8$, then are these Numbers Geometrical Proportionals; and, in order to express the Proportions the better, they are thus set, viz. $2 : 4 :: 8 : 16$; and thus read, As 2 is to 4, so is 8 to 16; and what has been said concerning Numbers, are likewise to be understood of the Products of Lines.

L E M. III.

Lines are said to be multiply'd one by another, when there is a right-angled Parallelogram made of the two Lines; as suppose the two Lines A B and B C were to be multiply'd, the Product of that Multiplication is the Parallelogram A B C D, let $A B = 3$, and $B C = 6$, then $A B \times B C = 6 \times 3 = 18$, the Number of Squares (whose Sides are one of those little equal Parts) that is contain'd in that Product, or Superficies A B C D.



L E M. IV.

If what has been said in the two first *Lemmas* be understood, it is easy to conceive, that any Equation may be made a Proportion by dividing each Side of the Equation into 4 Terms, in such a Manner, as that when the first and last of the Terms are multiply'd together, and the second and third the same, the Products of these Multiplications may produce the given Equation; as suppose $2 \times 16 = 4 \times 8$, which may be divided thus, as $2 : 4 :: 8 : 16$, or thus, as $2 : 8 :: 4 : 16$, or thus, as $8 : 2 :: 16 : 4$, or thus, as $8 : 16 :: 2 : 4$, and it would have been the same if it had been Lines.

PROP.

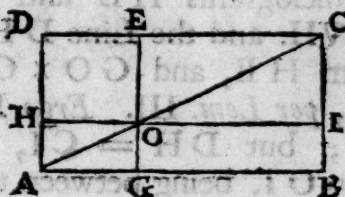
PRO P. XVII.

The Complements of a Parallelogram are equal.

N. B. When there is a Line, as HI, drawn parallel to one of the Sides of the Parallelogram, so as to cut the Diagonal in O, and thro' the Point G, another Line EG is drawn parallel to the other Side of the Parallelogram; these two Lines EG and HI, divide the Parallelogram in 4 other Parallelograms, thro' two of which the Diagonal AC passes, as thro' EI and HG, and thro' the other two, viz. HE and GI, the Diagonal does not pass; and these are called the Complements of the Parallelogram ABCD, and are equal to one another.

DEMONSTRATION.

THE Triangle ADC = Triangle ACB, and $\triangle OEC = \triangle OIC$, and $\triangle HOA = \triangle OGA$, per Prop. XII. Now if from the $\triangle$'s ADC and ACB, there be taken the equal Triangles EOC, OIC, AOH, and AOG, the Remains are the Parallelograms HOED and GBIO, which are equal, per Axiom III.



P R O P. XVIII.

If a Line, as OI (see preceding Figure) is drawn parallel to the Side AB of the Triangle ABC, it cuts off a Triangle OIC, equiangular or similar to the Triangle ABC, and the like Sides of similar Triangles are proportional.

DEMONSTRATION.

THE $\angle OAG = \angle COI$, per Prop. V. and $\angle ACB$ common to both the Triangles, and then the $\angle CIO = \angle CBA$ (per Cor. VII. Prop. VIII.) And in like Manner it may be proved, that the $\triangle AOG$ is similar to the $\triangle COI$, supposing OI parallel to AB , and OG parallel to CB .

And (for the second Part of the Proposition) compleat the Parallelogram ABCD, draw HI and EG parallel to the Sides of the Parallelogram; I say, in the Triangles AOG and OIC, which have been proved to be similar, it will be as $OG : AG :: CI : OI$.

DEMONSTRATION.

The Parallelograms HE and GI are equal, per Prop. XVII. and the Line $DH \times HO$, is = Parallelogram HE , and $GO \times GB$ = Parallelogram GI , per Lem. III. Ergo $DH \times HO = GO \times GB$; but $DH = CI$, $HO = AG$, and $GB = OI$, being between the same Parallels; therefore $CI \times AG = GO \times OI$; therefore $GO : AG :: CI : OI$ (per Lem. IV.)
Q. E. D.

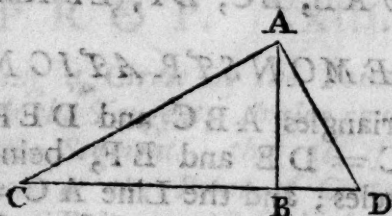
P R O P.

P R O P. XIX.

If a Perpendicular AB, be let fall from the right Angle A, of right-angled Triangle ADC, it divides the Triangle into two similar Triangles.

DEMONSTRATION.

IN the Triangles CAD and CAB, there is $\angle CAD = \angle ABC$, being both right Angles, and the $\angle C$ common to both; therefore the Angle $CAD = ADC$, per Cor. VII. Prop. VIII. In the same Manner may the Triangle ADB be proved similar to CAD, and of Consequence, the Triangle CAB, similar to Triangle ADB, per Ax. I.

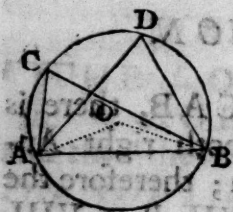


PROP.

P R O P. XX.

All Angles in the same, or equal Circles, that have the same or equal Arches for their Base, are equal; that is, the $\angle ACB = \angle ADB$, having the Arch AB for their Bases, draw the Radii AO , BO .

DEMONSTRATION.



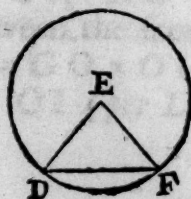
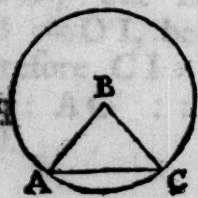
THE $\angle$'s ACB and ADB , are each of them the Halves of the $\angle AOB$ per Prop. XV. and are therefore, by Axiom I. equal to one another. $\angle$ E. D.

P R O P. XXI.

Equal Lines within equal Circles, answer to equal Arches; that is, if the Circles B and E are equal, and the Line $AC = DF$, then the Arch $DF = AC$, draw AB , BC , DE , EF Radius's.

DEMONSTRATION.

IN the Triangles ABC and DEF , the Sides AB , $BC = DE$ and EF , being Radii of the equal Circles; and the Line $AC = DF$, by Supposition there, per Prop. XI. $\angle B = \angle E$, and the Arches AC and DF being the Measure of these Angles (per Prob. VII. Sect. I.) are consequently equal.



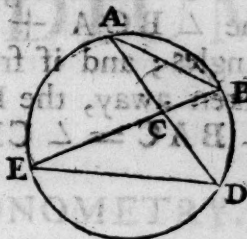
P R O P.

P R O P. XXII.

If two Lines cut or cross each other in a Circle, the Products of the Parts of one Line, are equal to the Products of the other; that is, $AC \times CD = EC \times CB$. Draw AB and ED.

DEMONSTRATION.

IN the Triangles ACB and ECD, there is $\angle ACB = \angle ECD$, by Prop. II. and the $\angle ADE = \angle ABE$, by Prop. XX. Ergo, the two Triangles are similar, consequently their like Sides, viz.



AC, CB, and EC, CD, are Proportionals, by Prop. XVIII. that is, as $AC : CB :: EC : CD$, and being Proportionals, per Lem. II. $AC \times CD = EC \times CB$. Q. E. D.

P R O P. XXIII.

A Line cutting a Circle at the Point of Contact, makes with the Tangent, Angles equal to those in the alternate Segments; that is, if AB meets the Tangent at the Point of Contact B, I say, that the $\angle EBA = \angle BDA$, and if upon the Line AB there is made an Isosceles Triangle ABC, I say the Angles CBG, ABC, and BAC are equal, draw the Diameter DB, and the Line AD.

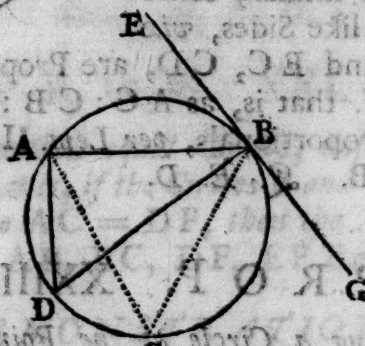
DEMONSTRATION.

THE $\angle A$ is a right Angle, per Prop. XVI. and per Prop. VIII. the $\angle D$, and the $\angle ABD$ together, are equal to a right Angle, and the $\angle DBE$ is a right Angle, by being a Tangent

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gent to the Diameter BD; now, if out of each, the $\angle ABD$ be taken, there remains the $\angle EBD = BDA$. *Q. E. D.*

Secondly, The $\angle BDA = \angle ACB$, by *Prop. XX.* and the $\angle CBA = \angle BAC$, per *Prop. IV.* but if instead of $\angle EBA$, we use its equal $\angle BCA$, then the $\angle BCA + \angle ABC + \angle GBC =$ two right Angles, per *Cor. Prop. I.* but the $\angle BCA + \angle ABC + \angle BAC =$ two right Angles; and if from both, the equal Angles be taken away, the Remainders will be equal, *viz.* $\angle BAC = \angle CBG$. *Q. E. D.*



A Line cutting a Circle in the Point of Contact, makes with the Tangent, Angles equal to those in the alternate Segments, that is, if A B meets the Tangent at the Point of Contact B, I say, that the $\angle EBA = \angle BDA$, and if from the Line A B, there is made an Angle $\angle ABC$, and $\angle BAC$ for the Angles $\angle BCG$, and $\angle BAC$ are equal, draw the Diameter BD, and the Line AD.

COMPOSITION

THE $\angle A$ is a right Angle, per *Prop. XVI.* and per *Prop. VIII.* the $\angle D$ and the $\angle ABD$ together, are equal to a right Angle, and the $\angle DBE$ is a right Angle, by being a Tan-



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Right-lined TRIGONOMETRY.

S E C T. III.



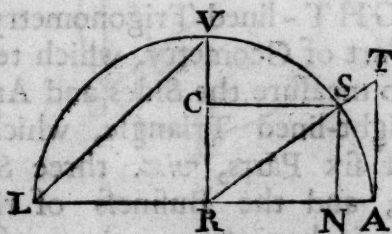
RIGHT-lined Trigonometry, is that Part of Geometry, which teacheth us to measure the Sides and Angles of a right-lined Triangle, which consists of six Parts, *viz.* three Sides, and three Angles; and the Business of right-lined Trigonometry, is having any three of the said three Things given (except the three Angles of a Triangle) to find the other three.

But the Measure of an Angle being the Arch of a Circle described about the angular Point, and because there has been yet no exact Proportion found between the Diameter of a Circle, and the Periphery, it has been hitherto the Custom to substitute certain right Lines in the Room of the Arches or Angles of the Triangle; to understand which, it is necessary to understand the following Definitions.

1. The

1. The Chord of an Arch is any Line drawn thro' each End of it, as $L V$ is the Chord of the Arch $L V$.

2. A right Sine of an Arch is a Perpendicular let fall from one End of the Arch, upon a Radius that passeth thro' the other End, as the Line $S N$ is the Sine of the Arches $S A$ and $L V S$, and $C S$ is the Cosine of the Arch $S A$; that is, it is the Sine of an Arch which is the Remainder of $S A$, taken from 90 Degrees; for $V R$ and $R A$ are Radius's, which are supposed perpendicular to one another, consequently the Arch $V A$ is a Quadrant; and when any Arch, as $S A$, is taken from 90 Degrees, the Remainder, as $V S$, is called its Complement; so that the Cosine of any Arch is no other but the Sine of the Complement of that Arch.



3. The Tangent of an Arch is a Line drawn perpendicular from one End of the Radius that passeth thro' one End of the Arch, 'till it meet with the Radius passing thro' the other. As the Tangent to the Arch $S A$ is, the Line $T A$ being perpendicular to $R A$, the Radius that passeth thro' one End of the Arch, and meeting the Radius $R S$, that passeth thro' the other End (being continued) to T .

4. The

Right-lined TRIGONOMETRY. 63

4. The Radius $R S$ continued to T , is called the Secant of the Arch $S A$.

5. The Difference of an Arch from a Semicircle, is called its Supplement, as the Arch $L S$ is the Supplement to the Arch $S A$.

6. The versed Sine of an Arch, is part of the Radius intercepted between the Arch and Sine, as $N A$ is the versed Sine of $S A$.

Having any three Things, except the Angles of a Triangle, the other Parts may be found mechanically by the Help of a Scale of equal Parts, Chords, Sines, &c. or by Calculation, by the Help of a Table calculated for the Lengths of the Sines, Tangents, &c. The Projection of which Scale, and Calculation of the Table, will be shewn in the following Problems.

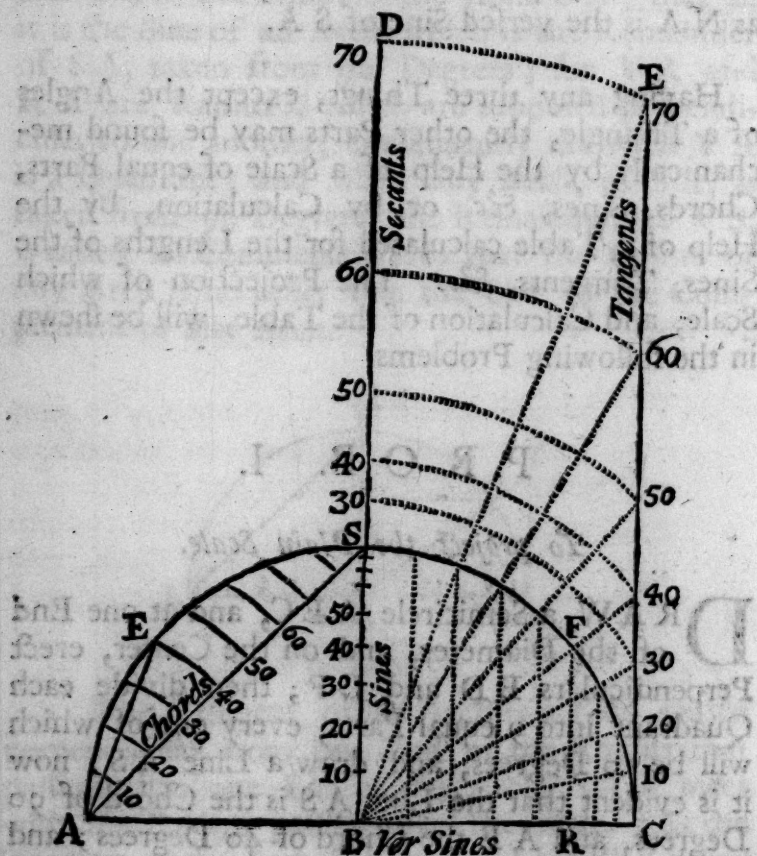
P R O B. I.

To project the Plain Scale.

DR A W a Semicircle $A E C$, and at one End of the Diameter, and on the Center, erect Perpendiculars $B D$ and $C F$; then divide each Quadrant into 9 equal Parts, every one of which will be 10 Degrees, and draw a Line $A S$; now it is evident that the Line $A S$ is the Chord of 90 Degrees, and $A E$ the Chord of 40 Degrees; and if we make $A 40 = A E$, then the Distance $A 40$, is the Chord of 40 Degrees to that Radius: And in the other Quadrant $C B S$, if you draw Lines from the Center to the Perpendicular $C E$, it will determine the Tangents of the several Arches;

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Arches ; and likewise the Secants by *Def. 3, & 4.*
 And make $40\ B = FR =$ right Sine of the Arch
 FC ; and so all those Lines being transferred,
 will become Sines to every Degree of the Qua-
 drant ; and the Secants are usually transferred to
 the End of the Line of Sines, for particular Uses,
 as will be shewn hereafter ; and these Lines, trans-
 ferred upon a thin Ruler, will be fit for Use.



PROB.

P R O B. II.

To make the Tables of Natural Sines, Tangents, &c.

THIS Method I shall take from my Compendium of Algebra, and shall only just take the Method from thence, where the Demonstration of each Rule may be shewn.

I have there shewn how to find the Periphery of a Circle to any assign'd Exactness; and the Periphery of a Circle, whose Radius is a Unite, is found to be 6.283185, from which I shall draw the following Rules; and, in order to perform them, it must be presumed, that the natural Sine of one Minute doth so insensibly differ from the Length of the Arch of one Minute, that it may be taken for the same: Consequently,

As the Periphery in Minutes,

Is to the Periphery in equal Parts of the Radius;

So is one Minute,

To the Parts agreeing to that Minute.

That is, $21600' : 6283185 :: 1 : 0.000290888$, equal the natural Sine of one Minute, which agrees with the largest Table of natural Sines I ever saw.

Having got the Sine of one Minute, its Cosine may be thus found;

Suppose $RA = RS$, the Radius of the Circle, and $SN =$ Sine of the Arch SA , then $RN = CS$, the Cosine of that Arch; but $\angle RS^q - SN^q = RN^q = CS^q$, per Prop. XVIII. Sect. II. which in Numbers is the Sine of $1'$, and is 000290885, its

F

Square

Square is .000000084612, and $1 - .000000084612 = 0.999999915388$, the Square Root of which is $.99999995 = \text{Cosine requir'd.}$

The Sine and Cosine of one Minute being obtained, all the rest of the Sines in the Quadrant may be gradually calculated by Mr. *Michael Dary's* Sinical Proportions, which I shall here insert to the same Effect as they are in his Miscellanies. If a Rank of Arches be equidistant, then,

As the Sine of any Arch in that Rank,

Is to the Sum of the Sines of any two Arches equally remote from it on each Side;

So is the Sine of any other Arch in the said Rank,

To the Sum of the Sines of two Arches next it on each Side, having the same common Distance.

I shall now explain this Proportion, and shew how they may be applied to Practice, having the Sine of one Minute, and its Cosine, as before; let the Radius be made the mean or middle Term between these Extrems, and the Proportion will run thus,

As the Radius,

Is to the double Cosine of one Minute;

So is the Sine of one Minute,

To the Sine of $2'$ and $00'$; And,

So is the Sine of $2'$,

To the Sum of the Sines of $3'$ and $1'$; And,

So is the Sine of $3'$,

To the Sum of the Sines of $4'$ and $2'$; and so on in a successive Order from Minute to Minute.

And

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And then if from the Sum of the Sines of $3'$ and $1'$, be taken the Sine of one Minute, the Remainder will be the Sine of $3'$, and the like; if from the Sum of $4'$ and $2'$, be taken the Sine of $2'$, the Remainder will be the Sine of $4'$, &c.

Proceeding on by this Method, all the natural Sines in the Quadrant may be calculated by Addition and Subtraction only; for the Radius or first Term of the Proportion being Unity, Division is wholly escaped; and because the second Term in the Proportion varies not, if a small Table be made thereof to all the nine Digits, then Multiplication is also escaped; for by the Help of that small Table, the whole Work may be perform'd by Addition and Subtraction only, until all the Sines are gradually made.

Having made the Table of Sines, the Tangents may be easily made by the following Proportion, *viz.*

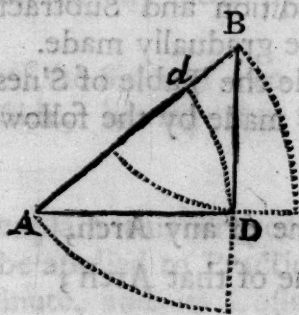
As the Cofine of any Arch,
Is to the Sine of that Arch;
So is the Radius,
To the Tangent of the same Arch:

That is, $RN : SN :: RA : TA$, and $RN : RS :: RA : RT =$ the Secant of that Arch, and so on for the rest of the Degrees and Minutes of the Quadrant.

Now, by the Help of a Table of Sines and Tangents, and the foregoing Definitions, and the following 4 Theorems, all the various Cases of right-lined Trigonometry, will appear easy to be done.

THEOREM I.

IN any right-angled Triangle, if either of the Legs be made Radius, it plainly appears, that the other Leg will be the Tangent of its opposite Angle; for if AD be made the Radius, then the Leg BD is the Tangent of the Arch $dD = \text{Angle } BAD$; and in like manner, if BD is made Radius, then AD is the Tangent of the Angle B ; but if the Hypothenufe be made Radius, then the Legs will be the Sines of their opposite Angles, as plainly appears from the Figure.



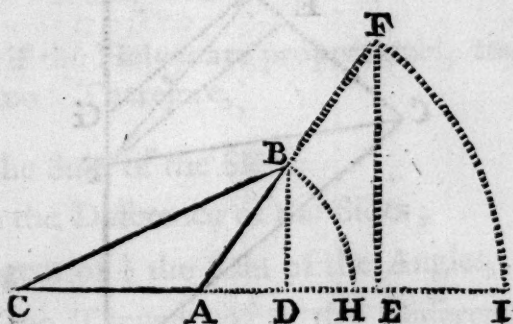
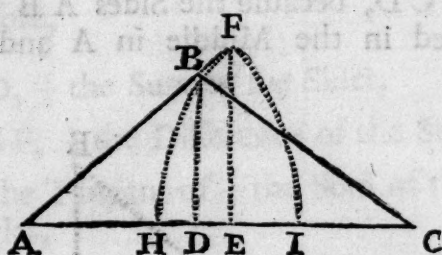
THEOREM II.

The Sides of any right-lined Triangle are proportional to one another, as the Sines of their opposite Angles.

IN the Triangle ABC , make $AF = BC$, and let fall Perpendiculars from F and B , to the Side AC ; then, having drawn the Arches HB and FI , 'tis evident, that BD is the Sine of the $\angle$ at C , and FE the Sine of the Angle at A , by *Def. II. Sect. III.*; and by *Prop. XVIII. Sect. II.* the Triangles ABD , AFE , are similar; therefore,

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therefore, *per Prop. XVIII. Sect. II.* $Af : BA :: FE : BD$, or as $AB : BD :: AF : FE$; and AF being equal to BC , and the two Perpendiculars being Sines, it will be, as the Side $BC : Side BA :: \text{Sine of } \angle A : \text{Sine of } \angle C$; as the Side $AB : \text{Sine } \angle C :: Side BC : \text{Sine } \angle A$; as in both the following Figures.



THEOREM III.

*As the Sum of the Legs about an Angle,
Is to their Difference;
So is the Tangent of $\frac{1}{2}$ the Sum of the other
two Angles,
To the Tangent of half their Difference.*

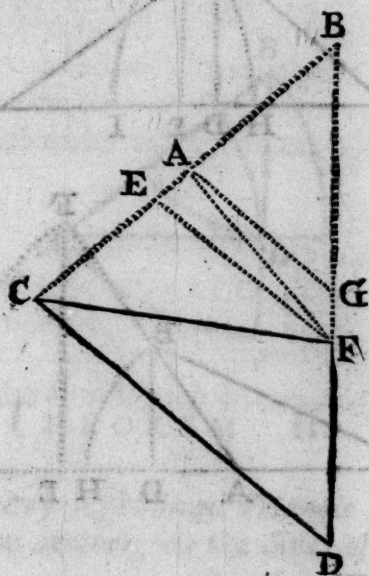
IN the Triangle CFD , produce the Side FD , and the Side $BF = CF$, then will BD be the Sum of the Legs, and GD half the Sum;

F 3

from

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from which, if you take the Leg FD , the Remainder $GF = \frac{1}{2}$ the Difference of the Legs, draw CB , and bisect it in A , and draw AF , which will be perpendicular to it, and the $\angle CFA = \angle BFA$, per *Prop. III. Sect. II.* but $\angle CFB = \angle FCD + \angle D$, per *Prop. VIII. Sect. II.* therefore $\angle CFA = \frac{1}{2}$ the Sum of the Angles $FCD + \angle D$, then draw AG , which will be parallel to CD , because the Sides AB and BD are bisected in the Middle in A and G , then



draw EF parallel to CD , which will likewise be parallel to AG , which will make the $\angle CFE =$ alternate $\angle FCD$, the lesser Angle of the Triangle, because $\angle FCB = \angle C + \angle D$, and $EFB = \angle D$, take $\angle D$ from both, then $CFE = \angle C$, which taken from $\angle CFA = \frac{1}{2}$ the Sum of the opposite Angles, leaves $EFA = \frac{1}{2}$ the Difference of the opposite Angles. Now let AF be Radius of a Circle, then is EA the Tangent

Right-lined TRIGONOMETRY: 71

gent of $\frac{1}{2}$ the Difference, and AC the Tangent of $\frac{1}{2}$ the Sum of the opposite Angles: And the Triangles BAG, BEF, BCD, are similar, *per Prop. XVIII. Sect. II.* and of consequence their Sides proportional: Therefore,

$$BG : GD :: AB : AC$$

$$BG : GF :: AB : AE$$

Therefore,

As GD, $\frac{1}{2}$ the Sum of the Sides,

Is to GF, $\frac{1}{2}$ the Difference of the Sides;

AC, the Tangent of $\frac{1}{2}$ the Sum of the opposite Angles,

To AE, the Tangent of $\frac{1}{2}$ the Difference.

And if the Halves are proportional, the Wholes are so too: Therefore,

As the Sum of the Sides,

Is to the Difference of the Sides;

Tangent of $\frac{1}{2}$ the Sum of the Angles,

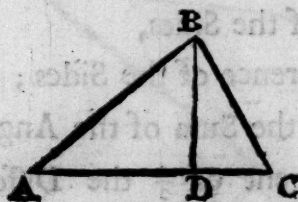
To the Tangent of $\frac{1}{2}$ the Difference of the Angles.

THEOREM IV.

In any Triangle whatsoever, the Square of the Side oppos'd to the acute Angle, with two Rectangles contained under the Side upon which the Perpendicular falls, and the Line which is between the Perpendicular and that Angle, is equal to the Squares of both the other Sides; that is, $BC^2 + 2AC \times AD = AB^2 + AC^2$.

DEMONSTRATION.

By the Figure	1	$AC - AD = DC$
per Pr. 14. S. 2.	2	$AB^2 = BD^2 + AD^2$
Ibid.	3	$BC^2 = BD^2 + AC^2 - 2AC \times AD + AD^2$
2 — 3	4	$AB^2 - BC^2 = 2AC \times AD - AC^2$
4 + $BC^2 + AC^2$	5	$AB^2 + AC^2 = BC^2 + 2AC \times AD$





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 OF

Right-angled TRIGONOMETRY.

S E C T. IV.

C A S E I.

In the right-angled Triangle ABC, right-angled at B,

Are given $\left\{ \begin{array}{l} \text{AC the Hypothenufe } 150, \\ \text{Angle A opposite to the Base } 40^{\circ} : 30' \end{array} \right.$

Required $\left\{ \begin{array}{l} \text{Perpendicular AB,} \\ \text{Base BC.} \end{array} \right.$

G E O M E T R I C A L L Y.

I.



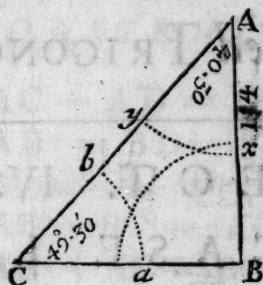
R A W the Line A B.

2. From the Scale, or Line of Chords, take in your Compasses the Chord of 60° , and from a Point in the Line A B, as A, describe the Arch $\propto y$.

3. From the Line of Chords take the given Angle $40^{\circ} - 30'$, and lay on the Arch from $\propto$ to y ; and from the Center A, thro' y , draw out the Line A C.

4. From

4. From a Scale of equal Parts, take the given Side AC 150, and lay from the Center A to C ; and from that Point C , let fall a Perpendicular on the Line AB , which compleats the Triangle; and the Length of the Sides AB and BC , may be found by applying them to the Line of equal Parts which the Side AC was laid down from; but if a more exact Answer be requir'd in Numbers, make AC the Radius, and the Proportion for finding the Side AB , will be, as $R : AC :: S \angle C : AB$, by *Prop. I. Sect. III.*



LOGARITHMETICALLY.

As Radius or Sine of $90^\circ - 00'$ - 10,0000000
 Is to the Hypothenuse AC 150 - 2,1760913
 So is the Sine of the Angle C $49^\circ - 30'$ 9,8810455

To the Side AB 114 - - - - 2,0571368

To find the Base BC , the Hypothenuse being Radius,
 the Proportion will be,

As Radius or Sine of $90^\circ - 00'$ - 10,0000000
 Is to the Hypothenuse AC 150 - 2,1760913
 So is the Sine of the Angle A $40^\circ - 30'$ 9,8125444

To the Base BC 97 - - - - 1,9886357

In

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In making the Proportions for finding the Sides or Angles of any right-angled Triangle, it must be observed, that every Side of a right-angled Triangle has two Denominations or Names, and that each Side has one of those Denominations fixed and unalterable, *viz.* the *Hypothenuſe*, the *Perpendicular*, and *Base*: The other Names are precarious according to the Side made Radius, and are called the Words on the ſeveral Sides; thus, When the Hypothenuſe is made Radius, then the Word on the Hypothenuſe is Radius, and the Word on the Base is the Sine of its oppoſite Angle; as alſo, the Word on the Perpendicular is the Sine of its oppoſite Angle; but when the Perpendicular is made Radius, then the Word on the Base is the Tangent of its oppoſite Angle, and the Word on the Hypothenuſe is the Secant of that ſame Angle; And when the Base is made Radius (and conſequently the Word on the Base) then the Word on the Perpendicular is the Tangent of its oppoſite Angle, and the Word on the Hypothenuſe is the Secant of that Angle. Theſe Things being obſerved, the Way to form a Proportion to find the Side of a Triangle, is thus,

Fiſt, having imagined one Side of the Triangle to be made a Radius, and thereby obſerving, as above, the Word (or Denomination) on the ſeveral Sides, the Proportion will be,

*As the Word or Denomination on the Side given,
Is to the given Side;*

*So is the Word or Denomination on the Side re-
quired,*

To the Side required.

Thus, in the preceding Caſe, the Hypothenuſe is imagined the Radius of the Circle, and if a Circle were drawn with that Radius, one Extre-
mity

mity of the Hypothenuſe being the Center, it would plainly appear, that the Baſe would be a right Sine, as alſo the Perpendicular to the reſpective Arches, which are the Meaſure of their oppoſite Angles; therefore the Proportion, as above, will be,

As the Word on the Side given, *viz.* Radius,
Is to the Side given, *viz.* the Hypothenuſe;
So is the Word on the Perpendicular, *viz.* the
Sine of the Angle at C,
To the Perpendicular in which the Hypothe-
nuſe is conſider'd as the Sine of a right An-
gle, *viz.* $90^{\circ} - 00'$;

And conſequently the Semidiameter of a Circle, which, in calculating the natural Sines, is ſuppoſ'd to be divided into 10,000000 of equal Parts, the Logarithm of which is the ſame 10,000000, and, in the preceding Caſe, is made the firſt Term in the Proportion; the Hypothenuſe is alſo conſider'd as a Line divided into 150 Yards, Feet, Inches, &c. the Logarithm of which is 2,1760913, and is made the ſecond Term in the Proportion; and the Perpendicular conſider'd as the right Sine of $49^{\circ} - 30'$, the Logarithm of which 9,8810455, is made the third Term in the Proportion; and here it is to be obſerved, that the Nature of the Logarithms, or their Proportion to one another, is ſuch, that Addition ſerves inſtead of Multiplication, and Subtraction for Division; therefore the Logarithms of the two laſt Terms being added together, and from that Sum the Logarithm of the firſt Term be ſubtracted, the Remainder 2,0571368, will be the Logarithm of the fourth Term; and the Number anſwering to that Logarithm, as appears from the Tables of

Loga-

Right-angled TRIGONOMETRY. 77

Logarithms, 114, is the Length of the Perpendicular consider'd as a Line of equal Parts; and hence it is evident, that no more than two Sides, with their several Denominations, are taken into the above Proportion, which consists of four Terms, *viz.*

As the Radius or Word on the Hypothenuſe,
Is to the Hypothenuſe ;

So is the Sine of the Angle C, or Word on
the Perpendicular,

To the Perpendicular.

C A S E II.

In the right-angled Triangle ABC

Are given $\left\{ \begin{array}{l} \text{The Perpendicular AB 124,} \\ \text{Angle A opposite to the Base } 40^{\circ}-30' \end{array} \right.$

Requir'd $\left\{ \begin{array}{l} \text{AC the Hypothenuſe,} \\ \text{BC the Base.} \end{array} \right.$

I. GEOMETRICALLY.

1. **D**R A W the Line AB, and from the Line of equal Parts lay 124, the given Side from A to B.

2. From B raise a Perpendicular, as BC.

3. With the Chord of 60° describe the Arch *a b*, and on that Arch lay $40^{\circ}-30'$, the given Angle.

4. From the Center C, thro' *b*, draw the Line AC, which compleats the Triangle.

2. LOGA-

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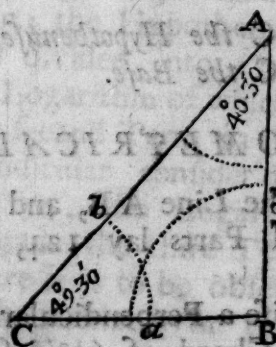
2. LOGARITHMETICALLY.

Making A B Radius, to find the Base.

As Radius or Sine of $90^{\circ} - 00'$	-	10,0000000
Is to the Perpendicular A B 124	-	2,0934217
So is the Tangent of the Angle at A $40^{\circ} - 30'$	-	9,9314989
To the Base B C 106	-	2,0249206

Again, To find the Hypotenuse.

As Radius or Sine of $90^{\circ} - 00'$	-	10,0000000
Is to the Perpendicular A B 124	-	2,0934217
So is the Secant of the Angle A $40^{\circ} - 30'$	-	10,1189645
To the Hypotenuse A C 163	-	2,2123862



3. By Scale and Compasses.

Extend the Compasses from the Sine of $49^{\circ} - 30'$ on the Line of Sines, to 124 on the Line of Numbers, and that Extent will reach from Radius or 90°

Right-angled TRIGONOMETRY. 79

90° on Sines, to the Hypothenuſe 163 on Numbers;
and the ſame Extent will reach from 40° — 30'
on Sines, to the Baſe, or the Line of Numbers.

4. *By Sliding Rule.*

Set 49° — 30' upon Sines, to 124 upon the
Line of Numbers, then againſt the Radius upon
Sines, is the Hypothenuſe upon Numbers; alſo
againſt 40° — 30' upon Sines, is the Baſe upon
the Line of Numbers.

In this preceding ſecond Caſe, the Perpendi-
cular being made, or ſuppoſed, Radius, the
Word on the Baſe is the Tangent of its oppoſite
Angle, and the Words on the Hypothenuſe, are
the Secant of that Angle; therefore, to find the
Baſe,

As the Word on the Side given,
Is to the Side given;
So is the Side required,
To the Side required.

That is,

As R,
Is to the Perpendicular A B;
So is the Tangent of the Angle at A,
To the Baſe B C: And,
So is the Secant of the Angle at A,
To the Hypothenuſe.

But in performing the Operation by the Scale,
the Hypothenuſe is made Radius, becauſe there
are no Lines of Secants put upon Scales for the
Calculation of Triangles: And in working by the
Scale and Compaſſes, obſerve to open the Com-
paſſes.

passes from the first Term of the Proportion to the second, and the same Length will reach from the third Term to the fourth required; and on the sliding Scale, the first Term being set directly opposite to the second, then against the third Term, you will find the fourth Term required.

C A S E III.

In the right-angled Triangle ABC,

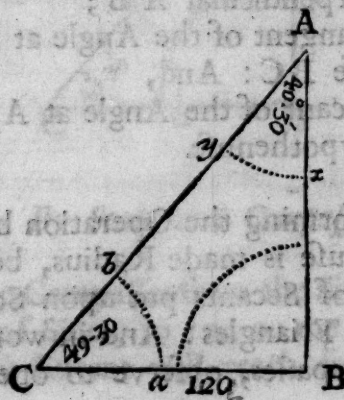
Are given $\left\{ \begin{array}{l} \text{The Base BC } 120, \\ \text{The Angle at the Base BCA } 40^\circ - 30'. \end{array} \right.$
 Required $\left\{ \begin{array}{l} \text{The Hypothenuse AC,} \\ \text{The Perpendicular AB.} \end{array} \right.$

I. GEOMETRICALLY.

1. **D**R A W the Line BC, and from the Line of equal Parts lay 120 from B to C, and upon the Point B raise a Perpendicular.

3. With the Chord of 60, from the Point C, describe the Arch ab ; and upon that Arch, from the Line of Chords, lay $40^\circ - 30'$, from the given Angle from a to b .

3. From the Center C, thro' b , draw the Line CA, which compleats the Triangle.



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2. LOGARITMETICALLY.

The Base BC being made Radius, to find the Hypothenufe.

As Radius or Sine of $90^\circ - 00'$ - 10,0000000
 Is to the Base BC 120 - - - - 2,0791812
 So is the Tangent of the Angle at }
 C $40^\circ - 30'$ - - - - } 9,9314989
 To the Perpendicular $102 \frac{7}{10}$ - - 2,5116801

To find the Hypothenufe.

As Radius or Sine of $90^\circ - 00'$ - 10,0000000
 Is to the Base BC 120 - - - - 2,0791812
 So is the Secant of the Angle $49^\circ - 30'$ 10,1874556
 To the Hypothenufe $180 \frac{6}{10}$ - - 2,2566368

3. By Scale and Compasses.

Extend the Compasses from $49^\circ - 30'$ upon the Sines, to 120 upon the Line of Numbers, and the same Extent will reach from $90^\circ - 00'$ upon Sines, to $157 \frac{8}{10}$ on the Line of Numbers; also, the same Extent will reach from $40^\circ - 30'$, to $102 \frac{7}{10}$ upon the Line of Numbers.

4. By the Sliding Rule.

Set $49^\circ - 30'$ upon Sines, to 120 upon Numbers; then against 90° upon Sines, is $157 \frac{8}{10}$ upon Numbers; and against $40^\circ - 30'$ upon Sines, there is $102 \frac{8}{10}$ upon Numbers.

82 A COMPENDIUM of

In this preceding third Case, the Base being made Radius, and being the Side given, then,

As the Word on the Side given,
Is to the Side given ;
So is the Word on the Side requir'd,
To the Side requir'd.

That is,

As the Radius,
Is to the Base ;
So is the Tangent of the Angle at C,
To the Perpendicular ; And,
So is the Secant of the Angle at C,
To the Hypothenufe.

C A S E IV.

In the right Angle Triangle ABC,

Are given $\left\{ \begin{array}{l} AC \text{ the Hypothenufe } 150, \\ AB \text{ the Perpendicular } 102, \end{array} \right.$

Required $\left\{ \begin{array}{l} \text{The Angle A, and C,} \\ \text{The Base BC.} \end{array} \right.$

1. GEOMETRICALLY.

1. **D**R A W the Line A B, and from the Line of equal Parts, lay 102 from A to B.
2. From B raise a Perpendicular B C.
3. From the Scale of equal Parts take 150, and with that, opening of the Compasses, with one Foot of the Compasses in A, with the other cross the Line B C, as in C, and from A to the Point C, draw the Line A C, which compleats the Triangle.

2. LOGA-

Right-angled TRIGONOMETRY. 83

2. LOGARITHMETICALLY.

The Hypothenufe being made Radius, to find the Angles.

As the Hypothenufe A C 150 - 2,1760913

Is to the Radius or Sine of 90° - 10,0000000

So is the Perpendicular A B 102 - 2,0086002

To the Sine of the Angle at C $41^\circ - 57'$ 12,0086002

9,8325089

Which subtracted from 90° , leaves $48^\circ - 03'$
the Angle at A, by Prop. VIII. Sect. II.

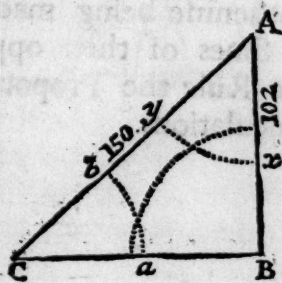
Again, To find the Base.

As the Radius or Sine of $90^\circ - 00'$ 10,0000000

Is to the Hypothenufe 150 - - 2,1760913

So is the Sine of the Angle at A $48^\circ - 03'$ 9,8714144

To the Base B C 111 $\frac{5}{10}$



G 2

3. By

3. *By the Scale and Compasses.*

Extend the Compasses from 150 upon Numbers, to 90° upon Sines; and the same Extent will reach from 102 upon Numbers, to $48^\circ - 57'$, the Angle at C upon Sines; also, the same Extent will reach from $48^\circ - 03'$ upon Sines, to $111 \frac{5}{16}$ upon Numbers.

4. *By the Sliding Rule.*

Set 150 in the Line of Numbers, to 90° in the Line of Sines; then against 102 in Numbers, is $41^\circ - 57'$ in Sines; also against $48^\circ - 03'$ in Sines, $111 \frac{5}{16}$ in Numbers.

In this preceding fourth Case, there are two Sides given to find an Angle; and in making the Proportion for finding an Angle, the following Rule must be observ'd, viz.

*As one of the Sides given,
Is to the Word on it;
So is the other given Side,
To the Word on it, which is the Angle requir'd.*

For the Hypothenufe being made Radius, the other Sides are Sines of their opposite Angles; and by the above Rule the Proportions are as in the preceding Calculations.

CASE V.

In the right-angled Triangle ABC,

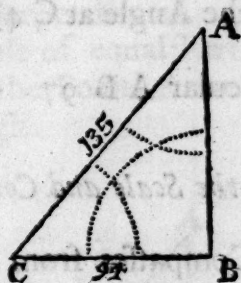
Are given $\begin{cases} AC \text{ the Hypotenuse } 135, \\ BC \text{ the Base } 94. \end{cases}$

Requir'd $\begin{cases} \text{The Angles A and C,} \\ \text{The Perpendicular.} \end{cases}$

By Geometrical Protraction.

1. DRAW the Line BC, on which lay 94 from a Scale of equal Parts, and from B raise a Perpendicular BA.

2. From a Scale of equal Parts take 135, and with that, opening of the Compasses, and one Foot in C, cross the Line BA with the other, as at A, then draw the Line AC, which compleats the Triangle.



G 3

LOGA

LOGARITHMETICALLY

The Hypothenuſe being made Radius, to find the Angle at A.

As the Hypothenuſe A C 135	-	2,1303338
Is to the Radius or Sine of 90°	-	10,0000000
So is the Baſe B C 94	-	1,9731279
To the Sine of the Angle at A 44°-07'	-	11,9731279
		9,8427941

Which, ſubtracted from 90°, leaves 45° — 53' the Angle at C.

Again, To find the Perpendicular A B.

As Radius or Sine of 90°	- - -	10,0000000
Is to the Hypothenuſe 135	- - -	2,1303336
So is the Sine of the Angle at C 45°-53'	- - -	9,8560784
To the Perpendicular A B 97	- - -	1,9864120

3. *By the Scale and Compaſſes.*

Extend the Compaſſes from 135 on the Line of Numbers, to 90° on the Line of Sines; and the ſame Extent will reach from 94 on Numbers, to 44° — 07' on Sines: Alſo the ſame Extent will reach from 45° — 53' on Sines, to 97 on Numbers.

4. *By the Sliding Rule.*

Set 135 in the Line of Numbers, to 90 in the Line of Sines, then against 94 on Numbers, will be found $44^{\circ} - 07'$ on Sines; and against $45^{\circ} - 53'$ on Sines, may be found 97, the Perpendicular on Numbers.

C A S E VI.

In the right-angled Triangle ABC,

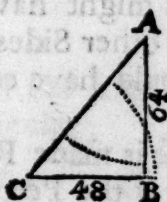
Are given $\begin{cases} AB \text{ the Perpendicular } 64, \\ BC \text{ the Base } 48. \end{cases}$

Requir'd $\begin{cases} \text{Angles } A \text{ and } C, \\ \text{And the Hypotenuse } AC. \end{cases}$

G E O M E T R I C A L L Y.

1. **D**R A W the Line BC, on which lay 48 from a Scale of equal Parts, and on the Point B raise a Perpendicular.

2. From a Scale of equal Parts, lay 64 from B to A, and join the Points A and C, which compleats the Triangle.



LOGARITHMETICALLY.

The Perpendicular being made Radius, to find the Angle at A, the Proportion will be, by the third Part of Prop. I. of Sect. III. As AB : R :: BC : TA; that is,

As the Perpendicular A B 64	-	-	1,80618
Is to the Radius 90°	-	-	10,00000
So is the Base B C 48	-	-	1,68124

To the Tangent at the Angle A 36° -- 52' 9,87506

Again, To find the Hypothenufe, the Perpendicular being made Radius, the Proportion will be, As R : A B :: Sec. ∠ A : A C; that is,

As Radius or Sine of 90°	-	-	10,00000
Is to the Perpendicular A B 64	-	-	1,80618
So is the Secant of Angle A 36° — 52'	-	-	10,09689

To the Hypothenufe 80 - - 1,90307

In the Solution of the preceding six Cases of right-angled Trigonometry, it is to be observ'd, that the Proportions might have been varied by making either of the other Sides Radius, and those Things requir'd, would have come out the same; thus, in *Case I.*

The Hypothenufe is made Radius, to find the Perpendicular; but if the Perpendicular had been made Radius to find itself, the Word on the Hypothenufe (the given Side) would have been the Secant of the Angle at A; therefore the Proportion would have been,

As

Right-angled TRIGONOMETRY. 89

As the Secant of the Angle at A,
Is to the Hypothenuſe AC;
So Radius or Sine of 90° ,
To the Perpendicular.

And this Proportion would bring out the ſame Answer with the Proportion made in Page 74; or the Perpendicular might be found by making the Baſe Radius, and the Proportion would be,

As the Secant of the Angle at C,
Is to the Hypothenuſe;
So the Tangent of the Angle at C,
To the Perpendicular.

Alſo in *Caſe II.* The Hypothenuſe might be found by making itſelf Radius; and the Proportion would be,

As the Sine of the Angle at C,
Is to the Perpendicular;
So the Radius,
To the Hypothenuſe.

And, in *Caſe II.* the Baſe might be found by making itſelf Radius; then the Proportion would be,

As the Tangent of the Angle at C,
Is to the Perpendicular;
So is the Radius,
To the Baſe.

Alſo, in *Caſe III.* the Perpendicular might be found by making either itſelf, or the Hypothenuſe, Radius, by ſtill obſerving the general Rule
to

90 *A COMPENDIUM of, &c.*

to find the Word on each of the other Sides, and by saying,

As the Word on the Side given,
Is to the Side given ;
So the Word on the Side requir'd,
To the Side requir'd.

And when the Sides are given to find the Angles, the Proportions may still be varied by making either of the Sides Radius ; but the Proportions must always be,

As one of the Sides given,
Is to the Word on it ;
So the other given Side,
To the Angle requir'd.

As, in *Case IV.* where the Hypothenufe and Perpendicular are given to find the Angles, the Angle at C is found by making the Hypothenufe Radius ; but it might have been found by making the Perpendicular Radius ; then the Proportion would have been,

As the Perpendicular,
Is to the Radius ;
So the Hypothenufe,
To the Secant of the Angle at A.

By observing these Rules, all the Varieties of the Proportions in right-angled plain Trigonometry will become very easy to the Learner.



A
COMPENDIUM
OF
Oblique plain **TRIGONOMETRY.**

S E C T. V.

*Containing the SOLUTION of the four CASES of
Oblique-angled plain Triangles.*

C A S E I.

*Two Sides, and an Angle opposite to one of them,
being given, to find the other opposite Angle, and
the third Side.*

E X A M P L E.

In the Oblique-angled plain Triangle ABC,

Are given $\begin{cases} AB & 120, \\ BC & 70, \\ \text{Angle } C & 40^\circ - 30'. \end{cases}$

Requir'd $\begin{cases} \text{Angles } B \text{ and } A, \\ \text{Also the Side } AC. \end{cases}$

i. GEOMETRICALLY.

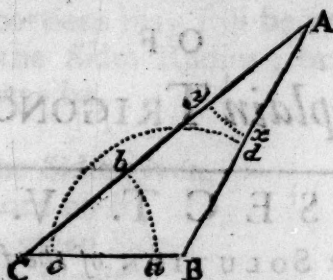
1.  **RAW** the Line AC, and from a Point in it, as at C, with the Chord of 60° , describe the Arch *ab*.

2. From the Chords lay $40^\circ - 30'$ the given Angle from *b* to *a*, and thro' *a* draw the
Line

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Line CB, and upon it lay 70 from the Scale of equal Parts, from B to C.

3. With the Distance of 120 between the Points of the Compasses, setting one Foot in B, with the other cross the Line AC in A, and join the Points B and A, and the Triangle is concluded, and the Angles A and B may be measured by the Line of Chords.



2. LOGARITHMETICALLY.

To find the Angles by Calculation, it will be, by Prop. II. of Trigonometry, $AB : BC :: S \angle C : S \angle A$; that is,

As the Side AB (opposite to the }
given Angle) 120 - - - } 2,0791812

To the Side BC, opposite to the }
Angle sought 70 - - - } 1,8450980

So is the Sine of the Angle at C $40^\circ - 30'$ 9,8125444

To the Sine of the Angle at A $22^\circ - 16'$ 11,6576424

9,5784612

The

Oblique Plain TRIGONOMETRY: 93.

The Angles A and C being known, the Angle at B may be found by *Prop. VIII. Sect. II.* thus,

From the Sine of the three Angles	-	180° - 00'
Take the Sum of the Angles A and B	62 — 46	
Remains the Angle at B	- -	117 — 14

To find the Length of the Side A C, the Proportion will be,

As the Sine of the Angle C 40° — 30'	9,8125444	
Is to the Side A B 120	- -	2,0791812
So is the Sine of the Angle B 117° - 14'	9,9489752	
To the Side A C 164 $\frac{2}{10}$	-	12,0281564
		2,2156120

N. B. In taking the Sine of the obtuse Angle B, observe, that by *Def. II. Sect. III.* the Sine of 117° — 14' is also the Sine of 62° — 46', the one being the Complement of the other to 180'; therefore, from the Table of Sines, you are to take the Sine of 62° — 46', because every right Sine is the Sine of two Arches, the one being the Complement of the other to a Semicircle.

N. B. The Parts requir'd in the several Cases of Oblique Trigonometry, may be found by the Scale and Compasses, by extending the Compasses from the first Term of the Proportion, to the second, and the same Extent will reach from the third Term to the fourth requir'd: Also the Parts requir'd may be found on the
Sliding

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Sliding Rule, by setting the first Term against the second; then opposite to the third Term, the fourth Term may be found.

CASE II.

The Angles, and one Side of an oblique-angled plain Triangle being given, to find either of the other Sides.

EXAMPLE.

In the oblique-angled plain Triangle ABC;

Are given Angles $\left\{ \begin{array}{l} A \quad 28^{\circ} - 30', \\ B \quad 110^{\circ} - 20', \\ C \quad 41^{\circ} - 10', \end{array} \right.$
And the Side CB 110.

Requir'd, The Sides BA and AC.

GEOMETRICALLY.

1. **M**AKE CB equal to 110, by a Line of equal Parts.
2. Make the Angle at B equal to $110^{\circ} - 20'$ and the Angle at C equal to $41^{\circ} - 10'$, by the Line of Chords.
3. Draw out the Lines BA and CA, 'till they intersect each other in the Point C, and the Triangle is formed; and the Sides may be measured by the Line of equal Parts.

The Proportion for finding the Length of the Side AC, will, by Prop. II. of Trigonometry, be,
As $S \angle C : S \angle B :: AB : AC$.

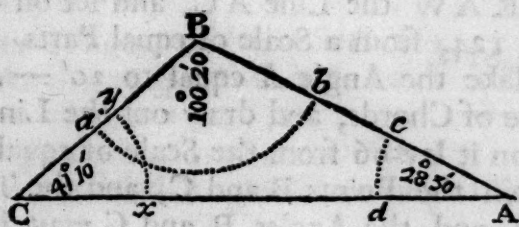
As

Oblique Plain TRIGONOMETRY. 95

As the Sine of the Angle at A $28^{\circ} - 30'$ 9,6786629
 To the Sine of the Angle B $110^{\circ} - 20'$ 9,9720579
 So is the Side BC 110 - - - 2,0413927
 To the Side AC $216 \frac{2}{10}$ - - 12,0134506
 2,3347877

Again, By the same Proportion, to find the Side AB.

As the Sine of the Angle at C $41^{\circ} - 10'$ 9,6786629
 Is to Sine of the Angle at A $28^{\circ} - 30'$ 9,8183919
 So is the Side BC 110 - - 2,0413927
 To the Side AB $152 \frac{1}{10}$ - 11,8607846
 2,1821217



CASE

C A S E III.

Two Sides, and the contained Angle of an oblique-angled plain Triangle being given, to find the opposite Angles, and the other Side.

E X A M P L E.

In the oblique-angled plain Triangle ABC,

Are given $\begin{cases} AB\ 96, \\ AC\ 124, \\ \text{Angle } A\ 30^\circ - 40'. \end{cases}$

Requir'd $\begin{cases} \text{Angles at B and C,} \\ \text{And the Side BC.} \end{cases}$

G E O M E T R I C A L L Y.

1. **D**R A W the Line AC, and set off upon it 124, from a Scale of equal Parts.
2. Make the Angle A equal to $30^\circ - 40'$, by the Line of Chords, and draw out the Line AB, and upon it lay 96 from the Scale of equal Parts.
3. Join the Points B and C, and the Triangle is made, and the Angles B and C may be measured by the Line of Chords.

The Proportion for finding the Angles in this Case, depends on *Prop. III. of Trigon.* for having found the Sum and Difference of the two Sides, subtract the given Angle from 180° , and take half the Remainder, which will be the Half Sum of the unknown Angles, by *Prop. VIII. Sect. II.* Thus,

Oblique Plain TRIGONOMETRY. 97

$$AC \quad - \quad - \quad - \quad 124$$

$$AB \quad - \quad - \quad - \quad 96$$

$$\text{Sum of the Sides} \quad - \quad - \quad 220$$

$$\text{Difference of the Sides} \quad - \quad 28$$

$$\text{Sum of all the Angles} \quad - \quad - \quad 180^{\circ} - 00'$$

$$\text{The Angle at A} \quad - \quad - \quad 30^{\circ} - 40'$$

$$\text{The Sum of the Angles at B and C} \quad - \quad 149^{\circ} - 20'$$

$$\text{Half the Sum of the Angles at B \& C} \quad 74^{\circ} - 40'$$

Then, by the third *Axiom*,

$$\text{As the Sum of the two Sides } AC \} \text{ and } AB \ 220 \quad - \quad - \quad - \quad 2,3424227$$

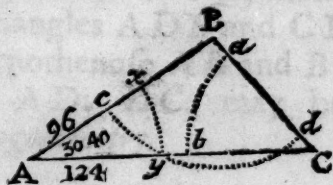
$$\text{So their Difference } 28 \quad - \quad - \quad - \quad 1,4471580$$

$$\text{So is the Tangent of } \frac{1}{2} \text{ the Sum of } \} \text{ the Angles at B and C } 74^{\circ} - 10' \} \quad 10,5619413$$

$$\text{To the Tangent of half the Differ-} \} \text{ ence of the said Angles from the } \} \quad 12,0090993$$

$$\text{half Sum } 24^{\circ} - 54' \quad - \quad - \quad -$$

$$9,6366766$$



H

To

To the half Sum of the Angles at B & C $74^{\circ} - 40'$
 Add half their Difference - - - $24 - 54$

 The Sum is the greater Angle, viz. the } $99 - 34$
 Angle at B. - - - - - }

Also from half the Sum of the Angles $74^{\circ} - 40'$
 Subtract half their Difference - - - $24 - 54$

 Remains the less, viz. the Angle at C $49 - 46$

Then, by *Axiom II.* the Side BC may be found ; thus,

As the Sine of the Angle at C $49^{\circ} - 46'$ 9,8827638
 Is to the Side AB 96 - - - - 1,9822712
 So is the Sine of the Angle at A $30' - 40$ 9,7076064

 To the Side BC 64 - - - - 11,6898776

 1,8071138

The Proposition for finding the Angles in a Triangle, depends on Prop. III. of *Trigon.* for having found the Sum and Difference of the two Sides, subtract the given Angle from the Sum, and take half the Remainder, and the Half Sum of the unknown Angles, will be the same.



CASE

Oblique Plain TRIGONOMETRY. 99

CASE IV.

Given the three Sides, to find the Angles.

EXAMPLE.

In the Oblique-angled Triangle ABC,

Are given $\left\{ \begin{array}{l} AB \ 110, \\ AC \ 150, \\ BC \ 86. \end{array} \right.$

Requir'd, The Angles A B and C.

GEOMETRICALLY.

1. **D**RAW the Line AC, on which lay 150 from a Line of equal Parts from A to C.
2. Take from the same Scale 110, and with one Foot in A, describe an Arch, as at B.
3. Take from the Scale 86, and with one Foot of the Compasses in C, cross the former Arch, as at B; and from A and C to the Intersecting, draw out the Lines AB and CB, which compleats the Triangle; and the Angles may be measured by the Line of Chords. But, in order to determine their Quantity by Calculation, from the Angle B let fall the Perpendicular BD, then will the Triangle ABC be reduced into two right-angled Triangles ADB and CDB, in each of which the Hypotenuse AB and BC are given, and the Bases AD, DC, may be found by Prop. IV. of Trigonometry.

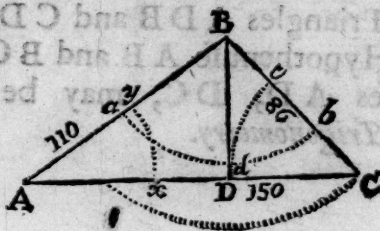
100. A COMPENDIUM of

As the Base A C 150	-	-	2,1760913
To the Sum of the Sides A B, B C 196			2,2922561
So is the Difference of the Sides A B and B C 24			1,3802112
To A e, the Difference of the Seg- ments of the Base A D and D C			3,6724673
31	-	-	
			1,4963760

Which 31 subtracted from A C 150, the whole Base, leaves e C 119, the Half of which is 59 - 5 D C, the lesser Segment; and this lesser 59 - 5, subtracted from the whole Base 150, leaves 90 - 5, the greater Segment A D.

The Segments A D and D C, or Bases of the Triangles, and the Hypotenuse, being known, the Angles may easily be found by Case V. of Right-angled plain Triangles; thus,

As the Base A D 90 - 5	-	-	1,9563425
Is to the Side A B 110	-	-	2,0413927
So is the Radius 90	-	-	10,0000000
To the Secant of the Angle at A 34 - 42			10,0850502



Again,

Oblique Plain TRIGONOMETRY. 101

Again, In the Triangle CDB.

As the Base CD 59 — 5	- - - -	1,7739420
Is to the Side CB 86	- - - -	1,9344985
So is Radius 90	- - - -	10,0000000
To the Secant of the Angle at C 46-17		19,1605565

Now, if to the Complement of the	}	55 — 18
Angle A - - - -		
Be added the Complement of the An-	}	43 — 43
gle at C - - - -		
The Sum will be the Angle ABC	-	99 — 01

N. B. The Angles A B C of the Triangle ABC, may be found, if the Perpendicular be let fall from any other Angle.





THE
APPLICATION
OF
TRIGONOMETRY
IN TAKING
HEIGHTS *and* DISTANCES.

SECT. VI.



THE Object to be measured may be either accessible, or inaccessible; or it may be upon level Ground, or standing upon a Hill.

EXAMPLE I.

To Measure an accessible Altitude.

LET AB represent a Tower, or Steeple, whose Height is requir'd.

First, with the Quadrant, or any other Instrument, find the Quantity of the Angle C, which suppose to be $52^{\circ} - 30'$; then measure the Distance AC, which suppose to be 85 Feet; then, by *Case I.* of plain Triangles,

As

The Application of, &c. 103

As S. of the Angle $37^{\circ} - 30'$ C B A }
C. R. - - - - - } 0,215553

To the Base A C 85 Feet Log. 1,929419

So is S. of the Angle C $52^{\circ} - 30'$ - 9,899466

To the Altitude A B 110,8 - - - 2,044438

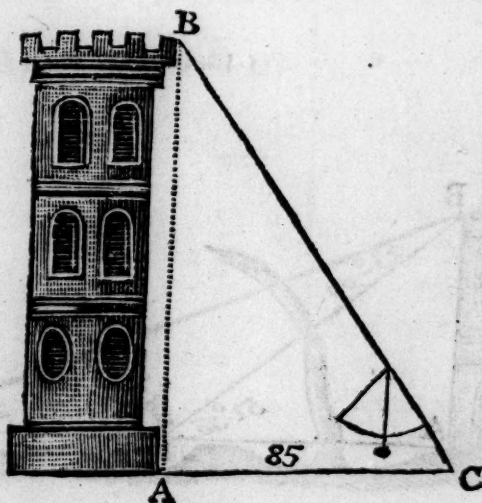
Or thus,

As the Radius - - - - - 10,000000

To the Base A C 85 Feet - - - 1,929419

So is T. of the Angle C $52^{\circ} - 30'$ 10,115019

To the Altitude 110,8 - - - 2,044438



Note, That in this, and all such Cases, you must add the Height of your Eye, or Instrument, to the Altitude before found.

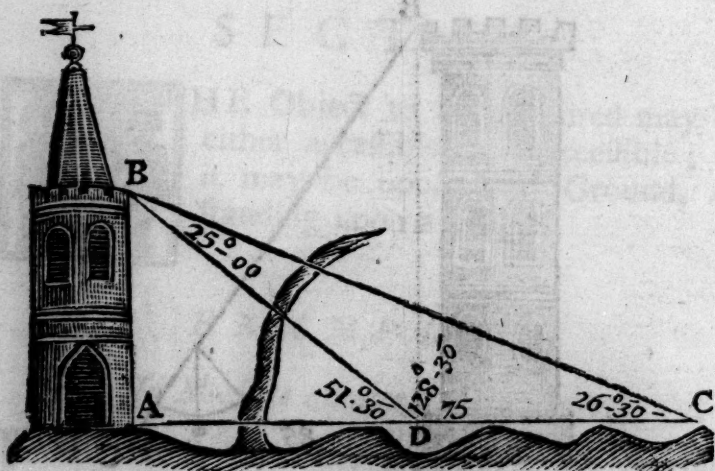
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EXAMPLE II.

To Measure an inaccessible Altitude.

LET AB in the following Figure, be a Church-Steeple, whose Height is requir'd, but, by reason of a River, or some other Obstacle, it is inaccessible; that is, you cannot come to the Foot of it at A .

First, with your Quadrant at C , take the Angle of Altitude, which suppose to be $26^{\circ} - 30'$, then measure in a Line towards the Steeple, to D , which suppose to be 75 Feet; and at D observe again the Angle of Altitude, which let be $51^{\circ} - 30'$.



Now the two visual Lines CB and DB , and the measured Distance CD , do form the oblique-angled plain Triangle CBD , wherein are given all the Angles, and the Side CD , the Angle BCD being $26^{\circ} - 30'$, and the Complement of ADB

in taking Heights and Distances. 105

A D B $51^{\text{d}} - 30'$, to 180 , is the obtuse Angle
B D C $128^{\circ} - 30'$, and consequently the third
Angle C B D is 25° : Then by *Case I.* of Oblique-
angled plain Triangles, find the Side D B; thus,

As S. of 25° C B D	C. A. R.	-	0,374052
To the Distance of C D 75	-	-	1,875067
So is S. of $26^{\circ} - 30'$ to the Angle C			<u>9,649527</u>
To the visual Line B D 79,18	-	-	1,898640

Then, by *Case I.* of right-angled plain Tri-
angles,

As Radius	-	-	10,000000
Is to B D 79,18	-	-	1,898640
So is S. of A D B $51^{\circ} - 30'$	-		<u>9,893544</u>
To the Altitude A B 61,97	-		1,792184

E X-

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EXAMPLE III.

Let BCD represent three Objects, whose Distances are known, and A, E, two such Stations, that from A may be seen the other Station E, and the Objects B and C, but not D: Also from E may be seen the Station A, and the Objects D and C, but not B, consequently the Angles BAC, CAE, CEA, and CED, may be found by Observation; but the Distance of the two Stations cannot be measured; 'tis required to know the Distances AB, AC, AE, CE, and ED.

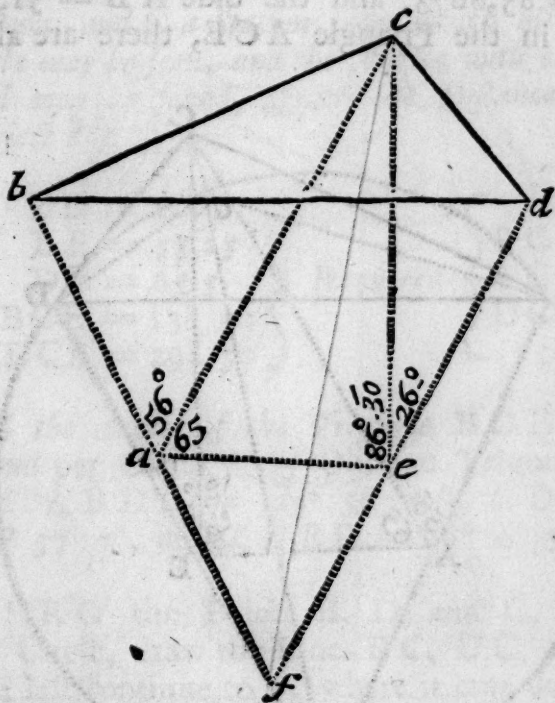
Let $BD = 88,25$, $BC = 71,25$, and $DC = 37,75$; then $\angle BCD = 103^\circ 50' 10''$, $\angle BDC = 51^\circ 37' 21''$, and $\angle DBC = 24^\circ 39' 29''$, and the other Angles, as in the Figure.

DRAW the Figure $abcde$ similar to the Figure ABCDE, and continue the Lines ba and de 'till they meet in the Point f . It is evident, that if the Figures are similar, the Figure $abcdef$ may be so enlarged, or contracted, that the Line ae may be of any assumed Length, and yet the Angles, in every respect, will be the same; therefore let us imagine the Line ae to be $= 10$, then in the Triangle aef there is given all the Angles, and the Side ae , to find $af = 11,493$; and in the Triangle ace there is given the same, to find $ca = 20,918$.

In order to go on with the remaining Part of the Calculation, continue AB and DE to F, thro' the Points BD and F, draw a Circle, and draw the Lines CF, and BG, and GD; then, in the Triangle acf there is given the Sides ac , af , and the included Angle caf , to find (by Case III. of Oblique

in taking Heights and Distances. 107

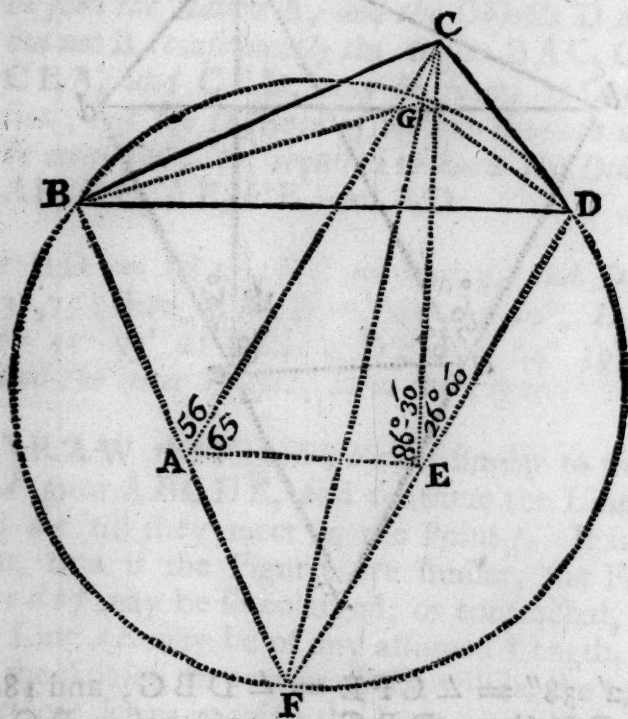
Oblique Trigonometry) the Angles $afc = 36^\circ 47' 22''$ ($= \angle AFC$, and by *Prop. XX. Sect. II.* $= \angle BDG$) and the $\angle acf = 19^\circ 12' 38''$ ($= \angle ACF$); and the $\angle afe - \angle afc =$



$16^\circ 42' 38'' = \angle CFE = \angle DBG$; and $180^\circ - \angle BDG - \angle DBG = 126^\circ 30' = \angle BGD$, per VIII. of *Sect. II.* then in the Triangle BGD there is given all the Angles, and the Side BD , to find (by *Case I. Oblique Trigonometry*) the Side $GD = 31,56674$; then in the Triangle CGD there is given the Sides GD , CD , and the included Angle GDC , to find (by *Case III. of Oblique Trigonometry*) the Angle $DGC = 117^\circ 3' 49'' 5'''$, and the $\angle DCG = 48^\circ 6' 11'' 55'''$; then $180^\circ - \angle DGC = 62^\circ 56' 10'' 35''' = \angle DGF = \angle DBF$, and $\angle BGD - \angle DGF =$
63°

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$63^{\circ} 33' 49'' 5''' = \angle BGF = \angle BDF$, and $180^{\circ} - \angle ABC - \angle BAC = 36^{\circ} 24' 20'' 5''' = \angle BCA$: Then in the Triangle BCA there are given all the Angles, and the Side BC , to find (by *Case I. of Oblique Trigonometry*) the Side $AC = 85,8673$, and the Side $AB = 51,007$: Again, in the Triangle ACE , there are all the



Angles, and the Side AC , given to find (by the same *Case*) the Side $AE = 41,0489$, and the Side $EC = 77,96764$: And then, $\angle BDC + \angle BDF = 115^{\circ} 11' 10'' 5''' = \angle FDC$, and $180 - \angle EDC - \angle DEC = 38^{\circ} 48' 49'' 55''' = \angle DCE$. Lastly, in the Triangle DCE there is given the Side CD , and all the Angles, to find (by the last *Case* mentioned) the Side $DE = 53,9758$, and the Side $EC = 77,749$.
Q. E. F.

EX.

EXAMPLE IV.

Let BED be three Objects, whose Distances are known, and C a Station from which all the Objects may be seen, and the Angles with each Object may be found, Quære the Distance of each Object?

$$\left. \begin{array}{l} BD = 106 \\ BE = 53,25 \\ DE = 63,5 \\ \angle BCE = 130^\circ 30' \\ \angle DCE = 29^\circ 50' \end{array} \right\} \text{Required} \left\{ \begin{array}{l} BC \\ EC \\ DC \end{array} \right.$$

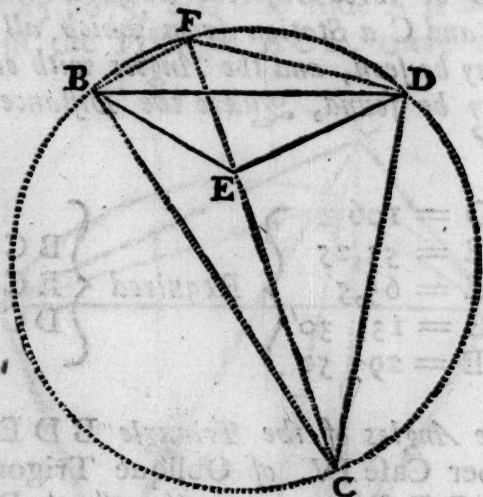
All the Angles of the Triangle BDE may be found per Case IV. of Oblique Trigonometry, viz. $\angle BDE = 23^\circ 56' 50''$, $\angle DBE = 29^\circ 57' 7''$, and $\angle BED = 26^\circ 6' 3''$.

THRO' the Points B , D , and C , draw a Circle, draw the Lines BC , DC , and EC , which last continue to F , where it cuts the Circle, draw BF and DF . Then $180^\circ - \angle BCD = \angle BFD = 136^\circ 40'$, per VIII. of Sect. II. and the $\angle BDF = \angle BCF$, per XX. of Sect. II. and by the same $\angle DBF = \angle DCF$. In the Triangle BFD there are given all the Angles, and the Side BD , to find $BF = 36,059$, and $DF = 76,843$, per Case I. of Oblique Trigonometry: Then the $\angle FDB + \angle BDE = \angle FDE$; then, in the Triangle FDE there are given the Sides ED , FD , and the included Angle FDE , to find the $\angle FED = 84^\circ 30' 24''$, and $180 - \angle FED = 95^\circ 29' 36''$; then, in the Triangle EDC there are all the Angles, and the Side DE , given to find the Side $EC = 107,42$, and $DC =$

131,05.

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131,05. Lastly, In the Triangle BDC there are given all the Angles, and the Sides BD and DC, to find $BC = 151,3$.



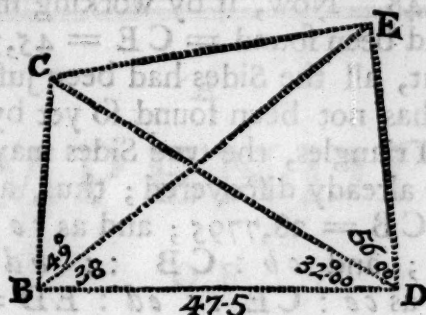
EXAMPLE V.

Suppose BD, two Stations whose Distance is 47,5 Yards, from whence the two Objectts C and E, may be seen, and their Angles found by Observation, viz. $CBE = 49^\circ$, $EBD = 38^\circ$, $CDB = 32^\circ$, and $CDE = 56^\circ$, it is required to find BC, BE, DC, DE, and CE.

IN the Triangle CBD, there is the Side BD, and all the Angles, given to find $BC = 28,7795$, and $DC = 54,2349$, which may be done by Case I. of Oblique Trigonometry, and in the Triangle BED, there are the same Things given to find $BE = 58,6775$, and $DE = 36,1475$, per the same Case. Lastly, In the Triangle BCE there are given BC, and BE, and the

intaking Heights and Distances. 111

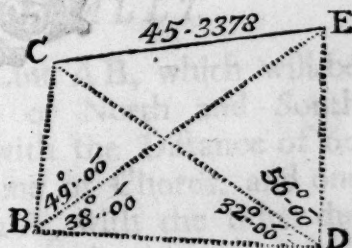
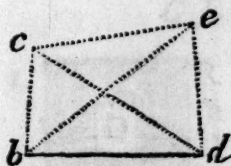
the included Angle CBE, to find $CE = 45,3378$,
per Case III. in Oblique Trigonometry.



EXAMPLE VI.

Let CE be two Objects, whose Distance is known, and let B and D represent two Stations, from whence they may be both seen, and the horizontal Angles CBE and DBE , CDE and CDB , found by Observation; but the Distance of the two Stations cannot be measured; 'tis required to find the Distance of the two Stations, and the Objects from each Station.

DR A W another Figure, as $cedb$ (whose Side bd , suppose to be any Number, as 10) and similar to the Figure $CEDB$; now, upon this Supposition, that bd is 10, and the little Figure



similar

†

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similar to the great one, we may, by the Method of the last Problem, find $de = 7,61$, $be = 12,353116$, $bc = 6,05885$, $dc = 11,41787$, and $ce = 9,5448$. Now, if by working in this Manner, ce had been found $= CE = 45,3378$, then it is evident, all the Sides had been justly found; but, as it has not been found so yet by the Similarity of Triangles, the true Sides may be found from those already discovered; thus, as $ce : CE :: cb : CB = 28,7795$; and as $ce : CE :: cd : CD$; and $cb : CB :: bd : BD = 47,5$; and as $ce : CE :: ed : ED = 36,147$; and so of the rest.





A
COMPENDIUM
OF
PLAIN SAILING.

SECT. VII.

CASE I.

*Course and Distance being given, to find the
Difference of Latitude and Departure.*

EXAMPLE.

*Admit a Ship from the Latitude $50^{\circ} 10'$ N.
sails S. S. W. 48,5 Miles, I require the Latitude
she is in, and her Departure from the Meridian.*

GEOMETRICALLY.

1.  DRAW the Line AB, which will be
a Meridian, or North and South
Line, and with the Distance of 60
from the Line of Chords, and one
Foot of the Compasses in A, with the other de-
scribe the Arch αy .

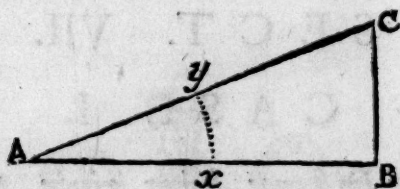
I

2. From

2. From a Line of Rhumbs take two Points, the Distance of S. S. W. from the S. (which is equal to $22^{\circ} 30'$) the given Course, and set it from x to y , and from the Center A draw the Line A y C.

3. From a Line of equal Parts take 48.5, the given Distance fail'd, and set it from A to C, and from C let fall the Perpendicular C B, which compleats the Triangle.

4. In the Triangle A B C, A B represents the Difference of Latitude, A C the Distance failed, B A the Westing, or Departure from the Meridian A B. The Angle A represents the Course, or the Angle of the rhumb Line failed on, makes with the Meridian, and the Angle at C is the Complement of the Course.



N. B. That the Course subtracted from, $90^{\circ} 00'$ leaves the Complement of the Course, which, in this Case, is $67^{\circ} 30'$.

N. B. That the Parts required may be found three different Ways by the Trigonometrical Rules; thus, 1st. If the Distance be made Radius, the Difference of the Latitude will be the Sine Complement of the Course, and the Departure, the Sine of the Course. 2dly, If the Difference of Latitude be made Radius, the Departure is the Sine of the Course, and the Distance failed is the Secant of the Course. 3dly, If the Departure be made Radius, the Difference of

PLAIN SAILING. 115

of Latitude is the Sine Complement of the Course, and the Distance sailed, the Secant Complement of the Course, per Axiom I.

If a Side be requir'd, begin the Proportion with an Angle ; that is, the Name of the given Side ; then say,

As the Name of the given Side,
Is to the Side given ;
So is the Name of the Side requir'd,
To the Side requir'd.

In the Application to Navigation, the Distance is commonly made Radius.

By the Logarithms.

As Radius $90^{\circ} 00'$	-	10,00000
Is to the Distance AC 48,5	-	1,68574
So is the Sine of the Angle A $22^{\circ} 30'$		9,58283
To the Departure BC = 18,5		<u>1,26857</u>

Then,

As Radius $90^{\circ} 00'$	-	10,00000
Is to the Distance AC 48,5	-	1,68574
So is $\angle C$ the Cosine of Course $67^{\circ} 30'$		9,96561
To the Difference of the Latitude 44,8		<u>1,65135</u>

From the Latitude departed from	56° 10'
Subtract the Difference of Latitude	00 44,8
Remains the present Latitude	49 25,2

By the Gunter Scale.

Extend the Compasses from 90 on the Sines, to 22° 30', and the same Extent will reach from 45,5 to 18,5 on the Line of Numbers; and the Extent from 90° to 67° 30' on the Sines, will reach from 48,5 to 44,8 on the Line of Numbers, which is the Difference of Latitude.

By the Sliding Rule.

Draw the middle Part towards the right Hand, 'till 90 on the Sines is opposite to 48,5 on the Numbers; then will 22° 30' on the Sines, stand against 18,5 on the Numbers, the Departure: Also without altering the Scale, against 67° 30' on the Sines, will stand 44,8 on the Numbers, the Difference of Latitude.

CASE

CASE II.

The Course and Difference of Latitude being given, to find the Departure and Distance sailed.

EXAMPLE.

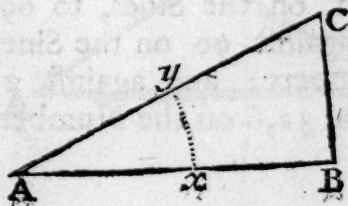
Admit a Ship sail from the Latitude $48^{\circ} 30'$ North, and sails S. S. E. $\frac{1}{4}$ Easterly, 'till she be in the Latitude $47^{\circ} 21'$, I demand the Departure and Distance sailed.

GEOMETRICALLY.

1. **D**RAW the Line AB, a North and South Line, representing the Meridian of $48^{\circ} 30'$ North, and subtracting the Latitudes one from another, it leaves $69'$ for the Difference of Latitude, which taken from a Scale of equal Parts, must be set from A to B.

2. With the Chord of 60 , describe the Arch αy from the Center A, on which set off the Chord of $25^{\circ} 19'$ (answering to S. S. E. $\frac{1}{4}$ Easterly) from α to y , and draw A y C.

3. On the Point B erect a Perpendicular 'till it cut A C in C, which compleats the Triangle.



By the Logarithms.

As $\angle C = 64^\circ 41'$ the Cosine of Course	9,956148
Is to the Difference of Lat. 69 = A B	1,838849
So is the Radius 90°	- - 18,000000
To the Dist. sail'd 76,3 Miles = B C	1,882701

Again,

As $\angle C = 64^\circ 41'$ the Cosine of Course	9,956148
Is to the Difference of Lat. 69 = A B	1,838849
So is the $\angle A 25^\circ 19'$ the Course	- 9,631058
To the Departure A B = 32,6 Miles	1,513759

By the Gunter Scale.

The Distance between $64^\circ 41'$, and 90 on the Line of Sines, will reach from 69 on the Line of Numbers, to 76,3 = Distance sailed; and the Distance between $64^\circ 41'$, and $25^\circ 19'$ on the Sines, will reach from 69 on the Line of Numbers, to 32,6 = the Departure from the Meridian.

By the Sliding Scale.

Set $64^\circ 41'$ on the Sines, to 69 on the Numbers; then against 90 on the Sines, will be 76,3 on the Numbers; and against $25^\circ 19'$ on the Sines, will be 32,6 on the Numbers.

CASE III.

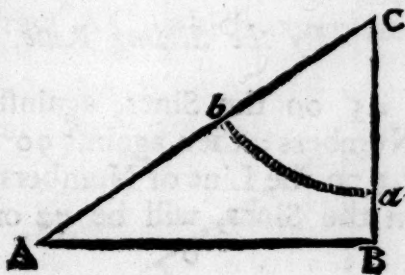
The Course and Departure given, to find the Distance and Difference of Latitude.

EXAMPLE.

A Ship, from the Latitude of $48^{\circ} 30'$ North, sails N. E. by N. 'till her Departure from the Meridian be 48,5, what is the Distance sailed, and Difference of Latitude.

GEOMETRICALLY.

1. **D**RAW a Line, as BC , and set thereon, from a Scale of equal Parts, 48.5, which is the Departure given.
2. With the Chord of 60, from C describe the Arch ab , on which, from the Line of Chords, lay $56^{\circ} 15'$, the Complement of the Course, and draw out the Line CbA .
3. From B erect a Perpendicular to CB , 'till it cut AC in A , which finisheth the Triangle.



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By the Logarithms.

As Sine of the $\angle A$ $33^{\circ} 45'$ the Course	9,744739
Is to the Departure BC 48,5 - -	1,685741
So is the Radius 90° - - -	10,000000
To the Distance AC 87,3 Miles -	1,941002

Then,

As the Radius 90° - -	10,000000
Is to the Distance AC 87,3 -	1,941002
So is Sine of $\angle C$ $56^{\circ} 15'$ Comp. of Course	9,919842
To the Difference of Lat. AB 72 Miles	1,860844

By the Gunter Scale.

The Distance from $33^{\circ} 45'$, to 90° upon the Line of Sines, will reach from 48,5 upon the Line of Numbers, to 87,3 = Distance required: And the Distance from 90° , to $56^{\circ} 15'$ on the Line of Sines, will reach from 87,3 on the Line of Numbers, to 72, the Difference of Latitude requir'd.

By the Sliding Rule.

Set $33^{\circ} 45'$ on the Sines, against 48,5 on the Line of Numbers; then against 90° on the Sines, will be 87,3 on the Line of Numbers; and against $56^{\circ} 15'$ on the Sines, will be 72 on the Line of Numbers.

CASE

CASE IV.

The Distance and Difference of Latitude being given, to find the Course and Departure.

EXAMPLE.

Admit a Ship sail from the Latitude $42^{\circ} 15'$ North, one some Point of the Compass between the North and West 88 Miles, then finds herself in the Latitude of $43^{\circ} 30'$ North; I demand the Course and Departure?

From $43^{\circ} 30'$
Subtract $42 \ 15$

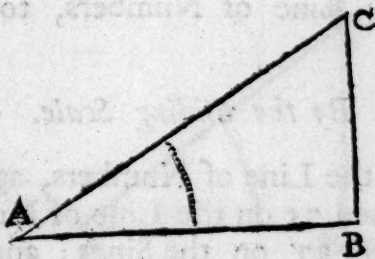
Miles

Remains $1 \ 15 = 75 = \text{Difference of Lat.}$

GEOMETRICALLY.

1. **D**R A W the Meridian A B, upon which, from a Scale of equal Parts, lay 75 from A to B, and make B C perpendicular to A B.

2. Take from the Scale of equal Parts 88, the given Distance, and with one Foot of the Compasses in A, and the other will cross the Line B C in C, then draw A C, which compleats the Triangle.



Then

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Then is the $\angle A$, the Angle of the Ship's Course $= 31^\circ 33' = \text{N. N. W. } \frac{3}{4} \text{ West}$, because the Ship sails between North and West, and the $\angle C$ is the Complement of the Ship's Course.

By the Logarithms.

As the Distance $AC = 88$	- - -	1,944482
Is to Radius 90°	- - -	10,000000
So is the Difference of Lat. $AB = 75$		1,875061
To the Comp. of Ship's Course $= 58^\circ 27'$		9,930579

Then,

As Radius 90°	- - -	10,000000
Is to the Distance $AC = 88$	- -	1,944482
So is the Ship's Course $31^\circ 33'$	- -	9,718703
To the Difference of Departure 46,04		1,663185

By Gunter's Scale.

The Distance between 88 and 75 upon the Line of Numbers, will reach from 90 upon the Line of Sines, to $58^\circ 27'$, which is the Complement of the Course: And the Distance between 90 and $31^\circ 33'$ upon the Sines, will reach from 88 upon the Line of Numbers, to 46,04, the Departure.

By the Sliding Scale.

Set 88 on the Line of Numbers, against 90 on the Sines; then 75 on the Line of Numbers, will be against $58^\circ 27'$ on the Sines; and $31^\circ 33'$ is against 46,04 on the Line of Numbers.

CASE

CASE V.

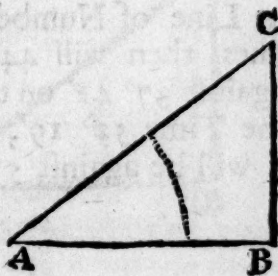
The Distance and Departure given, to find the Course and Difference of Latitude.

EXAMPLE.

Admit a Ship sails 72 Miles between South and West until her Departure is 44 Miles, I demand the Course and Difference of Latitude ?

GEOMETRICALLY.

1. **D**RAW a Line, as CB, and from a Scale of equal Parts take 44, the given Departure, and set it from B to C.
2. On B erect a Perpendicular of any Length.
3. With the Distance given taken from a Scale of equal Parts, setting one Foot of the Compasses in C, with the other cross the Line AB in A, then draw AC, which compleats the Triangle.
4. Then is AB the Difference of Latitude, and the Angle A = $37^{\circ} 41'$, the Angle of the Ship's Course between South and West, viz. S. W. by S. $3^{\circ} 56'$ Westerly ; as will be seen in the following Proportion.



By

By the Logarithms.

As the Distance sailed A C 72 Miles 1,857332

Is to the Radius 90° - - - 10,000000

So is the Departure C B 44 Miles - 1,643452

To the Angle of Ships Course $A = 37^\circ 41'$ 9,786120

Then,

As Radius 90° - - - - 10,000000

Is to the Distance sail'd A C 72 Miles 1,857332

So is Angle C the Comp. of Course $52^\circ 19'$ 9,898396

To the Diff. of Lat. A B = 57 Miles 1,755728

By the Gunter Scale.

The Distance between 72 and 44 on the Line of Numbers, applied to the Line of Sines, will reach from 90° to $37^\circ 41'$, the Angle of the Ship's Course: And the Distance between 90° and $52^\circ 19'$ on the Line of Sines, will reach from 75 to 57 on the Line of Numbers.

By the Sliding Scale.

Set 72 on the Line of Numbers, against 90 on the Line of Sines, then will 44 on the Line of Numbers, be against $37^\circ 41'$ on the Line of Sines; and at the same Time $52^\circ 19'$, the Complement of the Course, will be against 57 on the Line of Numbers.

CASE

CASE VI.

The Difference of Latitude and Departure given, to find the Course and Distance.

EXAMPLE.

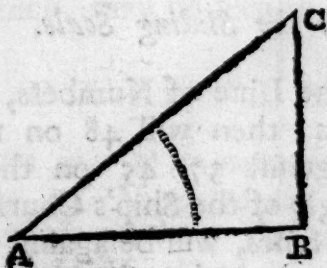
Admit a Ship sails between South and West until her Departure is 48 Miles, and her Difference of Latitude is 62 Miles, I demand her Course and Distance?

GEOMETRICALLY.

1. **D**R A W a Meridian, or North and South Line, as A B, then from a Scale of equal Parts, take 62, the Difference of Latitude, and set from A to B.

2. On B erect a Perpendicular, and set 48 equal Parts from B to C, and draw A C to compleat the Triangle.

3. Then will the $\angle A$ be found $= 37^{\circ} 45'$, as in the following Proportion, = Ship's Course = S. W. by S. 4° Westerly, and A C the Distance sailed.



By

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By the Logarithms.

As A B 62 Miles	-	-	-	1,7923917
Is to Radius 90°	-	-	-	10,0000000
So is C B 48 Miles	-	-	-	1,6812412
To the Tangent of the Course 37° 45'				9,8888495

Then,

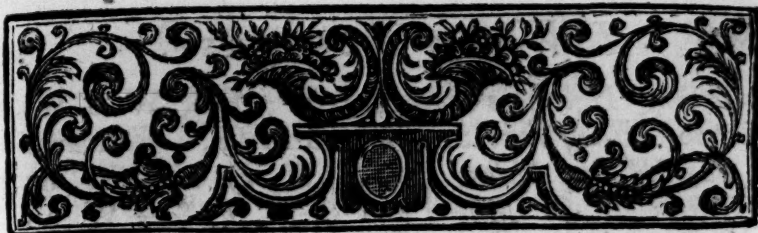
As the $\angle$ A the Ship's Course 37° 45'				9,786905
Is to the Departure C B 48 Miles	-			1,681241
So is the Radius 90°	-	-	-	10,0000000
To the Distance A C = 78,4 Miles				1,894336

By the Gunter Scale.

The Distance between 62 and 48 on the Line of Numbers, apply'd to the Radius upon the Tangents (which is 45°) will reach to 37° 45', the Angle of the Ship's Course; and the Distance between 37° 45', and 90° on the Sines, will reach from 48 on the Line of Numbers, to 78,4, the Distance failed.

By the Sliding Scale.

Place 62 on the Line of Numbers, against 45° on the Tangents; then will 48 on the Line of Numbers, be against 37° 45' on the Tangents, which is the Angle of the Ship's Course; and 90° on the Line of Sines, will be against 78,4 on the Line of Numbers, which is the Distance sought.



To WORK A
T R A V E R S E.

S E C T. VIII.



WHEN a Ship, failing from one Port to another, is forced by contrary Winds, or any other Accident, from the direct Course, and is oblig'd to make two, or more, different Courses, before she can attain the designed Port, then is that Sailing called a *Traverse*; the Method of working which, may be seen in the following

E X A M P L E.

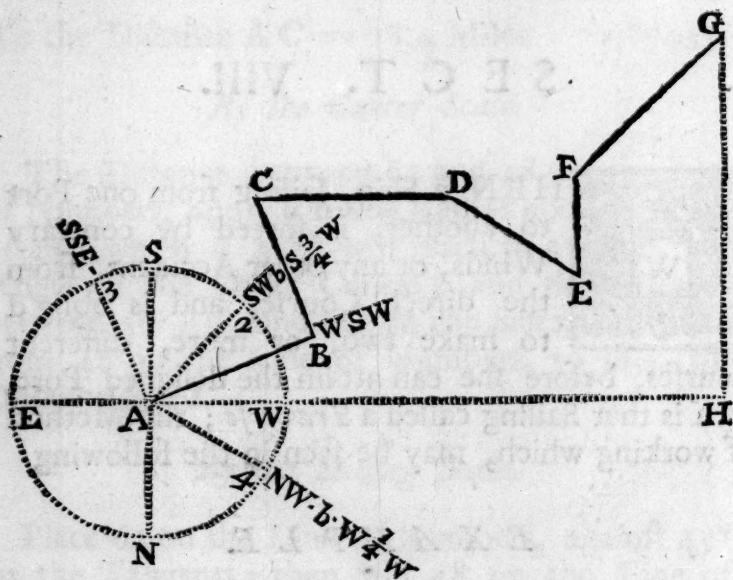
Suppose a Voyage was to be made from the Lizard, Latitude $50^{\circ} 10'$ North, to the Island of Bermudas, whose Latitude is $32^{\circ} 25'$ North, and Difference of the Meridians is 3360 Miles West of the Lizard; and, in order to attain the said Port, the Ship, by contrary Winds, sails the following

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lowing Distances and Courses; first, W. S. W. 100 Miles; then S. S. E. 78 Miles, afterwards due W. 95 Miles, then N. W. by W. $\frac{1}{4}$ W. 67 Miles, S. 58 Miles, and S. W. by S. $\frac{3}{4}$ W. 90 Miles, 'tis requir'd to find what Latitude the Ship is in, and her Departure from the Meridian, and also the direct Course and Distance to Bermudas?

GEOMETRICALLY.

Draw a Circle by the Radius 60, taken from the Line of Chords, as N E S W, and divide it into four Quarters, with the Lines E W and S N, let N the North, S the South, E the East, W the West.



From the Line of Rhumbs set off two Points = W S W from W to 1, and two Points from S to 3 = SSE, and $2\frac{1}{4}$ Points from W to 4 = N W b W $\frac{1}{4}$ W; lastly, Set $4\frac{1}{4}$ Points from W to 2 = S W b S $\frac{3}{4}$ W; and from these Points draw Lines

To Work a TRAVERSE. 129

Lines to the Center A, as in the Figure, which, to prevent Mistakes, may be marked with their respective Names.

Then continue A I 'till it be = 100 = Distance failed upon W S W Course, from the Point B draw B C, in Length 78, and parallel to A 3, which will be the Distance failed upon the S S E Course; draw C D parallel to E W = 95, the Distance failed upon the West Course; draw D E parallel to A 4 = 67 = Distance failed upon the N W $\frac{1}{4}$ W Course; draw E F parallel to S N = 58 = Distance failed upon the South Course; draw F G parallel to A 2 = 90 = Distance failed upon the last Course mentioned. It is evident from the Nature of the Thing and Figure, that if E W be continued, G H be made perpendicular to it, then will A H = 272 Miles = Difference of Latitude, and G H = 203,5 = Difference of Departure from the Point A, which represents the *Lizard*.

By Calculation.

For the Ease of Calculations of this Kind, make a Table as follows, and in the first Column towards the left Hand, set the Courses given; in the next, set the Number of Points that each Course is distant from the North, or South; and in the next, the Number of Degrees corresponding to those Points; and in the fourth, Distance failed upon the respective Courses. Make four other Columns, and mark them as you see.

Then in the Table of Latitude and Departure, look out six Points, or $67^{\circ} 30'$, and against 100 you will find 38,2 for Difference of Latitude, and 92,4 for Difference of Departure, which put under the Words South and West, because the Course consists of South and West. Again, Look

K

for

130 To Work a TRAVERSE.

for two Points, or $22^{\circ} 30'$, and against 78 you will find 72 Difference of Latitude, and 30 Difference of Departure, which place under the Words South and East, because the Course is S and E; and so proceed with all the Courses and Distances, 'till you have completed the following TABLE.

The Courses	Points	Angl. with the Merid	Dist. Sailed	Diff of Lat.		Diff. of Dep	
				North	South	East	West
W. S. W.	6	67,30	100	- -	38,2	- -	92,4
S S E	2	22,30	78	- -	72,1	30,0	- -
W	8	90,00	95	- -	- -	- -	95,0
NW $\frac{1}{4}$ W	$5 \frac{1}{4}$	59,04	67	34,4	- -	- -	57,5
S	0	00,00	58	- -	58,0	- -	- -
SW $\frac{1}{2}$ W	$3 \frac{1}{2}$	39,02	90	- -	69,6	- -	57,1

34,4 | 237,9 | 30,0 | 302,0
34,4 | | | 30,0

Difference of Lat. 203,5 S. 272,0

Add each respective Column, and place their respective Sums underneath, then subtract the least Difference of Latitude from the greatest, and the Remains are the true Difference of Latitude of the Name of the greatest; and do the same of the Departure, by which it is found, that the Difference of Latitude is 203,5 Miles South = $3^{\circ} 23' 30''$, and the Departure is 272 Miles West; then,

$$50^{\circ} 10' 00'' = \text{Latitude Lizard.}$$

$$3 \ 23 \ 30 = \text{Difference of Latitude.}$$

$$46 \ 46 \ 30 = \text{Latitude of the Ship. And,}$$

To Work a TRAVERSE. 131

46° 46' = Latitude of the Ship.

32 25 = Latitudes of *Bermudas*.

14 12 = 861 Miles = Difference of Lat.

Again,

3360 = Difference of the Meridians between the
Lizard and *Bermudas*.

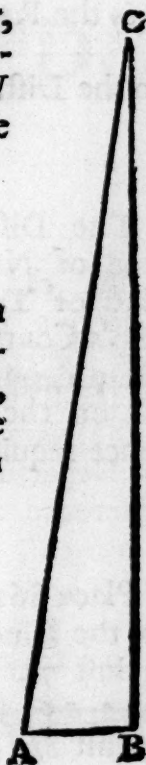
272 = the Ship's Departure.

3088 Miles = Difference of the Meridians.

Then, by *Case VI. in Plain Sailing*,
may be found the Course and Di-
stance to *Bermudas*, Course W S W
+ 6° 55' Westerly, and the Distance
3132 Miles.

Thus Geometrically.

Draw a Line, as A B, and from a
Scale of equal Parts set 861, the Dif-
ference of Latitude from A to B,
on B erect a Perpendicular, the
Length of 3088, equal Parts, then
draw A C to compleat the Triangle.



132 To Work a TRAVERSE.

By the Logarithms.

As the Difference of Lat. 861 Miles	2,9350032
Is to the Radius 90° - - - -	10,0000000
So is the Diff. of Merid. 3088 Miles	3,4896773
To the Tangent of the Course 74° 25'	10,5546741

And,

As the Cosine of the Course 15° 35'	9,4291701
Is to the Difference of the Lat. 861	2,9350032
So is the Radius 90° - - - -	10,0000000
To the Distance requir'd 3132 Miles	3,4958331

By the Gunter Scale.

The Distance between 861 and 3088 on the Line of Numbers, will reach from 45° on the Line of Tangents, to 74° 25', the Angle of the Ship's Course; and the Distance between 15° 25' and 90° on the Line of Sines, will reach from 861 on the Line of Numbers, to 3132, the Distance required.

By the Sliding Scale.

Place 861 on the Line of Numbers, against 45° on the Line of Tangents, and then 3088 will be against 74° 20' on the Line of Tangents; then place 15° 35', the Complement of the Course, against 861 on the Line of Sines, and 90° on the Sines, will be against 3132 on the Numbers.

A TABLE

A
T A B L E
O F
D I F F E R E N C E
O F
L A T I T U D E
A N D
D E P A R T U R E.



L O N D O N:
Printed in the Year M.DCC.XXIX,

134 *A Table of Difference of*

Diff.	1 Degree		2 Degrees		1 Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	01,0	00,0	01,0	00,0	01,0	00,0
2	02,0	00,0	02,0	00,1	02,0	00,1
3	03,0	00,1	03,0	00,1	03,0	00,1
4	04,0	00,1	04,0	00,1	04,0	00,2
5	05,0	00,1	05,0	00,2	05,0	00,2
6	06,0	00,1	06,0	00,2	06,0	00,3
7	07,0	00,1	07,0	00,2	07,0	00,3
8	08,0	00,1	08,0	00,3	08,0	00,4
9	09,0	00,2	09,0	00,3	09,0	00,4
10	10,0	00,2	10,0	00,4	10,0	00,5
11	11,0	00,2	11,0	00,4	11,0	00,5
12	12,0	00,2	12,0	00,4	12,0	00,6
13	13,0	00,2	13,0	00,5	13,0	00,6
14	14,0	00,2	14,0	00,5	14,0	00,7
15	15,0	00,3	15,0	00,5	15,0	00,7
16	16,0	00,3	16,0	00,6	16,0	00,8
17	17,0	00,3	17,0	00,6	17,0	00,8
18	18,0	00,3	18,0	00,6	18,0	00,9
19	19,0	00,3	19,0	00,7	19,0	00,9
20	20,0	00,4	20,0	00,7	20,0	01,0
21	21,0	00,4	21,0	00,7	21,0	01,0
22	22,0	00,4	22,0	00,8	22,0	01,1
23	23,0	00,4	23,0	00,8	23,0	01,1
24	24,0	00,4	24,0	00,8	24,0	01,2
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	89 Degrees		88 Degrees		7 $\frac{3}{4}$ Points	

3 Degrees		4 Degrees		5 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
01,0	00,0	01,0	00,1	01,0	00,1	1
02,0	00,1	02,0	00,1	02,0	00,2	2
03,0	00,2	03,0	00,2	03,0	00,3	3
04,0	00,2	04,0	00,3	04,0	00,3	4
05,0	00,3	05,0	00,3	05,0	00,4	5
06,0	00,3	06,0	00,4	06,0	00,5	6
07,0	00,4	07,0	00,5	07,0	00,6	7
08,0	00,4	08,0	00,6	08,0	00,7	8
09,0	00,5	09,0	00,6	09,0	00,8	9
10,0	00,5	10,0	00,7	10,0	00,9	10
11,0	00,6	11,0	00,8	11,0	01,0	11
12,0	00,6	12,0	00,9	12,0	01,0	12
13,0	00,7	13,0	00,9	12,9	01,1	13
14,0	00,7	14,0	01,0	13,9	01,2	14
15,0	00,8	15,0	01,0	14,9	01,3	15
16,0	00,8	16,0	01,1	15,9	01,4	16
17,0	00,9	17,0	01,2	16,9	01,5	17
18,0	00,9	17,9	01,3	17,9	01,6	18
19,0	01,0	18,9	01,3	18,9	01,7	19
20,0	01,0	19,9	01,4	19,9	01,7	20
21,0	01,1	20,9	01,5	20,9	01,8	21
22,0	01,1	21,9	01,5	21,9	01,9	22
23,0	01,2	22,9	01,6	22,9	02,0	23
24,0	01,3	23,9	01,7	23,9	02,1	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
87 Degrees		86 Degrees		85 Degrees		

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Diff.	1 Degree		2 Degrees		$\frac{1}{4}$ Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	25,0	00,4	25,0	00,9	25,0	01,2
26	26,0	00,5	26,0	00,9	26,0	01,3
27	27,0	00,5	27,0	00,9	27,0	01,3
28	28,0	00,5	28,0	01,0	28,0	01,4
29	29,0	00,5	29,0	01,0	29,0	01,4
30	30,0	00,5	30,0	01,1	30,0	01,5
31	31,0	00,5	31,0	01,1	31,0	01,5
32	32,0	00,6	32,0	01,1	31,9	01,6
33	33,0	00,6	33,0	01,2	32,9	01,6
34	34,0	00,6	34,0	01,2	33,9	01,7
35	35,0	00,6	35,0	01,2	34,9	01,7
36	36,0	00,6	36,0	01,3	35,9	01,8
37	37,0	00,7	37,0	01,3	36,9	01,8
38	38,0	00,7	38,1	01,3	37,9	01,9
39	39,0	00,7	39,0	01,4	38,9	01,9
40	40,0	00,7	40,9	01,4	39,9	02,0
41	41,0	00,7	41,0	01,4	40,9	02,0
42	42,0	00,7	42,0	01,5	41,9	02,1
43	43,0	00,8	43,5	01,5	42,9	02,1
44	44,0	00,8	44,0	01,5	43,9	02,2
45	45,0	00,8	45,0	01,6	44,9	02,2
46	46,0	00,8	46,0	01,6	45,9	02,2
47	47,0	00,8	47,0	01,6	46,9	02,3
48	48,0	00,8	48,0	01,7	47,9	02,3
49	49,0	00,9	49,0	01,7	48,9	02,4
50	50,0	00,9	50,0	01,8	49,9	02,4
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	89 Degrees		88 Degrees		$7\frac{3}{4}$ Point	

3 Degrees		4 Degrees		5 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
25,0	01,3	24,9	01,7	24,9	02,2	25
26,0	01,4	25,9	01,8	25,9	02,3	26
27,0	01,4	26,9	01,9	26,9	02,4	27
28,0	01,5	27,9	02,0	27,9	02,4	28
29,0	01,5	28,9	02,0	28,9	02,5	29
30,0	01,6	29,9	02,1	29,9	02,6	30
31,0	01,6	30,9	02,2	30,9	02,7	31
31,9	01,7	31,9	02,2	31,9	02,8	32
32,9	01,7	32,9	02,3	32,9	02,9	33
33,9	01,8	33,9	02,4	33,9	03,0	34
34,9	01,8	34,9	02,4	34,9	03,1	35
35,9	01,9	35,9	02,5	35,9	03,1	36
36,9	01,9	36,9	02,6	36,9	03,2	37
37,9	02,0	37,9	02,7	37,9	03,3	38
38,9	02,0	38,9	02,7	38,9	03,4	39
39,9	02,1	39,9	02,8	39,8	03,5	40
40,9	02,1	40,9	02,9	40,8	03,6	41
41,9	02,2	41,9	02,9	41,8	03,7	42
42,9	02,2	42,9	03,0	42,8	03,8	43
43,9	02,3	43,9	03,1	43,8	03,8	44
44,9	02,4	44,9	03,1	44,8	03,9	45
45,9	02,4	45,9	03,2	45,8	04,0	46
46,9	02,5	46,9	03,3	46,8	04,1	47
47,9	02,5	47,9	03,4	47,8	04,2	48
48,9	02,6	48,9	03,4	48,8	04,3	49
49,9	02,6	49,9	03,5	49,8	04,4	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
87 Degrees.		86 Degrees		85 Degrees		

Diff.	1 Degree		2 Degrees		$\frac{1}{4}$ Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	51,0	00,9	51,0	01,8	50,9	02,5
52	52,0	00,9	52,0	01,8	51,9	02,5
53	53,0	00,9	53,0	01,8	52,9	02,6
54	54,0	00,9	54,0	01,9	53,9	02,6
55	55,0	01,0	55,0	01,9	54,9	02,7
56	56,0	01,0	56,0	02,0	55,9	02,7
57	57,0	01,0	57,0	02,0	56,9	02,8
58	58,0	01,0	58,0	02,0	57,9	02,8
59	59,0	01,0	59,0	02,1	58,9	02,9
60	60,0	01,0	60,0	02,1	59,9	02,9
61	61,0	01,1	61,0	02,1	60,9	03,0
62	62,0	01,1	62,0	02,2	61,9	03,0
63	63,0	01,1	63,0	02,2	62,9	03,1
64	64,0	01,1	64,0	02,2	63,9	03,1
65	65,0	01,1	65,0	02,3	64,9	03,2
66	66,0	01,1	66,0	02,3	65,9	03,2
67	67,0	01,2	67,0	02,3	66,9	03,3
68	68,0	01,2	68,9	02,4	67,9	03,3
69	69,0	01,2	68,9	02,4	68,9	03,4
70	70,0	01,2	69,9	02,4	69,9	03,4
71	71,0	01,2	70,9	02,5	70,9	03,5
72	72,0	01,3	71,9	02,5	71,9	03,5
73	73,0	01,3	72,9	02,5	72,9	03,6
74	74,0	01,3	73,9	02,6	73,9	03,6
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	89 Degrees		88 Degrees		7 $\frac{3}{4}$ Point	

Latitude and Departure. 139

3 Degrees		4 Degrees		5 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
50,9	02,7	50,9	03,6	50,8	04,4	51
51,9	02,7	51,9	03,6	51,8	04,5	52
52,9	02,8	52,9	03,7	52,8	04,6	53
53,9	02,8	53,9	03,8	53,8	04,7	54
54,9	02,9	54,9	03,8	54,8	04,8	55
55,9	02,9	55,9	03,9	55,8	04,9	56
56,9	03,0	56,9	04,0	56,8	05,0	57
57,9	03,0	57,9	04,1	57,8	05,1	58
58,9	03,1	58,8	04,1	58,8	05,2	59
59,9	03,1	59,8	04,2	59,8	05,2	60
60,9	03,2	60,8	04,3	60,8	05,3	61
61,9	03,3	61,8	04,3	61,8	05,4	62
62,9	03,3	62,8	04,4	62,8	05,5	63
63,9	03,4	63,8	04,5	63,8	05,6	64
64,9	03,4	64,8	04,5	64,7	05,7	65
65,9	03,5	65,8	04,6	65,7	05,8	66
66,9	03,5	66,8	04,7	66,7	05,9	67
67,9	03,6	67,8	04,8	67,7	05,9	68
68,9	03,6	68,8	04,8	68,7	06,0	69
69,9	03,7	69,8	04,9	69,7	06,1	70
70,9	03,7	70,8	05,0	70,7	06,2	71
71,9	03,8	71,8	05,0	71,7	06,3	72
72,9	03,8	72,8	05,1	72,7	06,4	73
73,9	03,9	73,8	05,2	73,7	06,5	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
87 Degrees		86 Degrees		85 Degrees		

140 *A Table of Difference of*

Dif.	1 Degree		2 Degrees		$\frac{1}{4}$ Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	75,0	01,3	74,9	02,6	74,9	03,7
76	76,0	01,3	75,9	02,7	75,9	03,7
77	77,0	01,3	76,9	02,7	76,9	03,8
78	78,0	01,4	77,9	02,7	77,9	03,8
79	79,0	01,4	78,9	02,8	78,9	03,9
80	80,0	01,4	79,9	02,8	79,9	03,9
81	81,0	01,4	80,9	02,8	80,9	04,0
82	82,0	01,4	81,9	02,9	81,9	04,0
83	83,0	01,4	82,9	02,9	82,9	04,1
84	84,0	01,5	83,9	02,9	83,9	04,1
85	85,0	01,5	84,9	03,0	84,9	04,2
86	86,0	01,5	85,9	03,0	85,9	04,2
87	87,0	01,5	86,9	03,0	86,9	04,3
88	88,0	01,5	87,9	03,1	87,9	04,3
89	89,0	01,5	88,9	03,1	88,9	04,4
90	90,0	01,6	89,9	03,1	89,9	04,4
91	91,0	01,6	90,9	03,2	90,9	04,5
92	92,0	01,6	91,9	03,2	91,9	04,5
93	93,0	01,6	92,9	03,2	92,9	04,6
94	94,0	01,6	93,9	03,3	93,9	04,6
95	95,0	01,6	94,9	03,3	94,9	04,7
96	96,0	01,7	95,9	03,4	95,9	04,7
97	97,0	01,7	96,9	03,4	96,9	04,8
98	98,0	01,7	97,9	03,4	97,9	04,8
99	99,0	01,7	98,9	03,5	98,9	04,9
100	100,0	01,7	99,9	03,5	99,9	04,9
Dif.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	89 Degrees		88 Degrees		7 $\frac{3}{4}$ Point	

Latitude and Departure. 141

3 Degrees		4 Degrees		5 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
74,9	03,9	74,8	05,2	74,7	06,6	75
75,9	04,0	75,8	05,3	75,7	06,6	76
76,9	04,0	76,8	05,4	76,7	06,7	77
77,9	04,1	77,8	05,5	77,7	06,8	78
78,9	04,2	78,8	05,5	78,7	06,9	79
79,9	04,2	79,8	05,6	79,7	07,0	80
80,9	04,2	80,8	05,7	80,7	07,1	81
81,9	04,3	81,8	05,7	81,7	07,2	82
82,9	04,4	82,8	05,8	82,7	07,3	83
83,9	04,4	83,8	05,9	83,7	07,3	84
84,9	04,5	84,8	05,9	84,7	07,4	85
85,9	04,5	85,8	06,0	85,7	07,5	86
86,9	04,6	86,8	06,1	86,7	07,6	87
87,9	04,6	87,8	06,2	87,7	07,7	88
88,9	04,7	88,8	06,2	88,7	07,8	89
89,9	04,7	89,8	06,3	89,7	07,9	90
90,9	04,8	90,8	06,4	90,7	08,0	91
91,9	04,8	91,8	06,4	91,6	08,0	92
92,9	04,9	92,8	06,5	92,6	08,1	93
93,9	04,9	93,8	06,6	93,6	08,2	94
94,9	05,0	94,8	06,6	94,6	08,3	95
95,9	05,0	95,8	06,7	95,6	08,4	96
96,9	05,1	96,8	06,8	96,6	08,5	97
97,9	05,1	97,8	06,9	97,6	08,6	98
98,9	05,2	98,8	06,9	98,6	08,7	99
99,9	05,2	99,8	07,0	99,6	08,7	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
87 Degrees		86 Degrees		85 Degrees		

142 *A Table of Difference of*

Dif.	$\frac{1}{2}$ Point		6 Degrees		7 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	01,0	00,1	01,0	00,1	01,0	00,1
2	02,0	00,2	02,0	00,2	02,0	00,2
3	03,0	00,3	03,0	00,3	03,0	00,4
4	04,0	00,4	04,0	00,4	04,0	00,5
5	05,0	00,5	05,0	00,5	05,0	00,6
6	06,0	00,6	06,0	00,6	06,0	00,7
7	07,0	00,7	07,0	00,7	06,9	00,8
8	08,0	00,8	08,0	00,8	07,9	01,0
9	09,0	00,9	08,9	00,9	08,9	01,1
10	09,9	01,0	09,9	01,0	09,9	01,2
11	10,9	01,1	10,9	01,1	10,9	01,3
12	11,9	01,2	11,9	01,2	11,9	01,5
13	12,9	01,3	12,9	01,4	12,9	01,6
14	13,9	01,4	13,9	01,5	13,9	01,7
15	14,9	01,5	14,9	01,6	14,9	01,8
16	15,9	01,6	15,9	01,7	15,9	01,9
17	16,9	01,7	16,9	01,8	16,9	02,1
18	17,9	01,8	17,9	01,9	17,9	02,2
19	18,9	01,9	18,9	02,0	18,9	02,3
20	19,9	02,0	19,9	02,1	19,8	02,4
21	20,9	02,1	20,9	02,2	20,8	02,6
22	21,9	02,2	21,9	02,3	21,8	02,7
23	22,9	02,2	22,9	02,4	22,8	02,8
24	23,9	02,3	23,9	02,5	23,8	02,9
Dif.	D p.	Lat.	Dep.	Lat.	Dep.	Lat.
	$7 \frac{1}{2}$ Point		84 Degrees		83 Degrees	

8 Degrees		$\frac{3}{4}$ Point		9 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
01,9	00,1	01,0	00,1	01,0	00,2	1
02,9	00,3	02,0	00,3	02,0	00,3	2
03,9	00,4	03,0	00,4	03,0	00,5	3
04,9	00,6	04,0	00,6	03,9	00,6	4
04,9	00,7	04,9	00,7	04,9	00,8	5
05,9	00,8	05,9	00,9	05,9	00,9	6
06,9	01,0	06,9	01,0	06,9	01,1	7
07,9	01,1	07,9	01,2	07,9	01,2	8
08,9	01,2	08,9	01,3	08,9	01,4	9
09,9	01,4	09,9	01,5	09,9	01,6	10
10,9	01,5	10,9	01,6	10,9	01,7	11
11,9	01,7	11,9	01,8	11,9	01,9	12
12,9	01,8	12,9	01,9	12,8	02,0	13
13,9	01,9	13,8	02,1	13,8	02,2	14
14,8	02,1	14,8	02,2	14,8	02,3	15
15,8	02,2	15,8	02,3	15,8	02,5	16
16,8	02,4	16,8	02,5	16,8	02,7	17
17,8	02,5	17,8	02,6	17,8	02,8	18
18,8	02,6	18,8	02,8	18,8	03,0	19
19,8	02,8	19,8	02,9	19,7	03,1	20
20,8	02,9	20,8	03,1	20,7	03,3	21
21,8	03,1	21,8	03,2	21,7	03,4	22
22,8	03,2	22,7	03,4	22,7	03,6	23
23,8	03,3	23,7	03,5	23,7	03,8	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
82 Degrees		7 $\frac{1}{4}$ Points		81 Degrees		

Diff.	$\frac{1}{2}$ Point		6 Degrees		7 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	24.9	02.4	24.9	02.6	24.8	03.0
26	25.9	02.5	25.9	02.7	25.8	03.2
27	26.9	02.6	26.9	02.8	26.8	03.3
28	27.9	02.7	27.8	02.9	27.8	03.4
29	28.9	02.8	28.8	03.0	28.8	03.5
30	29.8	02.9	29.8	03.1	29.8	03.7
31	30.8	03.0	30.8	03.2	30.8	03.8
32	31.8	03.1	31.8	03.3	31.8	03.9
33	32.8	03.2	32.8	03.4	32.7	04.0
34	33.8	03.3	33.8	03.5	33.7	04.1
35	34.8	03.4	34.8	03.7	34.7	04.3
36	35.8	03.5	35.8	03.8	35.7	04.4
37	36.8	03.6	36.8	03.9	36.7	04.5
38	37.8	03.7	37.8	04.0	37.7	04.6
39	38.8	03.8	38.8	04.	38.7	04.8
40	39.8	03.9	39.8	04.2	39.7	04.9
41	40.8	04.0	40.8	04.3	40.7	05.0
42	41.8	04.1	41.8	04.4	41.7	05.1
43	42.8	04.2	42.8	04.5	42.7	05.2
44	43.8	04.3	43.7	04.6	43.7	05.4
45	44.8	04.4	44.7	04.7	44.7	05.5
46	45.8	04.5	45.7	04.8	45.7	05.6
47	46.8	04.6	46.7	04.9	46.6	05.7
48	47.8	04.7	47.7	05.0	47.6	05.9
49	48.8	04.8	48.7	05.1	48.6	06.0
50	49.8	04.9	49.7	05.2	49.6	06.1
Diff.	Dep. Lat.		Dep. Lat.		Dep. Lat.	
	$7 \frac{1}{2}$ Point		84 Degrees		83 Degrees	

8 Degrees		$\frac{3}{4}$ Point		9 Degrées		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
24,8	03,5	24,7	03,7	24,7	03,9	25
25,7	03,6	25,7	03,8	25,7	04,1	26
26,7	03,7	26,7	04,0	26,7	04,2	27
27,7	03,9	27,7	04,1	27,6	04,4	28
28,7	04,0	28,7	04,2	28,6	04,5	29
29,7	04,2	29,7	04,4	29,6	04,7	30
30,7	04,3	30,7	04,5	30,6	04,9	31
31,7	04,4	31,6	04,7	31,6	05,0	32
32,7	04,6	32,6	04,8	32,6	05,2	33
33,7	04,7	33,6	05,0	33,6	05,3	34
34,7	04,9	34,6	05,1	34,6	05,5	35
35,6	05,0	35,6	05,3	35,5	05,6	36
36,6	05,1	36,6	05,4	36,5	05,8	37
37,6	05,3	37,6	05,6	37,5	06,0	38
38,6	05,4	38,6	05,7	38,5	06,1	39
39,6	05,6	39,6	05,9	39,5	06,3	40
40,6	05,7	40,6	06,0	40,5	06,4	41
41,6	05,8	41,5	06,2	41,5	06,6	42
42,6	06,0	42,5	06,3	42,5	06,7	43
43,6	06,1	43,5	06,5	43,5	06,9	44
44,6	06,3	44,5	06,6	44,4	07,0	45
45,5	06,4	45,5	06,7	45,4	07,2	46
46,5	06,5	46,5	06,9	46,4	07,3	47
47,5	06,7	47,5	07,0	47,4	07,5	48
48,5	06,8	48,5	07,2	48,4	07,7	49
49,5	07,0	49,5	07,3	49,4	07,8	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
82 Degrees.		7 $\frac{1}{4}$ Points		81 Degrees		

Diff.	$\frac{1}{2}$ Point		6 Degrees		7 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	50,7	05,0	50,7	05,3	50,6	06,2
52	51,7	05,1	51,7	05,4	51,6	06,3
53	52,7	05,2	52,7	05,5	52,6	06,5
54	53,7	05,3	53,7	05,6	53,6	06,6
55	54,7	05,4	54,7	05,7	54,6	06,7
56	55,7	05,5	55,7	05,8	55,6	06,8
57	56,7	05,6	56,7	06,0	56,6	06,9
58	57,7	05,7	57,7	06,1	57,6	07,1
59	58,7	05,8	58,7	06,2	58,6	07,2
60	59,7	05,9	59,7	06,3	59,5	07,3
61	60,7	06,0	60,7	06,4	60,5	07,4
62	61,7	06,1	61,6	06,5	61,5	07,6
63	62,7	06,2	62,6	06,6	62,5	07,7
64	63,7	06,3	63,6	06,7	63,5	07,8
65	64,7	06,4	64,6	06,8	64,5	07,9
66	65,7	06,5	65,6	06,9	65,5	08,0
67	66,7	06,6	66,6	07,0	66,5	08,2
68	67,7	06,7	67,6	07,1	67,5	08,3
69	68,7	06,8	68,6	07,2	68,5	08,4
70	69,7	06,9	69,6	07,3	69,5	08,5
71	70,6	07,0	70,6	07,4	70,5	08,6
72	71,6	07,1	71,6	07,5	71,5	08,8
73	72,6	07,1	72,6	07,6	72,4	08,9
74	73,6	07,2	73,6	07,7	73,4	09,0
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	7 $\frac{1}{2}$ Points		84 Degrees		83 Degrees	

8 Degrees		$\frac{3}{4}$ Point		9 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
50,5	07,1	50,4	07,5	50,4	08,0	51
51,5	07,2	51,4	07,6	51,4	08,1	52
52,5	07,4	52,4	07,8	52,3	08,3	53
53,5	07,5	53,4	07,9	53,3	08,4	54
54,5	07,6	54,4	08,1	54,3	08,6	55
55,5	07,8	55,4	08,2	55,3	08,7	56
56,4	07,9	56,4	08,4	56,3	08,9	57
57,4	08,1	57,4	08,5	57,3	09,1	58
58,4	08,2	58,4	08,7	58,3	09,2	59
59,4	08,3	59,3	08,8	59,3	09,4	60
60,4	08,5	60,3	08,9	60,2	09,5	61
61,4	08,6	61,3	09,1	61,2	09,7	62
62,4	08,8	62,3	09,2	62,2	09,9	63
63,4	08,9	63,3	09,4	63,2	10,0	64
64,4	09,1	64,3	09,5	64,2	10,2	65
65,4	09,2	65,3	09,7	65,2	10,3	66
66,3	09,3	66,3	09,8	66,2	10,5	67
67,3	09,5	67,3	10,0	67,2	10,6	68
68,3	09,6	68,2	10,1	68,1	10,8	69
69,3	09,7	69,2	10,3	69,1	10,9	70
70,3	09,9	70,2	10,4	70,1	11,1	71
71,3	10,0	71,2	10,6	71,1	11,3	72
72,3	10,2	72,2	10,7	72,1	11,4	73
73,3	10,3	73,2	10,9	73,1	11,6	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
82 Degrees		$7\frac{1}{4}$ Points		81 Degrees		

148 *A Table of Difference of*

Dif.	$\frac{1}{2}$ Point		6 Degrees		7 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	74,6	07,3	74,6	07,8	74,4	09,1
76	75,6	07,4	75,6	07,9	75,4	09,3
77	76,6	07,5	76,6	08,0	76,4	09,4
78	77,6	07,6	77,6	08,1	77,4	09,5
79	78,6	07,7	78,6	08,3	78,4	09,6
80	79,6	07,8	79,6	08,4	79,4	09,8
81	80,6	07,9	80,5	08,5	80,4	09,9
82	81,6	08,0	81,5	08,6	81,4	10,0
83	82,6	08,1	82,5	08,7	82,4	10,1
84	83,6	08,2	83,5	08,8	83,4	10,2
85	84,6	08,3	84,5	08,9	84,4	10,3
86	85,6	08,4	85,5	09,0	85,4	10,4
87	86,6	08,5	86,5	09,1	86,3	10,5
88	87,6	08,6	87,5	09,2	87,3	10,7
89	88,6	08,7	88,5	09,3	88,3	10,9
90	89,6	08,8	89,5	09,4	89,3	11,0
91	90,6	08,9	90,5	09,5	90,3	11,1
92	91,6	09,0	91,5	09,6	91,3	11,2
93	92,6	09,1	92,5	09,7	92,3	11,3
94	93,5	09,2	93,5	09,8	93,3	11,5
95	94,5	09,3	94,5	09,9	94,3	11,6
96	95,5	09,4	95,5	10,0	95,3	11,7
97	96,5	09,5	96,5	10,1	96,3	11,8
98	97,5	09,6	97,5	10,2	97,3	12,0
99	98,5	09,7	98,5	10,3	98,3	12,1
100	99,5	09,8	99,4	10,4	99,2	12,2
Dif.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	7 $\frac{1}{2}$ Point		84 Degrees		83 Degrees	

Latitude and Departure. 149

8 Degrees		$\frac{3}{4}$ Point		9 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
74,3	10,4	74,2	11,0	74,1	11,7	75
75,3	10,6	75,2	11,1	75,1	11,9	76
76,2	10,7	76,2	11,3	76,0	12,0	77
77,2	10,9	77,1	11,4	77,0	12,2	78
78,2	11,0	78,1	11,6	78,0	12,4	79
79,2	11,1	79,1	11,7	79,0	12,5	80
80,2	11,3	80,1	11,9	80,0	12,7	81
81,2	11,4	81,1	12,0	81,0	12,8	82
82,2	11,5	82,1	12,2	82,0	13,0	83
83,2	11,7	83,1	12,3	83,0	13,1	84
84,2	11,8	84,1	12,5	83,9	13,3	85
85,2	12,0	85,1	12,6	84,9	13,4	86
86,1	12,1	86,0	12,8	85,9	13,6	87
87,2	12,2	87,0	12,9	86,9	13,8	88
88,1	12,4	88,0	13,1	87,9	13,9	89
89,1	12,5	89,0	13,2	88,9	14,1	90
90,1	12,7	90,0	13,4	89,9	14,2	91
91,1	12,8	91,0	13,5	90,9	14,4	92
92,1	12,9	92,0	13,6	91,8	14,5	93
93,1	13,1	93,0	13,8	92,8	14,7	94
94,1	13,2	94,0	13,9	93,8	14,9	95
95,1	13,4	95,0	14,1	94,8	15,0	96
96,0	13,5	95,9	14,2	95,8	15,2	97
97,0	13,6	96,9	14,4	96,8	15,3	98
98,0	13,8	97,9	14,5	97,8	15,5	99
99,0	13,9	98,9	14,7	98,8	15,6	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
82 Degrees		7 $\frac{1}{4}$ Points		81 Degrees		

Diff.	10 Degrees		11 Degrees		1 Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	01,0	00,2	01,0	00,2	01,0	00,2
2	02,0	00,3	02,0	00,4	02,0	00,4
3	02,9	00,5	02,9	00,6	02,9	00,6
4	03,9	00,7	03,9	00,8	03,9	00,8
5	04,9	00,9	04,9	00,9	04,9	01,0
6	05,9	01,0	05,9	01,1	05,9	01,2
7	06,9	01,2	06,9	01,3	06,9	01,4
8	07,9	01,4	07,8	01,5	07,8	01,6
9	08,9	01,6	08,8	01,7	08,8	01,8
10	09,8	01,7	09,8	01,9	09,8	01,9
11	10,8	01,9	10,8	02,1	10,8	02,1
12	11,8	02,1	11,8	02,3	11,8	02,3
13	12,8	02,3	12,8	02,5	12,7	02,5
14	13,8	02,4	13,7	02,7	13,7	02,7
15	14,8	02,6	14,7	02,9	14,7	02,9
16	15,7	02,8	15,7	03,0	15,7	03,1
17	16,7	02,9	16,7	03,2	16,7	03,3
18	17,7	03,1	17,7	03,4	17,7	03,6
19	18,7	03,3	18,6	03,6	18,6	03,7
20	19,7	03,5	19,6	03,8	19,6	03,9
21	20,7	03,6	20,6	04,0	20,6	04,1
22	21,7	03,8	21,6	04,2	21,6	04,3
23	22,6	04,0	22,6	04,4	22,6	04,5
24	23,6	04,2	23,6	04,6	23,5	04,7
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	80 Degrees		79 Degrees		7 Points	

12 Degrees		13 Degrees		14 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
01,0	00,2	01,0	00,2	01,0	00,2	1
02,0	00,4	01,9	00,4	01,9	00,5	2
02,9	00,6	02,9	00,7	02,9	00,7	3
03,9	00,8	03,9	00,9	03,9	01,0	4
04,9	01,0	04,9	01,1	04,8	01,2	5
05,9	01,2	05,8	01,3	05,8	01,4	6
06,8	01,5	06,8	01,6	06,8	01,7	7
07,8	01,7	07,8	01,8	07,8	01,9	8
08,8	01,9	08,8	02,0	08,7	02,2	9
09,8	02,1	09,7	02,2	09,7	02,4	10
10,8	02,3	10,7	02,5	10,7	02,7	11
11,7	02,5	11,7	02,7	11,6	02,9	12
12,7	02,7	12,7	02,9	12,6	03,1	13
13,7	02,9	13,6	03,1	13,6	03,4	14
14,7	03,1	14,6	03,4	14,5	03,6	15
15,6	03,3	15,6	03,6	15,5	03,9	16
16,6	03,5	16,6	03,8	16,5	04,1	17
17,6	03,7	17,5	04,0	17,5	04,4	18
18,6	03,9	18,5	04,2	18,4	04,6	19
19,6	04,2	19,5	04,5	19,4	04,8	20
20,5	04,4	20,5	04,7	20,4	05,1	21
21,5	04,6	21,4	04,9	21,3	05,3	22
22,5	04,8	22,4	05,2	22,3	05,6	23
23,5	05,0	23,4	05,4	23,3	05,8	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
78 Degrees		77 Degrees		76 Degrees		

Dif.	10 Degrees		11 Degrees		1 Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	24.6	04.3	24.5	04.8	24.5	04.9
26	25.6	04.5	25.5	05.0	25.5	05.1
27	26.6	04.7	26.5	05.1	26.5	05.3
28	27.6	05.9	27.5	05.3	27.5	05.5
29	28.6	05.0	28.5	05.5	28.4	05.7
30	29.5	05.2	29.4	05.7	29.4	05.8
31	30.5	05.4	30.4	05.9	30.4	06.0
32	31.5	05.5	31.4	06.1	31.4	06.2
33	32.5	05.7	32.4	06.3	32.4	06.4
34	33.5	05.9	33.4	06.5	33.3	06.6
35	34.5	06.1	34.4	06.7	34.3	06.8
36	35.4	06.2	35.3	06.9	35.3	07.0
37	36.4	06.4	36.3	07.1	36.3	07.2
38	37.4	06.6	37.3	07.2	37.3	07.4
39	38.4	06.8	38.3	07.4	38.2	07.6
40	39.4	06.9	39.3	07.6	39.2	07.8
41	40.4	07.1	40.2	07.8	40.2	08.0
42	41.4	07.3	41.2	08.0	41.2	08.2
43	42.3	07.5	42.2	08.2	42.2	08.4
44	43.3	07.7	43.2	08.4	43.1	08.6
45	44.3	07.8	44.2	08.6	44.1	08.8
46	45.3	08.0	45.2	08.8	45.1	09.0
47	46.3	08.1	46.1	09.0	46.1	09.2
48	47.3	08.3	47.1	09.2	47.1	09.4
49	48.3	08.5	48.1	09.3	48.1	09.6
50	49.2	08.7	49.1	09.5	49.0	09.8
Dif.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	80 Degrees		79 Degrees		7 Point	

12 Degrees		13 Degrees		14 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
24,4	05,2	24,3	05,6	24,3	06,0	25
25,4	05,4	25,3	05,8	25,2	06,3	26
26,4	05,6	26,3	06,1	26,2	06,5	27
27,4	05,8	27,3	06,3	27,2	06,8	28
28,4	06,0	28,2	06,5	28,1	07,0	29
29,3	06,2	29,2	06,7	29,1	07,3	30
30,3	06,4	30,2	07,0	30,1	07,5	31
31,3	06,6	31,2	07,2	31,0	07,7	32
32,3	06,9	32,1	07,4	32,0	08,0	33
33,2	07,1	33,1	07,6	33,0	08,2	34
34,2	07,3	34,1	07,9	34,0	08,5	35
35,2	07,5	35,1	08,1	34,9	08,7	36
36,2	07,7	36,0	08,3	35,9	09,0	37
37,2	07,9	37,0	08,5	36,9	09,2	38
38,1	08,1	38,0	08,8	37,8	09,4	39
39,1	08,3	39,0	09,0	38,8	09,7	40
40,1	08,5	39,9	09,2	39,8	09,9	41
41,1	08,7	40,9	09,4	40,7	10,2	42
42,1	08,9	41,9	09,7	41,7	10,4	43
43,0	09,1	42,9	09,9	42,7	10,6	44
44,0	09,4	43,8	10,1	43,7	11,0	45
45,0	09,6	44,8	10,3	44,6	11,1	46
46,0	09,8	45,8	10,6	45,6	11,4	47
47,0	10,0	46,8	10,8	46,6	11,6	48
47,9	10,2	47,7	11,0	47,5	11,9	49
48,9	10,4	48,7	11,2	48,5	12,1	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
78 Degrees		77 Degrees		76 Degrees		

154 *A Table of Difference of*

Diff.	10 Degrees		11 Degrees		1 Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	50.2	08.8	50.1	09.7	50.0	10.0
52	51.2	09.0	51.0	09.9	51.0	10.1
53	52.2	09.2	52.0	10.1	52.0	10.3
54	53.2	09.4	53.0	10.3	53.0	10.5
55	54.2	09.5	54.0	10.5	53.9	10.7
56	55.1	09.7	55.0	10.7	54.9	10.9
57	56.1	09.9	56.0	10.8	55.9	11.1
58	57.1	10.1	56.9	11.1	56.9	11.3
59	58.1	10.2	57.9	11.3	57.9	11.5
60	59.1	10.4	58.9	11.4	58.8	11.7
61	60.1	10.6	59.9	11.6	59.8	11.9
62	61.1	10.8	60.9	11.8	60.8	12.1
63	62.0	10.9	61.8	12.0	61.8	12.3
64	63.0	10.1	62.8	12.2	62.8	12.5
65	64.0	11.3	63.8	12.4	63.7	12.7
66	65.0	11.5	64.8	12.6	64.7	12.9
67	66.0	11.6	65.8	12.8	65.7	13.1
68	67.0	11.8	66.7	13.0	66.7	13.3
69	68.0	12.0	67.7	13.2	67.7	13.5
70	68.9	12.2	68.7	13.4	68.7	13.7
71	69.9	12.3	69.7	13.5	69.6	13.9
72	70.9	12.5	70.7	13.7	70.6	14.0
73	71.9	12.7	71.7	13.9	71.6	14.2
74	72.9	12.8	72.6	14.1	72.6	14.4
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	80 Degrees		79 Degrees		7 Points	

Latitude and Departure. 155

12 Degrees		13 Degrees		14 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
49.9	10.6	49.7	11.5	49.5	12.3	51
50.9	10.8	50.7	11.7	50.5	12.6	52
51.8	11.0	51.6	11.9	51.4	12.8	53
52.8	11.2	52.6	12.1	52.4	13.1	54
53.8	11.4	53.6	12.4	53.4	13.3	55
54.8	11.6	54.6	12.6	54.3	13.5	56
55.8	11.8	55.5	12.8	55.3	13.8	57
56.7	12.1	56.5	13.0	56.3	14.0	58
57.7	12.3	57.5	13.3	57.2	14.3	59
58.7	12.5	58.5	13.5	58.2	14.5	60
59.7	12.7	59.4	13.7	59.2	14.8	61
60.6	12.9	60.4	13.9	60.2	15.0	62
61.6	13.1	61.4	14.2	61.1	15.2	63
62.6	13.3	62.3	14.4	62.1	15.5	64
63.6	13.5	63.3	14.6	63.1	15.7	65
64.6	13.7	64.3	14.8	64.0	16.0	66
65.5	13.9	65.3	15.1	65.0	16.2	67
66.5	14.1	66.2	15.3	66.0	16.4	68
67.5	14.3	67.2	15.5	66.9	16.7	69
68.5	14.5	68.2	15.7	67.9	16.9	70
69.4	14.8	69.2	16.0	68.9	17.2	71
70.4	15.0	70.1	16.2	69.9	17.4	72
71.4	15.2	71.1	16.4	70.8	17.6	73
72.4	15.4	72.1	16.7	71.8	17.9	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
78 Degrees		77 Degrees		76 Degrees		

156 *A Table of Difference of*

Diff.	10 Degrees		11 Degrees		1 Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	73.9	13.0	73.6	14.3	73.6	14.6
76	74.8	13.2	74.6	14.5	74.5	14.8
77	75.8	13.4	75.6	14.7	75.5	15.0
78	76.8	13.5	76.6	14.9	76.5	15.2
79	77.8	13.7	77.5	15.1	77.5	15.4
80	78.8	13.9	78.5	15.3	78.5	15.6
81	79.8	14.1	79.5	15.5	79.4	15.8
82	80.8	14.2	80.5	15.6	80.4	16.0
83	81.7	14.4	81.5	15.8	81.4	16.2
84	82.7	14.6	82.5	16.0	82.4	16.4
85	83.7	14.8	83.4	16.2	83.4	16.6
86	84.7	14.9	84.4	16.4	84.3	16.8
87	85.7	15.1	85.4	16.6	85.3	17.0
88	86.7	15.3	86.4	16.8	86.3	17.2
89	87.6	15.4	87.4	17.0	87.3	17.4
90	88.6	15.6	88.3	17.2	88.3	17.6
91	89.6	15.8	89.3	17.4	89.2	17.8
92	90.6	16.0	90.3	17.6	90.2	17.9
93	91.6	16.1	91.3	17.7	91.2	18.1
94	92.6	16.3	92.3	17.9	92.2	18.3
95	93.5	16.5	93.3	18.1	93.2	18.5
96	94.5	16.7	94.2	18.3	94.2	18.7
97	95.5	16.8	95.2	18.5	95.1	18.9
98	96.5	17.0	96.2	18.7	96.1	19.1
99	97.5	17.2	97.2	18.9	97.1	19.3
100	98.5	17.4	98.2	19.1	98.1	19.5
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	80 Degrees		79 Degrees		7 Points	

Latitude and Departure. 157

12 Degrees		13 Degrees		14 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
73.4	15.6	73.1	16.9	72.8	18.1	75
74.3	15.8	74.0	17.1	73.7	18.4	76
75.3	16.0	75.0	17.3	74.7	18.6	77
76.3	16.2	76.0	17.5	75.7	18.9	78
77.3	16.4	77.0	17.8	76.6	19.1	79
78.2	16.6	77.9	18.6	77.6	19.3	80
79.2	16.8	78.9	18.2	78.6	19.4	81
80.2	17.0	79.9	18.4	79.6	19.8	82
81.2	17.2	80.9	18.7	80.5	20.1	83
82.2	17.5	81.8	18.9	81.5	20.3	84
83.1	17.7	82.8	19.1	82.5	20.6	85
84.1	17.9	83.8	19.3	83.4	20.8	86
85.1	18.1	84.8	19.6	84.4	21.0	87
86.1	18.3	85.7	19.8	85.4	21.3	88
87.1	18.5	86.7	20.0	86.4	21.5	89
88.0	18.7	87.7	20.2	87.3	21.8	90
89.0	18.9	88.7	20.5	88.3	22.0	91
90.0	19.1	89.6	20.7	89.3	22.2	92
91.0	19.3	90.6	20.9	90.2	22.5	93
91.9	19.5	91.6	21.1	91.2	22.7	94
92.9	19.7	92.6	21.4	92.2	23.0	95
93.9	20.0	93.5	21.6	93.1	23.2	96
94.9	20.2	94.5	21.8	94.1	23.5	97
95.9	20.4	95.5	22.0	95.1	23.7	98
96.8	20.6	96.5	22.3	96.1	23.9	99
97.8	20.8	97.4	22.5	97.0	24.2	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
78 Degrees		77 Degrees		76 Degrees		

158 *A Table of Difference of*

Dif.	1 $\frac{1}{4}$ Point		15 Degrees		16 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	01,0	00,2	01,0	00,3	01,0	00,3
2	01,9	00,5	01,9	00,5	01,9	00,5
3	02,9	00,7	02,9	00,8	02,9	00,8
4	03,9	01,0	03,9	01,0	03,8	01,1
5	04,8	01,2	04,8	01,3	04,8	01,4
6	05,8	01,5	05,8	01,5	05,7	01,6
7	06,8	01,7	06,8	01,8	06,7	01,9
8	07,8	01,9	07,7	02,1	07,7	02,2
9	08,7	02,2	08,7	02,3	08,6	02,5
10	09,7	02,4	09,7	02,6	09,6	02,8
11	10,7	02,8	10,6	02,8	10,6	03,0
12	11,6	02,9	11,6	03,1	11,5	03,3
13	12,6	03,2	12,6	03,4	12,5	03,6
14	13,6	03,4	13,5	03,6	13,5	03,9
15	14,5	03,6	14,5	03,9	14,4	04,1
16	15,5	04,0	15,5	04,1	15,4	04,4
17	16,5	04,1	16,4	04,4	16,3	04,7
18	17,5	04,4	17,4	04,7	17,3	05,0
19	18,4	04,6	18,4	04,9	18,3	05,2
20	19,4	04,9	19,3	05,2	19,2	05,5
21	20,4	05,1	20,3	05,4	20,2	05,8
22	21,3	05,3	21,2	05,7	21,1	06,1
23	22,3	05,6	22,2	06,0	22,1	06,3
24	23,3	05,8	23,2	06,2	23,0	06,6
Dif.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	6 $\frac{3}{4}$ Point		75 Degrees		74 Degrees	

Latitude and Departure. 159

$\frac{1}{2}$ Point		17 Degrees		18 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
01,0	00,3	01,0	00,3	00,9	00,3	1
01,9	00,6	01,9	00,6	01,9	00,6	2
02,9	00,9	02,9	00,9	02,8	00,9	3
03,8	01,2	03,8	01,2	03,8	01,2	4
04,8	01,5	04,8	01,5	04,8	01,5	5
05,8	01,7	05,7	01,7	05,7	01,8	6
06,7	02,0	06,7	02,0	06,7	02,2	7
07,7	02,3	07,6	02,3	07,6	02,5	8
08,6	02,6	08,6	02,0	08,6	02,8	9
09,6	02,9	09,6	02,9	09,5	03,1	10
10,5	03,2	10,5	03,2	10,5	03,4	11
11,5	03,5	11,5	03,5	11,4	03,7	12
12,4	03,8	12,4	03,8	12,4	04,0	13
13,4	04,1	13,4	04,1	13,3	04,3	14
14,4	04,4	14,3	04,4	14,3	04,6	15
15,3	04,6	15,3	04,7	15,2	04,9	16
16,3	04,9	16,3	05,0	16,2	05,2	17
17,2	05,2	17,2	05,3	17,1	05,6	18
18,2	05,5	18,2	05,5	18,1	05,9	19
19,1	05,8	19,1	05,8	19,0	06,2	20
20,1	06,1	20,1	06,1	20,0	06,5	21
21,0	06,4	21,0	06,4	20,9	06,8	22
22,0	06,7	22,0	06,7	21,9	07,1	23
23,0	06,8	22,9	07,0	22,8	07,4	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
6 $\frac{1}{2}$ Points		73 Degrees		72 Degrees		

160 *A Table of Difference of*

Dif.	1 $\frac{1}{4}$ Points		15 Degrees		16 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	24.2	06.0	24.1	06.5	24.0	06.9
26	25.2	06.3	25.1	06.7	24.9	07.2
27	26.2	06.6	26.1	07.0	25.9	07.4
28	27.2	06.8	27.0	07.2	26.9	07.7
29	28.1	07.0	28.0	07.5	27.8	08.0
30	29.1	07.3	29.0	07.8	28.8	08.3
31	30.1	07.5	29.9	08.0	29.8	08.5
32	31.0	07.7	30.9	08.3	30.7	08.8
33	32.0	08.0	31.9	08.5	31.7	09.1
34	33.0	08.3	32.8	08.8	32.7	09.4
35	33.9	08.5	33.8	09.0	33.6	09.6
36	34.9	08.7	34.8	09.3	34.6	09.9
37	35.9	09.0	35.7	09.6	35.6	10.2
38	36.9	09.2	36.7	09.8	36.5	10.5
39	37.8	09.5	37.7	10.1	37.5	10.7
40	38.8	09.7	38.6	10.3	38.4	11.0
41	39.8	10.0	39.6	10.6	39.4	11.3
42	40.7	10.2	40.6	10.9	40.4	11.6
43	41.7	10.4	41.5	11.1	41.3	11.8
44	42.7	10.7	42.5	11.3	42.3	12.1
45	43.6	10.9	43.5	11.6	43.2	12.4
46	44.6	11.2	44.4	11.9	44.2	12.7
47	45.6	11.4	45.4	12.2	45.2	12.9
48	46.6	11.7	46.4	12.4	46.1	13.2
49	47.5	11.9	47.3	12.7	47.1	13.5
50	48.5	12.1	48.3	12.9	48.1	13.8
Dif.	6 $\frac{3}{4}$ Points		75 Degrees		74 Degrees	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.

1 $\frac{1}{2}$ Point		17 Degrees		18 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
23,9	07,3	23,9	07,3	23,8	07,7	25
24,9	07,5	24,9	07,6	24,7	08,0	26
25,8	07,8	25,8	07,9	25,7	08,3	27
26,8	08,1	26,8	08,2	26,6	08,6	28
27,8	08,4	27,7	08,5	27,6	09,0	29
28,7	08,7	28,7	08,8	28,5	09,3	30
29,7	09,0	29,6	09,1	29,5	09,6	31
30,6	09,3	30,6	09,3	30,4	10,0	32
31,6	09,6	31,6	09,6	31,4	10,2	33
32,5	09,9	32,5	09,9	32,3	10,5	34
33,5	10,2	33,5	10,2	33,3	10,8	35
34,4	10,4	34,4	10,5	34,2	11,1	36
35,4	10,7	35,4	10,8	35,2	11,4	37
36,4	11,0	36,3	11,1	36,1	11,7	38
37,3	11,3	37,3	11,4	37,1	12,0	39
38,3	11,6	38,2	11,7	38,0	12,4	40
39,2	11,9	39,2	12,0	39,0	12,7	41
40,2	12,2	40,2	12,3	39,9	13,0	42
41,1	12,5	41,1	12,6	40,9	13,3	43
42,1	12,8	42,1	12,9	41,8	13,6	44
43,1	13,1	43,0	13,1	42,8	13,9	45
44,0	13,3	44,0	13,4	43,7	14,2	46
45,0	13,6	44,9	13,7	44,7	14,5	47
45,9	13,9	45,9	14,0	45,6	14,8	48
46,9	14,2	46,9	14,3	46,6	15,1	49
47,8	14,5	47,8	14,6	47,5	15,4	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
6 $\frac{1}{2}$ Points		73 Degrees.		72 Degrees		

Diff.	1 $\frac{1}{4}$ Point		15 Degrees		16 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	49,5	12,4	49,3	13,2	49,0	14,0
52	50,4	12,6	50,2	13,5	49,9	14,3
53	51,4	12,9	51,2	13,7	50,9	14,6
54	52,4	13,1	52,2	14,0	51,9	14,9
55	53,3	13,4	53,1	14,2	52,9	15,2
56	54,3	13,6	54,1	14,5	53,8	15,4
57	55,3	13,8	55,1	14,8	54,8	15,7
58	56,3	14,1	56,0	15,0	55,7	16,0
59	57,2	14,3	57,0	15,3	56,7	16,3
60	58,2	14,6	58,0	15,5	57,7	16,5
61	59,2	14,8	58,9	15,8	58,6	16,8
62	60,1	15,1	59,9	16,1	59,6	17,1
63	61,1	15,3	60,8	16,3	60,5	17,4
64	62,1	15,5	61,8	16,6	61,5	17,6
65	63,0	15,8	62,8	16,8	62,5	17,9
66	64,0	16,0	63,7	17,1	63,4	18,2
67	65,0	16,3	64,7	17,4	64,4	18,5
68	66,0	16,5	65,7	17,6	65,4	18,7
69	66,9	16,8	66,6	17,9	66,3	19,0
70	67,9	17,0	67,6	18,1	67,3	19,3
71	68,9	17,2	68,5	18,3	68,2	19,6
72	69,8	17,5	69,5	18,6	69,2	19,8
73	70,8	17,7	70,5	18,9	70,2	20,1
74	71,8	18,0	71,5	19,1	71,1	20,4
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	6 $\frac{3}{4}$ Points		75 Degrees		74 Degrees	

Latitude and Departure. 163

1 $\frac{1}{2}$ Point		17 Degrees		18 Degrees		Dif.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
48,8	14,8	48,8	14,9	48,5	15,8	51
49,7	15,1	49,7	15,2	49,4	16,1	52
50,7	15,3	50,7	15,5	50,4	16,4	53
51,7	15,7	51,6	15,8	51,3	16,7	54
52,6	16,0	52,6	16,1	52,3	17,0	55
53,6	16,2	53,5	16,4	53,3	17,3	56
54,5	16,5	54,5	16,7	54,2	17,6	57
55,5	16,8	55,5	17,0	55,2	17,9	58
56,5	17,1	56,4	17,2	56,1	18,2	59
57,4	17,4	57,4	17,5	57,1	18,5	60
58,4	17,7	58,3	17,8	58,0	18,8	61
59,3	18,0	59,3	18,1	59,0	19,2	62
60,3	18,3	60,2	18,4	59,9	19,5	63
61,2	18,6	61,2	18,7	60,9	19,8	64
62,2	18,9	62,2	19,0	61,8	20,1	65
63,2	19,2	63,1	19,3	62,8	20,4	66
64,1	19,4	64,1	19,6	63,7	20,7	67
65,1	19,7	65,0	19,9	64,7	21,0	68
66,0	20,0	66,0	20,2	65,6	21,3	69
67,0	20,3	66,9	20,5	66,6	21,6	70
67,9	20,6	67,9	20,8	67,5	21,9	71
68,9	20,9	68,8	21,0	68,5	22,2	72
69,8	21,2	69,8	21,3	69,4	22,6	73
70,8	21,5	70,8	21,6	70,4	22,9	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dif.
6 $\frac{1}{2}$ Points		73 Degrees		72 Degrees		

Diff.	1 $\frac{1}{4}$ Point		1 5 Degrees		1 6 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	72,7	18,2	72,4	19,4	72,1	20,7
76	73,7	18,5	73,4	19,7	73,0	20,9
77	74,7	18,7	74,4	19,9	74,0	21,2
78	75,7	18,9	75,3	20,2	75,0	21,5
79	76,6	19,2	76,3	20,4	75,9	21,8
80	77,6	19,4	77,3	20,7	76,9	22,0
81	78,6	19,7	78,2	21,0	77,9	22,3
82	79,5	19,9	79,2	21,2	78,8	22,6
83	80,5	20,3	80,2	21,5	79,8	22,9
84	81,5	20,4	81,1	21,7	80,8	23,1
85	82,4	20,7	82,1	22,0	81,7	23,4
86	83,4	20,9	83,1	22,3	82,7	23,7
87	84,4	21,1	84,0	22,5	83,6	24,0
88	85,4	21,4	85,0	22,8	84,6	24,2
89	86,3	21,6	86,0	23,0	85,6	24,5
90	87,3	21,9	86,9	23,3	86,5	24,8
91	88,3	22,1	87,9	23,5	87,5	25,1
92	89,2	22,4	88,9	23,8	88,4	25,3
93	90,2	22,6	89,8	24,1	89,4	25,6
94	91,2	22,8	90,8	24,3	90,4	25,9
95	92,1	23,1	91,8	24,6	91,3	26,2
96	93,1	23,3	92,7	24,8	92,3	26,4
97	94,1	23,6	93,7	25,1	93,2	26,7
98	95,1	23,8	94,7	25,4	94,2	27,0
99	96,0	24,1	95,6	25,6	95,2	27,3
100	97,0	24,3	96,6	25,9	96,1	27,6
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	6 $\frac{3}{4}$ Point		7 5 Degrees		7 4 Degrees	

Latitude and Departure. 165

1 Point		17 Degrees		18 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
71,8	21,8	71,7	21,9	71,3	23,2	75
72,7	22,1	72,7	22,2	72,3	23,5	76
73,7	22,3	73,6	22,5	73,2	23,8	77
74,6	22,6	74,6	22,8	74,2	24,1	78
75,6	22,9	75,5	23,1	75,1	24,4	79
76,6	23,2	76,5	23,4	76,1	24,7	80
77,5	23,5	77,5	23,7	77,0	25,0	81
78,5	23,8	78,4	24,0	78,0	25,3	82
79,4	24,1	79,4	24,3	78,9	25,6	83
80,4	24,4	80,3	24,5	79,9	26,0	84
81,3	24,7	81,3	24,8	80,8	26,3	85
82,3	25,0	82,2	25,1	81,8	26,6	86
83,3	25,2	83,2	25,4	82,7	26,9	87
84,2	25,5	84,1	25,7	83,7	27,2	88
85,2	25,8	85,1	26,0	84,6	27,5	89
86,1	26,1	86,1	26,3	85,6	27,8	90
87,1	26,4	87,0	26,6	86,5	28,1	91
88,0	26,7	88,0	26,9	87,5	28,4	92
89,0	27,0	88,9	27,2	88,4	28,7	93
90,0	27,3	99,9	27,5	89,4	29,0	94
90,9	27,6	90,8	27,8	90,3	29,3	95
91,9	27,9	91,8	28,1	91,3	29,7	96
92,8	28,2	92,8	28,4	92,3	30,0	97
93,8	28,4	93,7	28,6	93,2	30,3	98
94,7	28,7	94,7	28,9	94,2	30,6	99
95,7	29,0	95,6	29,2	95,1	30,9	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
6 $\frac{1}{2}$ Points		73 Degrees		72 Degrees		

166 *A Table of Difference of*

Diff.	19 Degrees		1 $\frac{3}{4}$ Point		20 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	00,9	00,3	00,9	00,3	00,9	00,3
2	01,9	00,0	01,9	00,7	01,9	00,7
3	02,8	01,0	02,8	01,0	02,8	01,0
4	03,8	01,3	03,8	01,3	03,8	01,4
5	04,7	01,6	04,7	01,7	04,7	01,7
6	05,7	01,9	05,6	02,0	05,6	02,0
7	06,6	02,3	06,6	02,4	06,6	02,4
8	07,6	02,6	07,5	02,7	07,5	02,7
9	08,5	02,9	08,5	03,0	08,5	03,1
10	09,5	03,3	09,4	03,4	09,4	03,4
11	10,4	03,6	10,4	03,7	10,3	03,8
12	11,3	03,9	11,3	04,0	11,3	04,1
13	12,3	04,2	12,2	04,4	12,2	04,4
14	13,2	04,6	13,2	04,7	13,2	04,8
15	14,2	04,9	14,1	05,1	14,1	05,1
16	15,1	05,2	15,1	05,4	15,0	05,4
17	16,1	05,5	16,0	05,7	16,0	05,8
18	17,0	05,9	16,9	06,1	16,9	06,3
19	18,0	06,1	17,9	06,4	17,9	06,5
20	18,9	06,5	18,8	06,7	18,8	06,8
21	19,9	06,8	19,8	07,1	19,7	07,2
22	20,8	07,2	20,7	07,4	20,7	07,5
23	21,7	07,5	21,7	07,7	21,6	07,9
24	22,7	07,8	22,6	08,1	22,5	08,2
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	71 Degrees		6 $\frac{1}{4}$ Points		70 Degrees	

Latitude and Departure. 167

21 Degrees		22 Degrees		2 Points		Dif.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
00,9	00,4	00,9	00,4	00,9	00,4	1
01,9	00,7	01,8	00,7	01,8	00,8	2
02,8	01,1	02,8	01,1	02,8	01,1	3
03,7	01,4	04,7	01,5	03,7	01,5	4
04,7	01,7	05,6	01,9	04,6	01,9	5
05,6	02,1	06,5	02,2	05,5	02,3	6
06,5	02,5	06,5	02,6	06,5	02,7	7
07,5	02,9	07,4	03,0	07,4	03,1	8
08,4	03,2	08,3	03,4	08,3	03,4	9
09,3	03,6	09,3	03,7	09,2	03,8	10
10,3	03,9	10,2	04,1	10,2	04,2	11
11,2	04,3	11,1	04,5	11,1	04,6	12
12,1	04,7	12,0	04,9	12,0	05,0	13
13,1	05,0	13,0	05,2	12,9	05,4	14
14,0	05,4	13,9	05,6	13,9	05,7	15
14,9	05,7	14,8	06,0	14,8	06,1	16
15,9	06,1	15,8	06,4	15,7	06,5	17
16,8	06,4	16,7	06,7	16,6	06,8	18
17,7	06,8	17,6	07,1	17,6	07,3	19
18,7	07,2	18,5	07,5	18,5	07,6	20
19,6	07,5	19,5	07,9	19,4	08,0	21
20,5	07,9	20,4	08,2	20,3	08,4	22
21,5	08,2	21,3	08,6	21,2	08,8	23
22,4	08,6	22,2	09,0	22,2	09,2	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dif.
69 Degrees		68 Degrees		6 Points		

168 *A Table of Difference of*

Diff.	19 Degrees		1 $\frac{3}{4}$ Point		20 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	23.6	08.1	23.5	08.4	23.5	08.5
26	24.6	08.5	24.5	08.8	24.4	08.9
27	25.5	08.8	25.4	09.1	25.4	09.2
28	26.5	09.1	26.4	09.4	26.3	09.6
29	27.4	09.4	27.3	09.8	27.2	09.9
30	28.4	09.8	28.2	10.1	28.2	10.3
31	29.3	10.1	29.2	10.4	29.1	10.6
32	30.3	10.4	30.1	10.8	30.1	10.9
33	31.2	10.7	31.1	11.1	31.0	11.3
34	32.1	11.1	32.0	11.5	31.9	11.6
35	33.1	11.4	33.0	11.8	32.9	12.0
36	34.0	11.7	33.9	12.1	33.8	12.3
37	35.0	12.1	34.1	12.5	34.8	12.6
38	35.9	12.4	35.4	12.8	35.7	13.0
39	36.9	12.6	36.6	13.1	36.6	13.3
40	37.8	13.0	37.0	13.5	37.6	13.7
41	38.8	13.3	38.3	13.8	38.5	14.0
42	39.7	13.7	39.7	14.1	39.5	14.4
43	40.7	14.0	40.0	14.5	40.4	14.7
44	41.6	14.3	41.4	14.8	41.3	15.0
45	42.6	14.6	42.4	15.2	42.3	15.4
46	43.5	15.0	43.3	15.5	43.2	15.7
47	44.4	15.3	44.2	15.8	44.2	16.1
48	45.4	15.6	45.2	16.2	45.1	16.4
49	46.3	15.9	46.1	16.5	46.0	16.8
50	47.3	16.3	47.1	16.8	47.0	17.1
Diff.	71 Degrees		6 $\frac{1}{4}$ Point		70 Degrees	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.

Latitude and Departure. 169

21 Degrees		22 Degrees		2 Point		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
23,3	09,0	23,2	09,4	23,1	09,6	25
24,3	09,3	24,1	09,7	24,0	09,9	26
25,2	09,7	25,0	10,1	24,9	10,3	27
26,1	10,0	26,0	10,5	25,9	10,7	28
27,1	10,4	26,9	10,9	26,8	11,1	29
28,0	10,7	27,8	11,2	27,7	11,5	30
28,9	11,1	28,7	11,6	28,6	11,9	31
29,9	11,5	29,7	12,0	29,6	12,2	32
30,8	11,8	30,6	12,4	30,5	12,6	33
31,7	12,2	31,5	12,7	31,4	13,0	34
32,7	12,5	32,4	13,1	32,3	13,4	35
33,6	12,9	33,4	13,5	33,3	13,8	36
34,5	13,3	34,3	13,9	34,2	14,2	37
35,5	13,6	35,2	14,2	35,1	14,5	38
36,4	14,0	36,2	14,6	36,0	14,9	39
37,3	14,3	37,1	15,0	36,9	15,3	40
38,3	14,7	38,0	15,3	37,9	15,7	41
39,2	15,1	38,9	15,7	38,8	16,1	42
40,1	15,4	39,9	16,1	39,7	16,5	43
41,1	15,8	40,8	16,5	40,6	16,8	44
42,0	16,1	41,7	16,8	41,6	17,2	45
42,9	16,5	42,6	17,2	42,5	17,6	46
43,9	16,8	43,6	17,6	43,4	18,0	47
44,8	17,2	44,5	18,0	44,3	18,4	48
45,7	17,6	45,4	18,3	45,3	18,7	49
46,7	17,9	46,4	18,7	46,2	19,1	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
69 Degrees		68 Degrees		6 Point		

170 *A Table of Difference of*

Diff.	19 Degrees		1 $\frac{3}{4}$ Point		20 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	48.2	16.6	48.0	17.2	47.9	17.4
52	49.2	16.9	49.0	17.5	48.9	17.8
53	50.1	17.3	49.9	17.9	49.8	18.1
54	51.1	17.6	50.8	18.2	50.7	18.5
55	52.0	17.9	51.8	18.5	51.7	18.8
56	52.9	18.2	52.7	18.9	52.6	19.2
57	53.9	18.6	53.7	19.2	53.6	19.5
58	54.8	18.9	54.6	19.5	54.5	19.8
59	55.8	19.2	55.5	19.9	55.4	20.2
60	56.7	19.5	56.5	20.2	56.4	20.5
61	57.7	19.9	57.4	20.5	57.3	20.9
62	58.6	20.2	58.4	20.9	58.3	21.2
63	59.6	20.5	59.3	21.2	59.2	21.5
64	60.5	20.8	60.3	21.6	60.1	21.9
65	61.5	21.2	61.2	21.9	61.1	22.2
66	62.4	21.5	62.1	22.2	62.0	22.6
67	63.3	21.8	63.1	22.6	63.0	22.9
68	64.3	22.1	64.0	22.9	63.9	23.3
69	65.2	22.4	65.0	23.2	64.8	23.6
70	66.2	22.8	65.9	23.6	65.8	23.9
71	67.1	23.1	66.8	23.9	66.7	24.3
72	68.1	23.4	67.8	24.2	67.7	24.6
73	69.0	23.8	68.7	24.6	68.6	25.0
74	70.0	24.1	69.7	24.9	69.5	25.3
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	71 Degrees		6 $\frac{1}{4}$ Points		70 Degrees	

Latitude and Departure. 171

21 Degrees		22 Degrees		2 Points		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
47.6	18.3	47.3	19.1	47.1	19.5	51
48.5	18.6	48.2	19.4	48.0	19.9	52
49.5	19.0	49.1	19.8	49.0	20.3	53
50.4	19.3	50.1	20.2	49.9	20.7	54
51.3	19.7	51.0	20.6	50.8	21.0	55
52.3	20.1	51.9	21.0	51.7	21.4	56
53.2	20.4	52.8	21.3	52.7	21.8	57
54.1	20.8	53.8	21.7	53.6	22.2	58
55.1	21.1	54.7	22.1	54.5	22.6	59
56.0	21.5	55.6	22.5	55.4	23.0	60
56.9	21.9	56.5	22.8	56.3	23.3	61
57.9	22.2	57.5	23.2	57.3	23.7	62
58.8	22.6	58.4	23.6	58.2	24.1	63
59.7	22.9	59.3	24.0	59.1	24.5	64
60.7	23.3	60.3	24.3	60.0	24.9	65
61.6	23.6	61.2	24.7	61.0	25.3	66
62.5	24.0	62.1	25.1	61.9	25.6	67
63.5	24.4	63.0	25.5	62.8	26.0	68
64.4	24.7	64.0	25.8	63.7	26.4	69
65.3	25.1	64.9	26.2	64.7	26.8	70
66.3	25.4	65.8	26.6	65.6	27.2	71
67.2	25.8	66.7	27.0	66.5	27.6	72
68.1	26.2	67.7	27.3	67.4	27.9	73
69.1	26.5	68.6	27.7	68.3	28.3	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
69 Degrees		68 Degrees		6 Points		

172 *A Table of Difference of*

Diff.	19 Degrees		1 3 Point		20 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	70.9	24.4	70.6	25.3	70.5	25.6
76	71.9	24.7	71.6	25.6	71.4	26.0
77	72.8	25.1	72.5	25.9	72.4	26.3
78	73.7	25.4	73.4	26.3	73.3	26.7
79	74.7	25.7	74.4	26.6	74.2	27.0
80	75.6	26.0	75.3	26.9	75.2	27.4
81	76.6	26.4	76.3	27.3	76.1	27.7
82	77.5	26.7	77.2	27.6	77.1	28.0
83	78.5	27.0	78.1	28.0	78.0	28.4
84	79.4	27.3	79.1	28.3	78.9	28.7
85	80.4	27.7	80.1	28.6	79.9	29.1
86	81.3	28.0	81.0	29.0	80.8	29.4
87	82.3	28.3	81.9	29.3	81.8	29.7
88	83.2	28.6	82.8	29.6	82.7	30.1
89	84.1	29.0	83.8	30.0	83.6	30.4
90	85.1	29.3	84.7	30.3	84.6	30.8
91	86.0	29.6	85.7	30.7	85.5	31.1
92	87.0	29.9	86.6	31.0	86.4	31.5
93	88.0	30.3	87.6	31.3	87.4	31.8
94	88.9	30.6	88.5	31.7	88.5	32.1
95	89.8	30.9	89.4	32.0	89.4	32.5
96	90.8	31.3	90.4	32.3	90.4	32.8
97	91.7	31.6	91.3	32.7	91.3	33.2
98	92.7	31.9	92.3	33.0	92.3	33.5
99	93.6	32.0	93.2	33.3	93.2	33.9
100	94.5	32.6	94.2	33.7	94.2	34.2
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	71 Degrees		7 $\frac{1}{4}$ Points		70 Degrees	

Latitude and Departure. 173

21 Degrees		22 Degrees		2 Point		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
70.0	26.9	69.5	28.1	69.3	28.7	75
70.9	27.2	70.5	28.5	70.2	29.1	76
71.9	27.6	71.4	28.8	71.1	29.5	77
72.8	27.9	72.3	29.2	72.0	29.8	78
73.7	28.3	73.2	29.6	73.0	30.2	79
74.7	28.7	74.2	30.0	73.9	30.6	80
75.6	29.0	75.1	30.3	74.8	31.0	81
76.5	29.4	76.0	30.7	75.7	31.4	82
77.5	29.7	76.9	31.1	76.7	31.8	83
78.4	30.1	77.9	31.5	77.6	32.1	84
79.3	30.5	78.8	31.8	78.5	32.5	85
80.3	30.8	79.7	32.2	79.4	32.9	86
81.2	31.2	80.7	32.6	80.4	33.3	87
82.1	31.5	81.6	33.0	81.3	33.7	88
83.1	31.9	82.5	33.3	82.2	34.1	89
84.0	32.3	83.4	33.7	83.1	34.4	90
84.9	32.6	84.4	34.1	84.1	34.8	91
85.9	33.0	85.3	34.5	85.0	35.2	92
86.8	33.3	86.2	34.8	85.9	35.6	93
87.7	33.7	87.2	35.2	86.8	36.0	94
88.7	34.0	88.1	35.6	87.8	36.3	95
89.6	34.4	89.0	35.9	88.7	36.7	96
90.5	34.8	89.9	36.3	89.6	37.1	97
91.5	35.1	90.9	36.7	90.5	37.5	98
92.4	35.5	91.8	37.1	91.5	37.9	99
93.4	35.8	92.7	37.5	92.4	38.3	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
69 Degrees		68 Degrees		6 Point		

174 *A Table of Difference of*

Diff.	23 Degrees		24 Degrees		25 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	00,9	00,4	00,9	00,4	00,9	00,4
2	01,8	00,8	01,8	00,8	01,8	00,8
3	02,8	01,2	02,7	01,2	02,7	01,3
4	03,7	01,6	03,6	01,6	03,6	01,7
5	04,6	01,9	04,6	02,0	04,5	02,1
6	05,5	02,3	05,5	02,4	05,4	02,5
7	06,4	02,7	06,4	02,8	06,3	03,0
8	07,4	03,1	07,3	03,2	07,2	03,4
9	08,3	03,5	08,2	03,7	08,2	03,8
10	09,2	03,9	09,1	04,1	09,1	04,2
11	10,1	04,3	10,0	04,5	10,0	04,6
12	11,0	04,7	11,0	04,9	10,9	05,1
13	12,0	05,1	11,9	05,3	11,8	05,5
14	12,9	05,5	12,8	05,7	12,7	05,9
15	13,8	05,9	13,7	06,1	13,6	06,3
16	14,7	06,2	14,6	06,5	14,5	06,8
17	15,6	06,6	15,5	06,9	15,4	07,2
18	16,6	07,0	16,4	07,3	16,3	07,6
19	17,5	07,4	17,4	07,7	17,2	08,0
20	18,4	07,8	18,3	08,1	18,1	08,4
21	19,3	08,2	19,2	08,5	19,0	08,9
22	20,2	08,6	20,1	08,9	19,9	09,3
23	21,2	09,0	21,0	09,3	20,8	09,7
24	22,1	09,4	21,9	09,8	21,7	10,1
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	67 Degrees		66 Degrees		65 Degrees	

2 $\frac{1}{4}$ Point		26 Degrees		27 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
00,9	00,4	00,9	00,4	00,9	00,4	1
01,8	00,9	01,8	00,9	01,8	00,9	2
02,7	01,3	02,7	01,3	02,7	01,4	3
03,6	01,7	03,6	01,7	03,6	01,8	4
04,5	02,1	04,5	02,2	04,5	02,3	5
05,4	02,6	05,4	02,6	05,3	02,7	6
06,3	03,0	06,3	03,1	06,2	03,2	7
07,2	03,4	07,2	03,5	07,1	03,6	8
08,1	03,8	08,1	03,9	08,0	04,1	9
09,0	04,3	09,0	04,4	08,9	04,5	10
09,9	04,7	09,9	04,8	09,8	05,0	11
10,8	05,1	10,8	05,3	10,7	05,4	12
11,7	05,6	11,7	05,7	11,6	05,9	13
12,7	05,9	12,6	06,1	12,5	06,4	14
13,6	06,4	13,5	06,6	13,4	06,8	15
14,5	06,6	14,4	07,0	14,3	07,3	16
15,4	07,3	15,3	07,4	15,1	07,7	17
16,3	07,7	16,2	07,9	16,0	08,2	18
17,2	08,1	17,1	08,3	16,9	08,6	19
18,1	08,5	18,0	08,8	17,8	09,1	20
19,0	09,0	18,9	09,2	18,7	09,5	21
19,9	09,4	19,8	09,9	19,6	10,0	22
20,8	09,8	20,7	10,1	20,5	10,4	23
21,7	10,3	21,6	10,5	21,4	10,9	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
5 $\frac{3}{4}$ Points		64 Degrees		63 Degrees		

176 *A Table of Difference of*

Dif.	23 Degrees		24 Degrees		25 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	23.0	09.8	22.8	10.2	22.7	10.6
26	23.9	10.2	23.7	10.6	23.6	11.0
27	24.8	10.5	24.7	11.0	24.5	11.4
28	25.8	10.9	25.6	11.4	25.4	11.8
29	26.7	11.3	26.5	11.8	26.3	12.3
30	27.6	11.7	27.4	12.2	27.2	12.7
31	28.5	12.1	28.3	12.6	28.1	13.1
32	29.5	12.5	29.2	13.0	29.0	13.5
33	30.4	12.9	30.1	13.4	29.9	13.9
34	31.3	13.3	31.1	13.8	30.8	14.4
35	32.2	13.7	32.0	14.2	31.7	14.8
36	33.1	14.1	32.9	14.6	32.6	15.2
37	34.1	14.4	33.8	15.0	33.8	15.6
38	35.0	14.8	34.7	15.4	34.7	16.0
39	35.9	15.2	35.6	15.9	35.6	16.5
40	36.8	15.6	36.5	16.3	36.5	16.9
41	37.7	16.0	37.5	16.7	37.5	17.3
42	38.7	16.4	38.4	17.1	38.4	17.7
43	39.6	16.8	39.3	17.5	39.3	18.2
44	40.5	17.2	40.2	17.9	40.2	18.6
45	41.4	17.6	41.1	18.3	41.1	19.0
46	42.3	18.0	42.0	18.7	42.0	19.4
47	43.3	18.4	42.9	19.1	42.9	19.9
48	44.2	18.8	43.8	19.5	43.8	20.3
49	45.1	19.2	44.8	19.9	44.8	20.7
50	46.0	19.5	45.7	20.3	45.7	21.1
Dif.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	67 Degrees		66 Degrees		65 Degrees	

2 $\frac{1}{4}$ Point		26 Degrees		27 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
22,6	10,7	22,5	11,0	22,3	11,3	25
23,5	11,1	23,4	11,4	23,2	11,8	26
24,4	11,5	24,3	11,8	24,1	12,3	27
25,3	12,0	25,2	12,3	24,9	12,7	28
26,2	12,4	26,1	12,7	25,8	13,2	29
27,1	12,8	27,0	13,1	26,7	13,6	30
28,0	13,3	27,9	13,6	27,6	14,1	31
28,9	13,7	28,8	14,0	28,5	14,5	32
29,8	14,1	29,7	14,4	29,4	15,0	33
30,7	14,5	30,6	14,9	30,3	15,4	34
31,6	15,0	31,5	15,3	31,2	15,9	35
32,5	15,4	32,4	15,8	32,1	16,3	36
33,4	15,8	33,2	16,2	33,0	16,8	37
34,3	16,2	34,1	16,6	33,9	17,2	38
35,3	16,7	35,0	17,1	34,7	17,7	39
36,2	17,1	35,9	17,5	35,6	18,2	40
37,1	17,5	36,8	18,0	36,5	18,6	41
38,0	18,0	37,7	18,4	37,4	19,1	42
39,9	18,4	38,6	18,8	38,3	19,5	43
39,8	18,8	39,5	19,3	39,2	20,0	44
40,7	19,2	40,4	19,7	40,1	20,4	45
41,6	19,7	41,3	20,2	41,0	20,9	46
42,5	20,1	42,2	20,6	41,9	21,3	47
43,4	20,5	43,1	21,0	42,8	21,8	48
44,3	20,9	44,0	21,5	43,7	22,2	49
45,2	21,4	44,9	21,9	44,5	22,7	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
5 $\frac{3}{4}$ Points		64 Degrees		63 Degrees		

178 *A Table of Difference of*

Diff.	23 Degrees		24 Degrees		25 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	46,9	19,9	46,6	20,7	46,2	21,5
52	47,9	20,3	47,5	21,1	47,1	22,0
53	48,8	20,7	48,4	21,5	48,0	22,4
54	49,7	21,1	49,3	22,0	48,9	22,8
55	50,6	21,5	50,2	22,4	49,8	23,2
56	51,5	21,9	51,2	22,8	50,7	23,7
57	52,5	22,3	52,1	23,2	51,7	24,1
58	53,4	22,7	53,0	23,6	52,6	24,5
59	54,3	23,0	53,9	24,0	53,5	24,9
60	55,2	23,4	54,8	24,4	54,4	25,4
61	56,1	23,8	55,7	24,8	55,3	25,8
62	57,1	24,2	56,6	25,2	56,2	26,2
63	58,0	24,6	57,5	25,6	57,1	26,6
64	58,9	25,0	58,5	26,0	58,0	27,0
65	59,8	25,4	59,4	26,4	58,9	27,5
66	60,7	25,8	60,3	26,8	59,8	27,9
67	61,7	26,2	61,2	27,2	60,7	28,3
68	62,6	26,6	62,1	27,7	61,6	28,7
69	63,5	27,0	63,0	28,1	62,5	29,2
70	64,4	27,3	63,9	28,5	63,4	29,6
71	65,4	27,7	64,9	28,9	64,3	30,0
72	66,3	28,1	65,8	29,3	65,2	30,4
73	67,2	28,5	66,7	29,7	66,2	30,8
74	68,1	28,9	67,6	30,1	67,1	31,3
Diff.	67 Degrees		66 Degrees		65 Degrees	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.

Latitude and Departure. 179

2 $\frac{1}{4}$ Point		26 Degrees		27 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
46,1	21,8	45,8	22,3	45,4	23,2	51
46,0	22,2	46,7	22,8	46,3	23,6	52
47,9	22,7	47,6	23,2	47,2	24,1	53
48,8	23,1	48,5	23,7	48,1	24,5	54
49,7	23,5	49,4	24,1	49,0	25,0	55
50,6	23,9	50,3	24,5	49,9	25,4	56
51,5	24,4	51,2	25,0	50,8	25,9	57
52,4	24,8	52,1	25,4	51,7	26,3	58
53,3	25,2	53,0	25,9	52,6	26,8	59
54,2	25,6	53,9	26,3	53,5	27,2	60
55,1	26,1	54,8	26,7	54,4	27,7	61
56,0	26,5	55,7	27,2	55,2	28,1	62
56,9	26,9	56,6	27,6	56,1	28,6	63
57,9	27,4	57,5	28,0	57,0	29,1	64
58,8	27,8	58,4	28,5	57,9	29,5	65
59,7	28,2	59,3	28,9	58,8	30,0	66
60,6	28,6	60,2	29,4	59,7	30,4	67
61,5	29,1	61,1	29,3	60,6	30,9	68
62,5	29,5	62,0	30,2	61,5	31,3	69
63,4	29,9	62,9	30,7	62,4	31,8	70
64,3	30,4	63,8	31,1	63,3	32,2	71
65,2	30,8	64,7	31,6	64,2	32,7	72
66,2	31,2	65,6	32,0	65,0	33,1	73
66,1	31,6	66,5	32,4	65,9	33,6	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
5 $\frac{3}{4}$ Points		64 Degrees		63 Degrees		

180 *A Table of Difference of*

Diff.	23 Degrees		24 Degrees		25 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	69,0	29,3	68,5	30,5	68,0	31,7
76	70,0	29,7	69,4	30,9	68,9	32,1
77	70,9	30,1	70,3	31,3	69,8	32,5
78	71,8	30,5	71,2	31,7	70,7	33,0
79	72,7	30,9	72,2	32,1	71,6	33,4
80	73,6	31,3	73,1	32,5	72,5	33,8
81	74,6	31,6	74,0	33,9	73,4	34,2
82	75,5	32,0	74,9	33,3	74,3	34,7
83	76,4	32,4	75,8	33,8	75,2	35,1
84	77,3	32,8	76,7	34,2	76,1	35,5
85	78,2	33,2	77,6	34,6	77,0	35,9
86	79,2	33,6	78,6	35,0	77,9	36,3
87	80,1	34,0	79,5	35,4	78,8	36,8
88	81,0	34,4	80,4	35,8	79,7	37,2
89	81,9	34,8	81,3	36,2	80,7	37,6
90	82,8	35,2	82,2	36,6	81,6	38,0
91	83,7	35,6	83,1	37,0	82,5	38,5
92	84,7	35,9	84,0	37,4	83,4	38,9
93	85,6	36,3	85,0	37,8	84,3	39,3
94	86,5	36,7	85,9	38,2	85,2	39,7
95	87,4	37,1	86,8	38,6	86,1	40,1
96	88,4	37,5	87,7	39,0	87,0	40,6
97	89,3	37,9	88,6	39,4	87,9	41,0
98	90,2	38,3	89,5	39,9	88,8	41,4
99	91,1	38,7	90,4	40,3	89,7	41,8
100	92,0	39,1	91,4	40,7	90,6	42,3
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	67 Degrees		66 Degrees		65 Degrees	

Latitude and Departure. 181

2 $\frac{1}{4}$ Point		26 Degrees		27 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
68,0	32,1	67,4	32,9	66,8	34,1	75
68,9	32,5	68,3	33,3	67,7	34,5	76
69,8	32,9	69,2	33,7	68,6	35,0	77
70,7	33,3	70,1	34,2	69,5	35,4	78
71,6	33,8	71,0	34,6	70,4	35,9	79
72,5	34,2	71,9	35,1	71,3	36,3	80
73,4	34,6	72,8	35,5	72,2	36,3	81
74,3	35,1	73,7	35,9	73,1	37,2	82
75,2	35,5	74,6	36,4	74,0	37,7	83
76,1	35,9	75,5	36,8	74,8	38,1	84
77,0	36,3	76,4	37,3	75,7	38,6	85
77,9	36,8	77,3	37,7	76,6	39,0	86
78,8	37,2	78,2	38,1	77,5	39,5	87
79,7	37,6	79,1	38,6	78,4	40,0	88
80,7	38,1	80,0	39,0	79,3	40,4	89
81,6	38,5	80,9	39,4	80,2	40,9	90
82,5	38,9	81,8	39,8	81,1	41,3	91
83,4	39,3	82,7	40,3	82,0	41,8	92
84,3	39,8	83,6	40,8	82,9	42,2	93
85,2	40,2	84,5	41,2	83,8	42,7	94
86,1	40,6	85,4	41,6	84,6	43,1	95
87,0	41,0	86,3	42,1	85,5	43,6	96
87,9	41,5	87,2	42,5	86,4	44,0	97
88,8	41,9	88,1	43,0	87,3	44,5	98
89,7	42,3	89,0	43,4	88,2	44,9	99
90,6	42,7	89,9	43,8	89,1	45,4	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
5 $\frac{3}{4}$ Points		64 Degrees		63 Degrees		

182 *A Table of Difference of*

Dif.	28 Degrees		2 $\frac{1}{2}$ Point		29 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	00,9	00,5	00,9	00,5	00,9	00,5
2	01,8	00,9	01,8	00,9	01,7	01,0
3	02,6	01,4	02,6	01,4	02,6	01,4
4	03,5	01,9	03,5	01,9	03,5	01,9
5	04,4	02,3	04,4	02,4	04,4	02,4
6	05,3	02,8	05,3	02,8	05,2	02,9
7	06,2	03,3	06,2	03,3	06,1	03,4
8	07,1	03,8	07,1	03,8	07,0	03,9
9	07,9	04,2	07,9	04,2	07,9	04,4
10	08,8	04,7	08,8	04,7	08,7	04,8
11	09,7	05,2	09,7	05,2	09,6	05,3
12	10,6	05,6	10,6	05,6	10,5	05,8
13	11,5	06,1	11,5	06,1	11,4	06,3
14	12,4	06,6	12,3	06,6	12,2	06,8
15	13,2	07,0	13,2	07,1	13,1	07,3
16	14,1	07,5	14,1	07,5	14,0	07,7
17	15,0	08,0	15,0	08,0	14,9	08,2
18	15,9	08,4	15,9	08,5	15,7	08,7
19	16,8	08,9	16,8	08,9	16,6	09,2
20	17,7	09,4	17,6	09,4	17,5	09,7
21	18,5	09,9	18,5	09,9	18,4	10,2
22	19,4	10,3	19,4	10,3	19,2	10,7
23	20,3	10,8	20,3	10,8	20,1	11,1
24	21,2	11,3	21,2	11,3	21,0	11,6
Dif.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	62 Degrees		5 $\frac{1}{2}$ Points		61 Degrees	

Latitude and Departure. 183

30 Degrees		2 $\frac{3}{4}$ Point		31 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
00,9	00,5	00,9	00,5	00,9	00,5	1
01,7	01,0	01,7	01,0	01,7	01,0	2
02,6	01,5	02,6	01,5	02,6	01,5	3
03,5	02,0	03,4	02,1	03,4	02,1	4
04,3	02,5	04,3	02,6	04,3	02,6	5
05,2	03,0	05,1	03,1	05,1	03,1	6
06,1	03,5	06,0	03,6	06,0	03,6	7
06,9	04,0	06,9	04,1	06,9	04,1	8
07,8	04,5	07,7	04,6	07,7	04,6	9
08,7	05,0	08,6	05,1	08,6	05,1	10
09,5	05,5	09,4	05,6	09,4	05,7	11
10,4	06,0	10,3	06,2	10,3	06,2	12
11,3	06,5	11,1	06,7	11,1	06,7	13
12,1	07,0	12,0	07,2	12,0	07,2	14
13,0	07,5	12,9	07,7	12,9	07,7	15
13,9	08,0	13,7	08,2	13,7	08,2	16
14,7	08,5	14,6	08,7	14,6	08,8	17
15,6	09,0	15,4	09,2	15,4	09,3	18
16,4	09,5	16,3	09,8	16,3	09,8	19
17,3	10,0	17,1	10,3	17,1	10,3	20
18,2	10,5	18,0	10,8	18,0	10,8	21
19,0	11,0	18,9	11,3	18,9	11,3	22
19,9	11,5	19,7	11,8	19,7	11,8	23
20,8	12,0	20,6	12,3	20,6	12,4	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
60 Degrees		6 $\frac{1}{4}$ Point		59 Degrees		

184 *A Table of Difference of*

Diff.	28 Degrees		2 $\frac{1}{2}$ Point		29 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	22.1	11.7	22.0	11.8	21.9	12.1
26	23.0	12.2	22.9	12.3	22.7	12.6
27	23.8	12.7	23.8	12.7	23.6	13.1
28	24.7	13.1	24.7	13.2	24.5	13.6
29	25.6	13.6	25.6	13.7	25.4	14.1
30	26.5	14.1	26.5	14.1	26.2	14.5
31	27.4	14.5	27.3	14.6	27.1	15.0
32	28.2	15.0	28.2	15.1	28.0	15.5
33	29.1	15.5	29.1	15.5	28.9	16.0
34	30.0	16.0	30.0	16.0	29.7	16.5
35	30.9	16.4	30.9	16.1	30.6	17.0
36	31.8	16.9	31.7	17.0	31.5	17.4
37	32.7	17.4	32.6	17.4	32.4	17.9
38	33.5	17.0	33.5	17.9	33.2	18.4
39	34.4	18.3	34.4	18.4	34.1	19.9
40	35.3	18.8	35.3	18.9	35.0	19.4
41	36.2	19.2	36.1	19.3	35.8	19.9
42	37.1	19.7	37.0	19.8	36.7	20.4
43	38.0	20.1	37.9	20.3	37.6	20.8
44	38.8	20.6	38.8	20.7	38.5	21.3
45	39.7	21.1	39.7	21.2	39.3	21.8
46	40.6	21.6	30.6	21.7	40.2	22.3
47	41.5	22.1	31.5	22.2	41.1	22.8
48	42.4	22.5	32.4	22.6	42.0	23.3
49	43.3	23.1	33.3	23.2	42.8	23.7
50	44.1	23.5	34.1	23.6	43.7	24.8
Diff.	62 Degrees		5 $\frac{1}{2}$ Point		61 Degrees	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.

Latitude and Departure. 185

30 Degrees		2 $\frac{3}{4}$ Point		22 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
21,6	12,5	21,4	12,8	21,4	12,9	25
22,5	13,0	22,3	13,4	22,3	13,4	26
23,4	13,5	23,1	13,9	23,1	13,9	27
24,2	14,0	24,0	14,4	24,0	14,4	28
25,1	14,5	24,9	14,9	24,9	14,9	29
26,0	15,0	25,7	15,4	25,7	15,4	30
26,8	15,5	26,6	15,9	26,6	16,0	31
27,7	16,0	27,4	16,4	27,4	16,5	32
28,6	16,5	28,3	17,0	28,3	17,0	33
29,5	17,0	29,2	17,5	29,1	17,5	34
30,3	17,5	30,0	18,0	30,0	18,0	35
31,2	18,0	30,9	18,5	30,9	18,5	36
32,0	18,5	31,7	19,0	31,7	19,1	37
32,9	19,0	32,5	19,5	32,6	19,6	38
33,8	19,5	33,4	20,0	33,4	20,1	39
34,6	20,0	34,3	20,6	34,3	20,6	40
35,5	20,5	35,2	21,1	35,1	21,1	41
36,4	21,0	36,0	21,6	36,0	21,6	42
37,2	21,5	36,9	22,1	36,9	22,1	43
38,1	22,0	37,7	22,6	37,7	22,6	44
39,0	22,5	38,6	23,1	38,6	23,2	45
39,8	23,0	39,5	23,6	39,4	23,7	46
40,7	23,5	40,3	24,2	40,3	24,2	47
41,6	24,0	41,2	24,7	41,1	24,7	48
42,4	24,5	42,0	25,2	42,0	25,2	49
43,3	25,0	42,9	25,7	42,9	25,7	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
60 Degrees		5 $\frac{1}{4}$ Points		59 Degrees		

186 *A Table of Difference of*

Diff.	28 Degrees		2 $\frac{1}{2}$ Point		29 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	45.0	23.9	45.0	24.0	44.6	24.7
52	45.9	24.4	45.9	24.5	45.5	25.2
53	46.8	24.9	46.7	25.0	46.3	25.7
54	47.7	25.3	47.6	25.5	47.2	26.2
55	48.6	25.8	48.5	25.9	48.1	26.7
56	49.4	26.3	49.4	26.4	49.0	27.1
57	50.3	26.8	50.3	26.9	49.8	27.6
58	51.2	27.2	51.2	27.3	50.7	28.1
59	52.1	27.7	52.0	27.8	51.6	28.6
60	53.0	28.2	52.9	28.3	52.5	29.1
61	53.9	28.6	53.8	28.7	53.3	29.6
62	54.7	29.1	54.7	29.2	54.2	30.1
63	55.6	29.6	55.6	29.7	55.1	30.5
64	56.5	30.0	56.4	30.2	56.0	31.0
65	57.4	30.5	57.3	30.5	56.8	31.5
66	58.3	31.0	58.2	31.1	57.7	32.0
67	59.2	31.4	59.1	31.6	58.6	32.5
68	60.0	31.9	60.0	32.0	59.5	33.0
69	60.9	32.4	60.8	32.5	60.3	33.4
70	61.8	32.9	61.7	33.0	61.2	33.9
71	62.7	33.3	62.6	33.5	62.1	34.4
72	63.6	33.8	63.5	33.9	63.0	34.9
73	64.4	34.3	64.4	34.4	63.8	35.4
74	65.3	34.7	65.3	34.9	64.7	35.9
Diff.	62 Degrees		5 $\frac{1}{2}$ Points		61 Degrees	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.

Latitude and Departure. 187

30 Degrees		2 $\frac{3}{4}$ Points		31 Degrees		Dif.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
44.2	25.5	43.7	26.2	43.7	26.3	51
45.0	26.0	44.6	26.7	44.6	26.8	52
45.9	26.5	45.5	27.2	45.4	27.3	53
46.8	27.0	46.3	27.8	46.3	27.8	54
47.6	27.5	47.2	28.3	47.1	28.3	55
48.5	28.0	48.0	28.8	48.0	28.8	56
49.4	28.5	48.9	29.3	48.9	29.4	57
50.2	29.0	49.7	29.8	49.7	29.9	58
51.1	29.5	50.6	30.3	50.6	30.4	59
52.0	30.0	51.5	30.8	51.4	30.9	60
52.8	30.5	52.3	31.4	52.3	31.4	61
53.7	31.0	53.2	31.9	53.1	31.9	62
54.6	31.5	54.0	32.4	54.0	32.4	63
55.4	32.0	54.9	32.9	54.9	33.0	64
56.3	32.5	55.7	33.4	55.7	33.5	65
57.2	33.0	56.6	33.9	56.6	34.0	66
58.0	33.5	57.5	34.4	57.4	34.5	67
58.9	34.0	58.3	35.0	58.3	35.0	68
59.7	34.5	59.2	35.5	59.1	35.5	69
60.6	35.0	60.0	36.0	60.0	36.0	70
61.5	35.5	60.9	36.5	60.9	36.6	71
62.2	36.0	61.8	37.0	61.7	37.1	72
63.3	36.5	62.6	37.5	62.6	37.6	73
64.1	37.0	63.5	38.0	63.4	38.1	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dif.
60 Degrees		5 $\frac{1}{4}$ Points		59 Degrees		

Diff.	28 Degrees		2 $\frac{1}{2}$ Point		29 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	66.2	35.2	66.1	35.4	65.6	36.4
76	67.1	35.7	67.0	35.8	66.5	36.8
77	68.0	36.1	67.9	36.3	67.3	37.3
78	68.9	36.6	68.8	36.8	68.2	37.8
79	69.7	37.1	69.7	37.2	69.1	38.3
80	70.6	37.6	70.5	37.7	70.0	38.8
81	71.5	38.0	71.4	38.2	70.8	39.8
82	72.4	38.5	72.3	38.6	71.7	39.7
83	73.3	39.0	73.2	39.1	72.6	40.2
84	74.2	39.4	74.1	39.6	73.5	40.7
85	75.0	39.9	75.0	40.1	74.3	41.2
86	75.9	40.4	75.8	40.5	75.2	41.7
87	76.8	40.8	76.7	41.0	76.1	42.2
88	77.7	41.3	77.6	41.5	77.0	42.7
89	78.6	41.8	78.5	41.9	77.8	43.1
90	79.5	42.2	79.4	42.4	78.7	43.6
91	80.3	42.7	80.2	42.9	79.6	44.1
92	81.2	43.2	81.1	43.4	80.5	44.6
93	82.1	43.6	82.0	43.8	81.3	45.1
94	83.0	44.1	82.9	44.3	82.2	45.6
95	83.9	44.6	83.8	44.8	83.1	46.1
96	84.8	45.1	84.7	45.2	84.0	46.5
97	85.6	45.5	85.5	45.7	84.8	47.0
98	86.5	46.0	86.4	46.2	85.7	47.5
99	87.4	46.5	87.3	46.7	86.6	48.0
100	88.3	46.9	88.2	47.1	87.5	48.5
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	62 Degrees		5 $\frac{1}{2}$ Points		61 Degrees	

Latitude and Departure. 189

30 Degrees		2 $\frac{3}{4}$ Point		31 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
64.9	37.5	64.3	38.6	64.3	38.6	75
65.8	38.0	65.2	39.1	65.1	39.1	76
66.7	38.5	66.0	39.6	66.0	39.7	77
67.5	39.0	66.9	40.1	66.9	40.2	78
68.4	39.5	67.8	40.6	67.7	40.7	79
69.3	40.0	68.6	41.1	68.6	41.2	80
70.1	40.5	69.5	41.6	69.4	41.7	81
70.9	41.0	70.3	42.2	70.3	42.2	82
71.8	41.5	71.2	42.7	71.1	42.7	83
72.7	42.0	72.1	43.2	72.0	43.3	84
73.6	42.5	72.0	43.7	72.9	43.8	85
74.5	43.0	73.8	44.2	73.7	44.3	86
75.3	43.5	74.6	44.7	74.4	44.8	87
76.2	44.0	75.5	45.2	75.6	45.3	88
77.1	44.5	76.3	45.8	76.3	45.8	89
77.9	45.0	77.2	46.3	77.1	46.3	90
78.8	45.5	78.1	46.8	78.0	46.9	91
79.7	46.0	78.9	47.3	78.9	47.4	92
80.5	46.5	79.8	47.8	79.7	47.9	93
81.4	47.0	80.6	48.3	80.6	48.4	94
82.3	47.5	81.5	48.8	81.4	48.9	95
83.1	48.0	82.3	49.3	82.3	49.4	96
84.0	48.5	83.2	49.9	83.1	50.0	97
84.9	49.0	84.1	40.4	84.0	50.5	98
85.7	49.5	84.9	40.9	84.9	51.0	99
86.6	40.0	85.8	41.4	85.7	51.5	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
60 Degrees		5 $\frac{1}{4}$ Point		59 Degrees		

190 *A Table of Difference of*

Diff.	32 Degrees		33 Degrees		3 Points	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	00,8	00,5	00,8	00,5	00,8	00,6
2	01,7	01,1	01,7	01,1	01,7	01,1
3	02,5	01,6	02,5	01,6	02,5	01,7
4	03,4	02,1	03,4	02,2	03,3	02,2
5	04,2	02,6	04,2	02,7	04,2	02,8
6	05,1	03,2	05,0	03,3	05,0	03,3
7	05,9	03,7	05,9	03,8	05,8	03,9
8	06,8	04,2	06,7	04,4	06,6	04,4
9	07,6	04,8	07,5	04,9	07,5	05,0
10	08,5	05,3	08,4	05,4	08,3	05,6
11	09,3	05,8	09,2	06,0	09,1	06,1
12	10,2	06,4	10,1	06,5	10,0	06,7
13	11,0	06,9	10,9	07,1	10,8	07,2
14	11,9	07,4	11,7	07,6	11,6	07,8
15	12,7	07,9	12,6	08,2	12,5	08,3
16	13,6	08,5	13,4	08,7	13,3	08,9
17	14,4	09,0	14,3	09,3	14,1	09,4
18	15,3	09,5	15,1	09,8	15,0	10,0
19	16,1	10,1	15,9	10,3	15,8	10,6
20	17,0	10,6	16,8	10,9	16,6	11,1
21	17,8	11,1	17,6	11,4	17,5	11,7
22	18,6	11,7	18,5	12,0	18,3	12,2
23	19,5	12,2	19,3	12,5	19,1	12,8
24	20,3	12,7	20,1	13,1	20,0	13,3
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	58 Degrees		57 Degrees		5 Point	

Latitude and Departure. 191

34 Degrees		35 Degrees		36 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
00,8	00,6	00,8	00,6	00,8	00,6	1
01,7	01,1	01,6	01,1	01,6	01,2	2
02,5	01,7	02,5	01,7	02,4	01,8	3
03,3	02,2	03,3	02,3	03,2	02,3	4
04,1	02,8	04,1	02,9	04,0	02,9	5
05,0	03,4	04,9	03,4	04,8	03,5	6
05,8	03,9	05,7	04,0	05,7	04,1	7
06,6	04,5	06,5	04,6	06,5	04,7	8
07,5	05,0	07,4	05,2	07,3	05,3	9
08,3	05,6	08,2	05,7	08,1	05,9	10
09,1	06,1	09,0	06,3	08,9	06,5	11
09,9	06,7	09,8	06,9	09,7	07,0	12
10,8	07,3	10,6	07,5	10,5	07,6	13
11,6	07,8	11,5	08,0	11,3	08,2	14
12,4	08,4	12,3	08,6	12,1	08,8	15
13,3	08,9	13,1	09,2	12,9	09,4	16
14,1	09,5	13,9	09,8	13,7	10,0	17
15,9	10,1	14,7	10,3	14,6	10,6	18
15,7	10,6	15,6	10,9	15,4	11,2	19
16,6	11,2	16,4	11,5	16,2	11,8	20
17,4	11,7	17,2	12,0	17,0	12,3	21
18,2	12,3	18,0	12,6	17,8	12,9	22
19,0	12,8	18,8	13,2	18,6	13,5	23
19,9	13,4	19,7	13,8	19,4	14,1	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
56 Degrees		55 Degrees		54 Degrees		

Dif.	32 Degrees		33 Degrees		3 Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	21.2	03.2	21.0	13.6	20.8	13.9
26	22.0	13.8	21.8	14.2	21.6	14.4
27	22.9	14.3	22.6	14.7	22.4	15.0
28	23.7	14.8	23.5	15.2	23.3	15.5
29	24.6	15.4	24.3	15.8	24.1	16.1
30	25.4	15.9	25.2	16.3	24.9	16.7
31	26.3	16.4	26.0	16.9	25.8	17.2
32	27.1	17.0	26.8	17.4	26.6	17.8
33	28.0	17.5	27.7	18.0	27.4	18.3
34	28.8	18.0	28.5	18.5	28.3	18.9
35	29.7	18.5	29.4	19.1	29.1	19.4
36	30.5	19.1	30.2	19.6	29.9	20.0
37	31.4	19.6	31.0	20.1	30.8	20.6
38	32.2	20.1	31.9	20.7	31.6	21.1
39	33.1	20.7	32.7	21.2	32.4	21.7
40	33.9	21.2	33.6	21.8	33.3	22.2
41	34.8	21.7	34.4	22.3	34.1	22.8
42	35.6	22.3	35.2	22.9	34.9	23.3
43	36.5	22.8	36.1	23.4	35.7	23.9
44	37.3	23.3	36.9	24.0	36.6	24.4
45	38.1	23.8	37.7	24.5	37.4	25.0
46	39.0	24.4	38.6	25.0	38.2	25.5
47	39.9	24.9	39.4	25.6	39.1	26.1
48	40.7	25.4	40.3	26.1	39.9	26.7
49	41.5	26.0	41.1	26.7	40.7	27.2
50	42.4	26.5	41.9	27.2	41.6	27.8
Dif.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	58 Degrees		57 Degrees		5 Point	

34 Degrees		35 Degrees		36 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
20,7	14,0	20,5	14,3	20,1	14,7	25
21,5	14,5	21,3	14,9	21,0	15,3	26
22,4	15,1	22,1	15,5	21,8	15,9	27
23,2	15,6	22,9	16,1	22,6	16,5	28
24,0	16,2	23,8	16,6	23,5	17,0	29
24,9	16,8	24,6	17,2	24,3	17,6	30
25,7	17,3	25,4	17,8	25,1	18,2	31
26,5	17,9	26,2	18,3	25,9	18,8	32
27,4	18,4	27,0	18,9	26,7	19,4	33
28,2	19,0	27,9	19,5	27,5	20,0	34
29,0	19,6	28,7	20,1	28,3	20,6	35
29,8	20,1	29,5	20,6	29,1	21,2	36
30,7	20,7	30,3	21,2	29,9	21,7	37
31,5	21,2	31,1	21,8	30,7	22,3	38
32,3	21,8	32,0	22,3	31,5	22,9	39
33,2	22,4	32,8	22,9	32,4	23,5	40
34,0	22,9	33,6	23,5	33,2	24,1	41
34,8	23,5	34,4	24,1	34,0	24,7	42
35,6	24,0	35,2	24,6	34,8	25,3	43
36,5	24,6	36,0	25,2	35,6	25,9	44
37,3	25,2	36,9	25,8	36,4	26,4	45
38,1	25,7	37,7	26,4	37,2	27,0	46
39,0	26,3	38,5	26,9	38,0	27,6	47
39,8	26,8	39,3	27,5	38,8	28,2	48
40,6	27,4	40,1	28,1	39,6	28,8	49
41,4	28,0	41,0	28,7	40,4	29,4	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
56 Degrees		55 Degrees		54 Degrees		

194 *A Table of Difference of*

Dif.	32 Degrees		33 Degrees		3 Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	43,2	27,0	42,8	27,8	42,4	28,3
52	44,1	27,6	43,6	28,3	43,2	28,9
53	44,9	28,1	44,5	28,9	44,1	29,4
54	45,8	28,6	45,3	29,4	44,9	30,0
55	46,6	29,1	46,1	30,0	45,7	30,6
56	47,5	29,7	47,0	30,5	46,6	31,1
57	48,3	30,2	47,8	31,0	47,4	31,7
58	49,2	30,7	48,7	31,6	48,2	32,2
59	50,0	31,3	49,5	32,1	49,0	32,8
60	50,9	31,8	50,3	32,7	49,9	33,3
61	51,7	32,3	51,2	33,2	50,7	33,9
62	52,6	32,9	52,0	33,8	51,5	34,4
63	53,4	33,4	52,9	34,3	52,4	35,0
64	54,3	33,9	53,7	34,9	53,2	35,5
65	55,1	34,4	54,5	35,4	54,0	36,1
66	56,0	35,0	55,3	35,9	54,9	36,7
67	56,8	35,5	56,2	36,5	55,7	37,2
68	57,7	36,0	57,0	37,0	56,5	37,8
69	58,5	36,6	57,9	37,6	57,4	38,3
70	59,4	37,1	58,7	38,1	58,2	38,9
71	60,2	37,6	59,6	38,7	59,0	39,4
72	61,0	38,1	60,4	39,2	59,8	40,0
73	61,9	38,7	61,2	39,8	60,7	40,6
74	62,7	39,2	62,1	40,3	61,5	41,1
Dif.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	58 Degrees		57 Degrees		5 Point	

Latitude and Departure. 195

34 Degrees		35 Degrees		36 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
42,3	28,5	41,8	29,2	41,3	30,0	51
43,1	29,1	42,6	29,8	42,1	30,6	52
43,9	29,6	43,4	30,4	42,9	31,2	53
44,5	30,2	44,2	31,0	43,7	31,7	54
45,6	30,7	45,1	31,5	44,5	32,3	55
46,4	31,3	45,9	32,1	45,3	32,9	56
47,3	31,9	46,7	32,7	46,1	33,5	57
48,1	32,4	47,5	33,3	46,9	34,1	58
48,9	33,0	48,3	33,8	47,7	34,7	59
49,7	33,5	49,1	34,4	48,5	35,3	60
50,6	34,1	50,0	34,9	49,3	35,9	61
51,4	34,7	50,8	35,6	50,2	36,4	62
52,2	35,2	51,6	36,1	51,0	37,0	63
53,1	35,8	52,4	36,7	51,8	37,6	64
53,9	36,3	53,2	37,3	52,6	38,2	65
54,7	36,9	54,1	37,9	53,4	38,8	66
55,5	37,5	54,9	38,4	54,2	39,4	67
56,4	38,0	55,7	39,0	55,0	40,0	68
57,2	38,6	56,5	39,6	55,8	40,6	69
58,0	39,1	57,3	40,1	56,6	41,1	70
58,9	39,7	58,2	40,7	57,4	41,7	71
59,7	40,3	59,0	41,3	58,2	42,3	72
60,5	40,8	59,8	41,9	59,1	42,9	73
61,3	41,4	60,6	42,4	59,9	43,5	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
56 Degrees		55 Degrees		54 Degrees		

196 *A Table of Difference of*

Diff.	32 Degrees		33 Degrees		3 Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	63,6	39,7	62,9	40,8	62,4	41,7
76	64,4	40,3	63,8	41,5	63,2	42,2
77	65,3	40,8	64,6	41,9	64,8	42,8
78	66,1	41,3	65,4	42,5	64,9	43,3
79	67,0	41,9	66,3	43,0	65,7	43,9
80	67,8	42,4	67,1	43,6	66,5	44,4
81	68,7	42,9	68,0	44,1	67,3	45,0
82	69,5	43,4	68,8	44,7	68,2	45,5
83	70,4	44,0	69,6	45,2	69,0	46,1
84	71,2	44,5	70,5	45,8	69,8	46,7
85	72,1	45,0	71,3	46,3	70,7	47,2
86	72,9	45,6	72,1	46,8	71,5	47,8
87	73,8	46,1	73,0	47,3	72,3	48,3
88	74,6	46,6	73,8	47,9	73,2	48,9
89	75,5	47,2	74,7	48,5	74,0	49,4
90	76,3	47,7	75,5	49,0	74,8	50,0
91	77,2	48,2	76,3	49,6	75,7	50,6
92	78,0	48,7	77,2	50,1	76,5	51,1
93	78,9	49,3	78,0	50,6	77,3	51,7
94	79,7	49,8	78,9	51,2	78,2	52,2
95	80,6	50,3	79,7	51,7	79,0	52,8
96	81,4	50,9	80,5	52,3	79,8	53,3
97	82,3	51,4	81,4	52,8	80,6	53,9
98	83,1	51,9	82,2	53,4	81,5	54,4
99	84,0	52,5	83,1	53,9	82,3	55,0
100	84,8	53,0	83,0	54,5	83,1	55,6
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	58 Degrees		57 Degrees		5 Points	

34 Degrees		35 Degrees		36 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
62,2	41,9	61,4	43,0	60,7	44,1	75
63,0	42,5	62,3	43,6	61,5	44,7	76
63,8	43,0	63,1	44,2	62,3	45,3	77
64,7	43,6	63,9	44,7	63,1	45,8	78
65,5	44,2	64,7	45,3	63,9	46,4	79
66,3	44,7	65,5	45,9	64,7	47,0	80
67,1	45,3	66,4	46,5	65,5	47,6	81
68,0	45,8	67,2	47,0	66,3	48,2	82
68,8	46,4	68,0	47,6	67,1	48,8	83
69,6	47,0	68,8	48,2	68,0	49,4	84
70,5	47,5	69,6	48,8	68,8	50,0	85
71,3	48,1	70,5	49,3	69,6	50,5	86
72,1	48,6	71,3	49,9	70,4	51,1	87
72,9	49,2	72,1	50,5	71,2	51,7	88
73,8	49,8	72,9	51,0	72,0	52,3	89
74,6	50,3	73,7	51,6	72,8	52,9	90
75,4	50,9	74,5	52,2	73,6	53,5	91
76,3	51,4	75,4	52,8	74,4	54,1	92
77,1	52,0	76,2	53,3	75,2	54,7	93
77,9	52,6	77,0	53,9	76,0	55,2	94
78,8	53,1	77,8	54,5	76,9	55,8	95
79,6	53,7	78,6	55,1	77,7	56,4	96
80,4	54,2	79,5	55,6	78,5	57,0	97
81,2	54,8	80,3	56,2	79,3	57,6	98
82,1	55,4	81,1	56,8	80,1	58,2	99
82,9	55,9	81,9	57,4	80,9	58,8	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
56 Degrees		55 Degrees		54 Degrees		

198 *A Table of Difference of*

Diff.	3 $\frac{1}{4}$ Point		37 Degrees		38 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	00,8	00,6	00,8	00,6	00,8	00,6
2	01,6	01,2	01,6	01,2	01,6	01,2
3	02,4	01,8	02,4	01,8	02,4	01,8
4	03,2	02,4	03,2	02,4	03,1	02,5
5	04,0	03,0	04,0	03,0	03,9	03,1
6	04,8	03,6	04,8	03,6	04,7	03,7
7	05,6	04,2	05,6	04,2	05,5	04,3
8	06,4	04,8	06,4	04,8	06,3	04,9
9	07,2	05,4	07,2	05,4	07,1	05,5
10	08,0	06,0	08,0	06,0	07,9	06,2
11	08,8	06,6	08,6	06,6	08,7	06,8
12	09,6	07,1	09,2	07,2	09,4	07,4
13	10,4	07,7	10,4	07,8	10,2	08,1
14	11,2	08,3	11,2	08,4	11,0	08,7
15	12,0	08,9	12,0	09,0	11,8	09,3
16	12,8	09,5	12,8	09,6	12,6	09,8
17	13,6	10,1	13,6	10,2	13,4	10,5
18	14,5	10,7	14,4	10,8	14,2	11,1
19	15,3	11,3	15,2	11,4	15,0	11,7
20	16,1	11,9	16,0	12,0	15,8	12,3
21	16,9	12,5	16,8	12,6	16,5	12,9
22	17,7	13,1	17,6	13,2	17,3	13,5
23	18,5	13,7	18,4	13,8	18,1	14,2
24	19,3	14,3	19,2	14,4	18,9	14,7
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	4 $\frac{3}{4}$ Points		53 Degrees		52 Degrees	

39 Degrees		3 $\frac{1}{2}$ Point		40 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
00,8	00,7	00,8	00,6	00,8	00,6	1
01,5	01,3	01,5	01,3	01,5	01,3	2
02,3	01,9	02,3	01,9	02,3	01,9	3
03,5	02,5	03,1	02,5	03,1	02,6	4
03,9	03,1	03,9	03,2	03,8	03,2	5
04,6	03,9	04,6	03,8	04,6	03,9	6
05,4	04,4	05,4	04,4	05,4	04,5	7
06,2	05,0	06,2	05,1	06,1	05,1	8
07,0	05,7	07,0	05,7	06,9	05,8	9
07,8	06,3	07,7	06,3	07,7	06,4	10
08,5	06,9	08,5	07,0	08,4	07,1	11
09,3	07,5	09,3	07,6	09,2	07,7	12
10,1	08,2	10,0	08,2	10,0	08,4	13
10,9	08,8	10,8	08,9	10,7	09,0	14
11,6	09,4	11,6	09,5	11,5	09,6	15
12,4	10,1	12,4	10,1	12,3	10,3	16
13,2	10,7	13,1	10,8	13,0	10,9	17
13,9	11,3	13,9	11,4	13,8	11,6	18
14,8	12,0	14,7	12,0	14,5	12,2	19
15,5	12,6	15,5	12,7	15,3	12,9	20
16,3	13,2	16,2	13,3	16,1	13,5	21
17,1	13,8	17,0	14,0	16,8	14,1	22
17,9	14,5	17,8	14,6	17,6	14,8	23
18,6	15,1	18,5	15,2	18,4	15,4	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
51 Degrees		4 $\frac{1}{2}$ Point		50 Degrees		

A Table of Difference of

Diff.	3 $\frac{1}{4}$ Point		37 Degrees		38 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	20.1	14.9	20.0	15.0	19.7	15.4
26	20.9	15.5	20.8	15.6	20.5	16.0
27	21.7	16.1	21.6	16.2	21.3	16.6
28	22.5	16.7	22.4	16.8	22.1	17.2
29	23.3	17.3	23.2	17.4	22.8	17.8
30	24.1	17.9	24.0	18.0	23.6	18.5
31	24.9	18.5	24.8	18.6	24.4	19.1
32	25.7	19.1	25.6	19.3	25.2	19.7
33	26.5	19.7	26.4	19.9	26.0	20.3
34	27.3	20.2	27.1	20.5	26.8	20.9
35	28.1	20.8	27.9	21.1	27.6	21.5
36	28.9	21.4	28.7	21.7	28.4	22.2
37	29.7	22.0	29.5	22.3	29.2	22.8
38	30.5	22.6	30.3	22.9	29.9	23.4
39	31.3	23.2	31.1	23.5	30.7	24.0
40	32.1	23.8	31.9	24.1	31.5	24.6
41	32.9	24.4	32.7	24.7	32.3	25.2
42	33.7	25.0	33.5	25.3	33.1	25.9
43	34.5	25.6	34.3	25.9	33.9	26.5
44	35.3	26.2	35.1	26.5	34.7	27.1
45	36.1	26.8	35.9	27.1	35.5	27.7
46	36.9	27.4	36.7	27.7	36.2	28.3
47	37.7	28.0	37.5	28.3	37.0	28.9
48	38.5	28.6	38.3	28.9	37.8	29.5
49	39.3	29.2	39.1	29.5	38.6	30.2
50	40.2	29.8	39.9	20.1	39.4	30.8
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	4 $\frac{3}{4}$ Point		53 Degrees		52 Degrees	

Latitude and Departure. 201

39 Degrees		3 $\frac{1}{2}$ Point		40 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
19,4	15,7	19,3	15,9	19,1	16,1	25
20,2	16,4	20,1	16,5	19,9	16,7	26
21,0	17,0	20,9	17,1	20,7	17,4	27
21,8	17,6	21,6	17,8	21,4	18,0	28
22,1	18,3	22,4	18,4	22,2	18,6	29
23,3	18,9	23,2	19,0	23,0	19,3	30
24,1	19,5	24,0	19,7	23,7	19,9	31
24,9	20,1	24,7	20,3	24,5	20,6	32
25,6	20,8	25,5	20,9	25,3	21,2	33
26,4	21,4	26,3	21,6	26,0	21,9	34
27,2	22,0	27,0	22,2	26,8	22,5	35
27,7	22,7	27,8	22,8	27,6	23,1	36
28,8	23,3	28,6	23,5	28,3	23,8	37
29,5	23,9	29,4	24,1	29,1	24,4	38
30,3	24,5	30,1	24,7	29,9	25,1	39
31,1	25,2	30,9	25,4	30,6	25,7	40
31,9	25,8	31,7	26,0	31,4	26,4	41
32,6	26,4	32,5	26,6	32,2	27,0	42
33,4	27,1	33,2	27,3	32,9	27,6	43
34,2	27,7	34,0	27,9	33,7	28,3	44
35,0	28,3	34,8	28,5	34,5	28,9	45
35,7	29,0	35,6	29,2	35,2	29,6	46
36,5	29,6	36,3	29,8	36,0	30,2	47
37,8	30,2	37,1	30,4	36,8	30,9	48
38,1	30,8	37,9	31,1	37,5	31,5	49
38,9	31,5	38,6	31,7	38,3	32,1	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
51 Degrees		4 $\frac{1}{2}$ Points		50 Degrees		

202 *A Table of Difference of*

Dif.	3 $\frac{1}{4}$ Point		37 Degrees		38 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	41.0	30.4	40.7	30.7	40.2	31.4
52	41.8	31.0	41.5	31.3	41.0	32.0
53	42.6	31.6	42.3	31.9	41.8	32.6
54	43.4	32.2	43.1	32.5	42.5	33.2
55	44.2	32.8	43.0	33.1	43.3	33.9
56	45.0	33.3	44.7	33.8	44.1	34.5
57	45.8	33.9	45.5	34.3	44.9	35.1
58	46.6	34.5	46.3	34.9	45.7	35.8
59	47.4	35.1	47.1	35.5	46.5	36.3
60	48.2	35.7	47.9	36.1	47.3	36.9
61	49.0	36.3	48.7	36.7	48.1	37.5
62	49.8	36.9	49.5	37.3	48.9	38.2
63	50.6	37.5	50.3	37.9	49.6	38.8
64	51.4	38.1	51.1	38.5	50.4	39.4
65	52.2	38.7	51.9	39.1	51.2	40.0
66	53.0	39.3	52.7	39.7	52.0	40.6
67	53.8	39.9	53.5	40.3	52.8	41.2
68	54.6	40.5	54.3	40.9	53.6	41.9
69	55.4	41.1	55.1	41.5	54.4	42.5
70	56.2	41.7	55.9	42.1	55.2	43.1
71	57.0	42.3	56.7	42.7	55.9	43.7
72	57.8	42.9	57.5	43.3	56.7	44.3
73	58.6	43.5	58.3	43.9	57.5	44.9
74	59.4	44.1	59.1	44.5	58.3	45.6
Dif.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	4 $\frac{3}{4}$ Points		53 Degrees		52 Degrees	

Latitude and Departure. 203

39 Degrees		3 $\frac{1}{2}$ Points		40 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
39.6	32.1	39.4	32.3	39.1	32.8	51
40.4	32.7	40.2	33.0	39.8	33.4	52
41.2	33.3	41.0	33.6	40.6	34.1	53
42.0	34.0	41.7	34.3	41.4	34.7	54
42.7	34.6	42.5	34.9	42.1	35.4	55
43.5	35.2	43.3	35.5	42.9	36.0	56
44.3	35.9	44.1	36.2	43.7	36.6	57
45.1	36.5	44.8	36.8	44.4	37.3	58
45.8	37.1	45.6	37.4	45.2	37.9	59
46.6	37.8	46.4	38.1	46.0	38.6	60
47.4	38.4	47.1	38.7	46.7	39.2	61
48.2	39.0	47.9	39.3	47.5	39.9	62
49.0	39.6	48.7	40.0	48.3	40.5	63
49.7	40.3	49.5	40.6	49.0	41.2	64
50.5	40.9	50.2	41.2	49.8	41.8	65
51.3	41.5	51.0	41.9	50.5	42.4	66
52.1	42.2	51.8	42.5	51.3	43.1	67
52.8	42.8	52.6	43.1	52.1	43.7	68
53.6	43.4	53.3	43.8	52.9	44.4	69
54.4	44.0	54.1	44.4	53.6	45.0	70
55.2	44.7	54.9	45.0	54.4	45.6	71
55.9	45.3	55.7	45.7	55.1	46.3	72
56.7	45.9	56.4	46.3	55.9	46.9	73
57.5	46.6	57.2	46.9	56.7	47.6	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
51 Degrees		4 $\frac{1}{2}$ Points		50 Degrees		

204 *A Table of Difference of*

Diff.	3 $\frac{1}{4}$ Point		37 Degrees		38 Degrees	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	60.2	44.7	59.9	45.1	59.2	46.2
76	61.0	45.3	60.7	45.7	60.0	46.8
77	61.8	45.9	61.5	46.3	60.5	47.4
78	62.7	46.5	62.3	46.9	61.7	48.0
79	63.5	47.1	63.1	47.5	62.2	48.6
80	64.3	47.7	63.9	48.1	63.0	49.3
81	65.1	48.3	64.7	48.7	63.8	49.9
82	65.9	48.8	65.5	49.3	64.6	50.5
83	66.7	49.4	66.3	49.9	65.4	51.1
84	67.5	50.0	67.1	50.5	66.2	51.7
85	68.3	50.6	67.9	51.1	67.0	52.3
86	69.1	51.2	68.7	51.7	67.8	52.9
87	70.9	51.8	69.5	52.4	68.6	53.6
88	70.7	52.4	70.3	53.0	69.3	54.2
89	71.5	53.0	71.1	53.6	70.1	54.8
90	72.3	53.6	71.9	54.2	70.9	55.4
91	73.1	54.2	72.7	54.8	71.7	56.0
92	73.9	54.8	73.5	55.4	72.5	56.6
93	74.7	55.4	74.3	56.0	73.3	57.3
94	75.5	56.0	75.1	56.6	74.1	57.9
95	76.3	56.6	75.9	57.2	74.9	58.5
96	77.1	57.2	76.7	57.8	75.6	59.1
97	77.9	57.8	77.5	58.4	76.4	59.7
98	78.7	58.4	78.3	59.0	77.2	60.3
99	79.5	59.0	79.1	59.6	78.0	60.9
100	80.3	59.6	79.9	60.2	78.8	61.6
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	4 $\frac{3}{4}$ Points		53 Degrees		52 Degrees	

Latitude and Departure. 205

39 Degrees		3 $\frac{1}{2}$ Point		40 Degrees		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
58.3	47.2	58.0	47.6	57.4	48.2	75
59.1	47.8	58.7	48.2	58.2	48.9	76
59.8	48.5	59.5	48.8	59.0	49.5	77
60.6	49.1	60.3	49.5	60.7	50.1	78
61.4	49.7	61.1	50.1	60.5	50.8	79
62.2	50.3	61.8	50.7	61.3	51.4	80
62.9	51.0	62.6	51.4	62.0	52.1	81
63.7	51.6	63.4	52.0	62.8	52.7	82
64.5	52.2	64.2	52.6	63.6	53.4	83
65.3	52.9	64.9	53.3	64.3	54.0	84
66.1	53.5	65.7	53.9	65.1	54.6	85
66.8	54.1	66.5	54.6	65.9	55.3	86
67.6	54.8	67.2	55.2	66.6	55.9	87
68.4	55.4	68.0	55.8	67.4	56.6	88
69.2	56.0	68.8	56.5	68.2	57.2	89
69.9	56.6	69.6	57.1	68.9	57.4	90
70.7	57.3	70.3	57.7	69.7	58.5	91
71.5	57.9	71.1	58.4	70.5	59.1	92
72.3	58.5	71.9	59.0	71.2	59.8	93
73.0	59.2	72.7	59.6	72.0	60.4	94
73.8	59.8	73.4	60.3	72.8	61.1	95
74.6	60.4	74.2	60.9	73.5	61.7	96
75.4	61.0	75.0	61.5	74.3	62.1	97
76.2	61.7	75.7	62.2	75.2	63.0	98
76.9	62.3	76.5	62.8	75.8	63.6	99
77.7	62.9	77.3	63.4	76.6	64.3	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
51 Degrees		4 $\frac{1}{2}$ Point		50 Degrees		

206 *A Table of Difference of*

Diff.	41 Degrees		42 Degrees		3 $\frac{3}{4}$ Points	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	00,7	00,7	00,7	00,7	00,7	00,7
2	01,5	01,3	01,5	01,3	01,5	01,3
3	02,3	02,0	02,2	02,0	02,2	02,0
4	03,0	02,6	03,0	02,7	03,0	02,7
5	03,8	03,3	03,7	03,3	03,7	03,4
6	04,5	03,9	04,5	04,0	04,4	04,0
7	05,3	04,6	05,2	04,7	05,2	04,7
8	06,0	05,2	05,9	05,3	05,9	05,4
9	06,8	05,9	06,7	06,0	06,7	06,0
10	07,5	06,6	07,4	06,7	07,4	06,7
11	08,3	07,2	08,2	07,4	08,1	07,4
12	09,1	07,9	08,9	08,0	08,9	08,1
13	09,8	08,5	09,7	08,7	09,6	08,7
14	10,6	09,2	10,4	09,4	10,4	09,4
15	11,3	09,8	11,1	10,0	11,1	10,1
16	12,1	10,5	12,9	10,7	11,9	10,9
17	12,8	11,1	12,6	11,4	12,6	11,4
18	13,6	11,8	13,4	12,0	13,3	12,1
19	14,3	12,5	14,1	12,7	14,1	12,8
20	15,1	13,1	14,9	13,4	14,8	13,4
21	15,8	13,8	15,6	14,0	15,6	14,1
22	16,6	14,4	16,3	14,7	16,3	14,8
23	17,4	15,1	17,1	15,4	17,0	15,4
24	18,1	15,7	17,8	16,1	17,8	16,1
Diff.	49 Degrees		48 Degrees		4 $\frac{1}{4}$ Point	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.

Latitude and Departure. 207

43 Degrees		44 Degrees		4 Point		Dift.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
00,7	00,7	00,7	00,7	00,7	00,7	1
01,5	01,4	01,4	01,4	01,4	01,4	2
02,2	02,0	02,2	02,1	02,1	02,1	3
02,9	02,7	02,9	02,8	02,8	02,8	4
03,6	03,4	03,6	03,5	03,5	03,5	5
04,4	04,1	04,3	04,2	04,2	04,2	6
05,1	04,8	05,0	04,9	04,9	04,9	7
05,8	05,5	05,7	05,6	05,7	05,7	8
06,6	06,1	06,5	06,2	06,4	06,4	9
07,3	06,8	07,2	06,9	07,1	07,1	10
08,0	07,5	07,9	07,6	07,8	07,8	11
08,8	08,2	08,6	08,3	08,5	08,5	12
09,5	08,9	09,3	09,0	09,2	09,2	13
10,2	09,5	10,1	09,7	09,9	09,9	14
11,0	10,2	10,8	10,4	10,6	10,6	15
11,7	10,6	11,5	11,1	11,3	11,3	16
12,4	11,6	12,2	11,8	12,0	12,0	17
13,2	12,3	12,9	12,5	12,7	12,7	18
13,9	13,0	13,7	13,2	13,4	13,4	19
14,6	13,6	14,4	13,9	14,1	14,1	20
15,4	14,3	15,1	14,6	14,8	14,8	21
16,1	15,0	15,8	15,3	15,5	15,5	22
16,8	15,7	16,5	16,0	16,3	16,3	23
17,5	16,4	17,3	16,7	17,0	17,0	24
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dift.
47 Degrees		46 Degrees		4 Point		

208 *A Table of Difference of*

Diff.	41 Degrees		42 Degrees		3 $\frac{3}{4}$ Point	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
25	18.9	16.4	18.6	16.7	18.5	16.8
26	19.6	17.1	19.3	17.4	19.3	17.4
27	20.4	17.7	20.1	18.1	20.0	18.1
28	21.1	18.4	20.8	18.7	20.7	18.8
29	21.9	19.0	21.5	19.4	21.5	19.5
30	22.6	19.7	22.3	20.1	22.2	20.1
31	23.4	20.3	23.0	20.7	23.0	20.8
32	24.1	21.0	23.8	21.4	23.7	21.5
33	24.9	21.6	24.5	22.1	24.4	22.2
34	25.6	22.3	25.3	22.7	25.2	22.8
35	26.4	23.0	26.0	23.4	25.9	23.5
36	27.2	23.6	26.7	24.1	26.7	24.2
37	27.9	24.3	27.5	24.7	27.4	24.8
38	28.7	24.9	28.2	25.4	28.2	25.5
39	29.4	25.6	29.0	26.1	28.9	26.2
40	30.2	26.2	29.7	26.8	29.6	26.9
41	31.0	26.9	30.5	27.4	30.4	27.5
42	31.7	27.5	31.2	28.1	31.1	28.2
43	32.5	28.2	31.9	28.8	31.9	28.9
44	33.2	28.9	32.7	29.4	32.6	29.5
45	34.0	29.5	33.4	30.1	33.3	30.2
46	34.7	30.2	34.2	30.8	34.1	30.9
47	35.5	30.8	34.9	31.4	34.8	31.6
48	36.3	31.5	35.7	32.1	35.6	32.2
49	37.0	32.1	36.4	32.8	36.3	32.9
50	37.7	32.8	37.2	33.5	37.0	33.6
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	49 Degrees		48 Degrees		4 $\frac{1}{4}$ Point	

43 Degrees		44 Degrees		4 Points		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
18.3	17.1	18.0	17.4	17.7	17.7	25
19.0	17.7	18.7	18.1	18.4	18.4	26
19.7	18.4	19.4	18.8	19.1	19.1	27
20.5	19.1	20.1	19.4	19.8	19.8	28
21.2	19.8	20.9	20.0	20.5	20.5	29
21.9	20.5	21.6	20.8	21.2	21.2	30
22.6	21.1	22.3	21.5	21.9	21.9	31
23.4	21.8	23.0	22.2	22.6	22.6	32
24.1	22.5	23.7	22.9	23.3	23.3	33
24.9	23.2	24.5	23.6	24.0	24.0	34
25.6	23.9	25.2	24.3	24.7	24.7	35
26.3	24.5	25.9	25.0	25.4	25.4	36
27.0	25.2	26.6	25.7	26.1	26.1	37
27.8	25.9	27.3	26.4	26.9	26.9	38
28.5	26.6	28.0	27.1	27.6	27.6	39
29.2	27.3	28.8	27.8	28.3	28.3	40
30.0	28.0	29.5	28.5	29.0	29.0	41
30.7	28.6	30.2	29.2	29.7	29.7	42
31.4	29.3	30.9	29.9	30.4	30.4	43
32.2	30.0	31.6	30.6	31.1	31.1	44
32.9	30.7	32.4	31.3	31.8	31.8	45
33.6	31.4	33.1	32.0	32.5	32.5	46
34.4	32.1	33.8	32.6	33.2	33.2	47
35.1	32.7	34.5	33.3	33.9	33.9	48
35.8	33.4	35.2	34.0	34.6	34.6	49
36.6	34.1	36.0	34.7	35.3	35.3	50
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
47 Degrees		46 Degrees		4 Points		

210 *A Table of Difference of*

Diff.	41 Degrees		42 Degrees		3 $\frac{3}{4}$ Points	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
51	38.5	33.5	37.9	34.1	37.8	34.2
52	39.2	34.1	38.6	34.8	38.5	34.9
53	40.0	34.8	39.4	35.5	39.3	35.6
54	40.8	35.4	40.1	36.1	40.0	36.3
55	41.5	36.0	40.9	36.8	40.7	36.9
56	42.3	36.7	41.6	37.5	41.5	37.6
57	43.0	37.4	42.4	38.1	42.2	38.3
58	43.8	38.1	43.1	38.8	43.0	38.9
59	44.5	38.7	43.8	39.5	43.7	39.6
60	45.3	39.4	44.6	40.1	44.5	40.3
61	46.0	40.0	45.3	40.8	45.2	41.0
62	46.8	40.7	46.1	41.5	45.9	41.6
63	47.6	41.3	46.8	42.2	46.7	42.3
64	48.3	42.0	47.5	42.8	47.4	43.0
65	49.1	42.6	48.3	43.5	48.2	43.6
66	49.8	43.3	49.0	44.2	48.9	44.3
67	50.6	44.0	49.8	44.8	49.6	45.0
68	51.3	44.6	50.5	45.5	50.4	45.7
69	52.1	45.3	51.3	46.2	51.1	46.3
70	52.8	45.9	52.0	46.8	51.9	47.0
71	53.6	46.6	52.8	47.5	52.6	47.7
72	54.3	47.2	53.5	48.2	53.3	48.3
73	55.1	47.9	54.2	48.8	54.1	49.0
74	55.9	48.5	55.0	49.5	54.8	49.7
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	45 Degrees		48 Degrees		4 $\frac{1}{4}$ Points	

Latitude and Departure. 211

43 Degrees		44 Degrees		4 Points		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
37.3	34.8	36.7	35.4	36.1	36.1	51
38.0	35.5	37.4	36.1	36.8	36.8	52
38.8	36.1	38.1	36.8	37.5	37.5	53
39.5	36.8	38.8	37.5	38.2	38.2	54
40.2	37.5	39.6	38.2	38.9	38.9	55
41.0	38.2	40.3	38.9	39.6	39.6	56
41.7	38.9	41.0	39.6	43.3	40.3	57
42.4	39.5	41.7	40.3	41.0	41.0	58
43.1	40.2	42.4	41.0	41.7	41.7	59
43.8	40.9	43.2	41.7	42.4	42.4	60
44.6	41.7	43.9	42.4	43.1	43.1	61
45.3	42.3	44.6	43.1	43.8	43.8	62
46.1	43.0	45.3	43.8	44.5	44.5	63
46.8	43.6	46.0	44.5	45.3	45.3	64
47.5	44.3	46.8	45.1	46.0	46.0	65
48.3	45.0	47.5	45.8	46.7	46.7	66
49.0	45.7	48.2	46.5	47.4	47.4	67
49.7	46.4	48.9	47.2	48.1	48.1	68
50.5	47.1	49.6	47.9	48.8	48.8	69
51.2	47.7	50.3	48.6	49.5	49.5	70
51.9	48.4	51.1	49.3	50.2	50.2	71
52.7	49.1	51.8	50.0	50.9	50.9	72
53.4	49.8	52.5	50.7	51.6	51.6	73
54.1	50.5	53.2	51.4	52.3	52.3	74
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
47 Degrees		46 Degrees		4 Points		

212 *A Table of Difference of*

Dif.	41 Degrees		42 Degrees		3 $\frac{3}{4}$ Points	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
75	56.6	49.2	55.7	50.2	55.6	50.4
76	57.4	49.9	56.5	50.9	56.3	51.0
77	58.1	50.5	57.2	51.5	57.1	51.7
78	58.9	51.2	58.0	52.1	57.8	52.4
79	59.6	51.8	58.7	52.8	58.5	53.1
80	60.4	52.5	59.4	53.5	59.3	53.7
81	61.1	53.1	60.2	54.2	60.0	54.4
82	61.5	53.8	60.9	54.9	60.8	55.1
83	62.6	54.5	61.7	55.5	61.5	55.7
84	63.4	55.1	62.4	56.2	62.2	56.4
85	64.2	55.9	63.2	56.9	63.0	57.1
86	64.9	56.4	63.9	57.5	63.7	57.7
87	65.7	57.1	64.7	58.2	64.5	58.4
88	66.4	57.7	65.4	58.9	65.2	59.1
89	67.2	58.4	66.1	59.6	65.9	59.8
90	67.9	59.0	66.9	60.2	66.7	60.4
91	68.7	59.7	67.6	60.9	67.4	61.1
92	69.4	60.4	68.4	61.6	68.2	61.8
93	70.2	61.0	69.1	62.2	68.9	62.4
94	71.0	61.7	69.9	62.9	69.6	63.1
95	71.7	62.3	70.6	63.6	70.4	63.8
96	72.5	63.0	71.3	64.2	71.1	64.5
97	73.2	63.6	72.1	64.9	71.9	65.1
98	74.0	64.3	72.8	65.6	72.6	65.8
99	74.7	65.0	73.6	66.2	73.4	66.5
100	75.5	65.6	74.3	66.9	74.1	67.2
Dif.	49 Degrees		48 Degrees		4 $\frac{1}{4}$ Points	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.

Latitude and Departure. 213

43 Degrees		44 Degrees		4 Points		Diff.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
54.8	51.1	53.9	52.1	53.0	53.0	75
55.6	51.8	54.7	52.8	53.7	53.7	76
56.3	52.5	55.4	53.5	54.4	54.4	77
57.0	53.2	56.1	54.2	55.2	55.2	78
57.8	53.9	56.8	54.9	55.9	55.9	79
58.5	54.6	57.5	55.6	56.6	56.6	80
59.2	55.2	58.3	56.3	57.3	57.3	81
60.0	55.9	59.0	57.0	58.0	58.0	82
60.7	56.6	59.7	57.6	58.7	58.7	83
61.4	57.3	60.4	58.3	59.4	59.4	84
62.2	58.0	61.1	59.0	60.1	60.1	85
63.0	58.6	61.9	59.7	60.8	60.8	86
63.6	59.3	62.6	60.4	61.5	61.5	87
64.4	60.0	63.3	61.1	62.2	62.2	88
65.1	60.7	64.0	61.8	62.9	62.9	89
65.8	61.4	64.7	62.5	63.6	63.6	90
66.5	62.1	65.5	63.2	64.3	64.3	91
67.3	62.7	66.2	63.9	65.0	65.0	92
68.0	63.4	66.9	64.6	65.8	65.8	93
68.7	64.1	67.6	65.3	66.5	66.5	94
69.5	64.8	68.3	66.0	67.2	67.2	95
70.2	65.5	69.1	66.7	67.9	67.9	96
70.9	66.1	69.8	67.4	68.6	68.6	97
71.7	66.8	70.5	68.1	69.3	69.3	98
72.4	67.5	71.2	68.8	70.0	70.0	99
73.1	68.2	71.9	69.5	70.7	70.7	100
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
47 Degrees		46 Degrees		4 Points		



A
COMPENDIUM
OF
CURRENT SAILING.

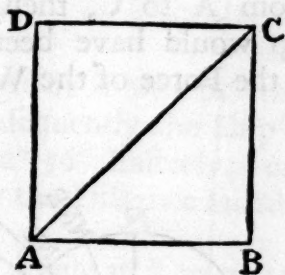
SECT. IX.



VERY Ship that sails, moves thro' two Mediums, Air and Water, and she has a Force (which gives her Motion) impelled upon her, sometimes from one, sometimes from the other, and sometimes from both: Now, when either Force acts upon her separately, she then moves according to the Direction of that Force directly; but when both Forces act upon her jointly, then she moves according to the Direction of neither, but receives a Motion compounded of both, as will appear from what follows.

Suppose a Ship at A, and the Wind blew directly to B, if the Water has no Motion, then will the Ship move on directly toward B, but if the Water has a Motion from A towards D, and the Force of the Sea equal to that of the Wind,
not-

notwithstanding the Ship (by the Steersman) is directed towards B, it will not arrive there; but, instead of the Point B, will arrive at the Point C; that is, the Ship's Track will be the Diagonal of a Square: But if the impelling Force of the one, be stronger than that of the other, then the Track described by the Ship, will be the Diagonal of a Parallelogram. The Motion of the Water is called the Current, and the estimating the Ship's Way from the two Forces jointly acting, called *Current Sailing*.



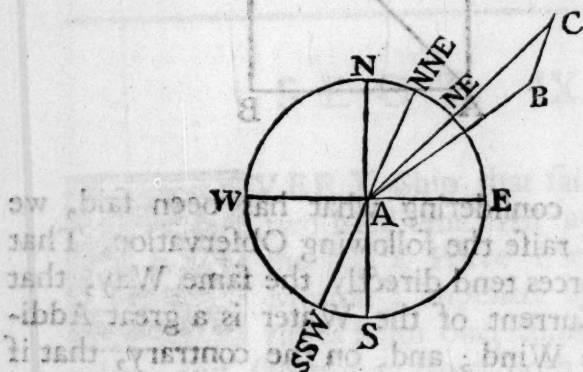
So that, considering what has been said, we may justly raise the following Observation, That if both Forces tend directly the same Way, that then the Current of the Water is a great Addition to the Wind; and, on the contrary, that if they are opposite, that the Current stops the Ship's Course. This being so very plain, a few Examples will be enough.

EXAMPLE I.

If a Ship sails N. E. 110 Miles in 15 Hours, and a Current is found to set S. S. W. at the Rate of two Miles an Hour, it is required to find the Ship's true Course and Distance.

GEOMETRICALLY.

Having drawn the Circle N E S W, and in it the Points N E and S S W, upon the N E Course set off 110 from A to C, then is C the Place where the Ship would have been, had she been acted upon by the Force of the Wind only.



Then from C draw CB ($= 30$ Miles the Velocity of the Current in 15 Hours) parallel to the S S W Line, then is B the true Station of the Ship when acted upon by the Force of both Wind and Water.

And AB the true Distance sailed, and the Angle NAB the true Angle of the Ship's Course.

By

By the Logarithms.

As the Sum of the Sides A C and C B }
 = 140 - - - - - } 2,1461180

Is to the Difference of the Sides }
 = 80 - - - - - } 1,9030900

So is the Tangent of $\frac{1}{2}$ the Sum of }
 the $\angle$ s A and B $78^{\circ} 45'$ - - - } 10,7013382

To the Tangent of $\frac{1}{2}$ their Differ- }
 ence = $70^{\circ} 49'$ - - - } 10,4583002

Hence the Angle at A will be found to be $7^{\circ} 56'$ Easterly, consequently the Ship's true Course will be North $52^{\circ} 56'$ Easterly, or N E $7^{\circ} 56'$ Easterly ; and for the Distance sailed,

As the Sine of the Angle at A = $7^{\circ} 56'$ 9,1399445

Is to the Drift of the Current = 30' 1,4771213

So is the Sine of the $\angle$ at C = $22^{\circ} 30'$ 9,5828397

To the Distance run 83,18 Miles - 1,9200165

By the Sliding Scale.

Place 140 on the Line of Numbers, against the Tangent of $78^{\circ} 45'$, and then 80 on the Line of Numbers, will be against $70^{\circ} 49'$ on the Tangents. Again,

Place the Sine of $7^{\circ} 56'$, against 30 on the Line of Numbers, and $22^{\circ} 30'$ will be against 83,18 Miles, the true Distance run.

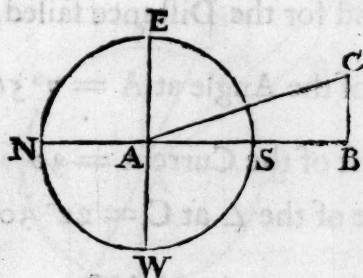
EXAMPLE II.

Suppose a Current setteth East two Miles an Hour, and a Ship sails South six Miles an Hour, what is her true Course and Distance?

GEOMETRICALLY.

Describe a Circle, as in the foregoing Problem, and supposing the Ship at A, set off from the Scale of equal Parts 6 Miles from A to B.

Then, from the same Scale, set off 2 Miles from B to C, in such a Manner, that BC may be parallel to WE; then draw AC, and your Projection is finished.

**TRIGONOMETRICALLY.**

In the right-angled Triangle ABC, there is given the Legs AB and BC, to find the Angle A, and the Hypothenufe AC: Thus,

As AB = 6	-	-	-	0,778151
Is to the Radius	-	-	-	10,000000
So is the Leg CB = 2	-	-	-	0,301030
To the Tangent of $\angle A = 18^\circ 26'$				9,522889
				Again,

Again,

As the $\angle A = 18^\circ 26'$	- - -	9,499963
Is to the Leg $BC = 2$	- - -	0,301030
So is the Radius	- - -	10,000000
To the Hypothenufe $AC = 6,33$		801067

By the Sliding Scale.

Place 6 on the Line of Numbers, against 45 on the Tangents, and 2 on the Numbers will be against $18^\circ 26'$ on the Tangents. Again,

Place $18^\circ 26'$ on the Line of Sines, against 2 on the Line of Numbers, and then 90° on the Sines, will be against 6,33 on the Numbers = the Distance sailed.

EXAMPLE III.

A Ship coming out from Sea in the Night, has Sight of Scilly Light, bearing NE by N, Distance four Leagues, it being then Flood Tide, setting ENE about two Miles an Hour, the Ship running after the Rate of 5 Knots an Hour, I demand upon what Course she must sail, and how far, to hit the Lizard, it bearing from Scilly $E \frac{1}{2} S$, Distance 17 Leagues?

GEOMETRICALLY.

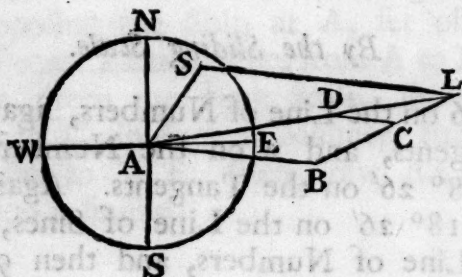
Having drawn the Circle NESW as before, let A represent the Place of the Ship at Sea, and draw NE by N, Line $AS = 12$ Miles, then will S represent Scilly.

From

226 A COMPENDIUM of

From S draw SL equal to 51 Miles, and parallel to the $E \frac{1}{2} S$ Line, then will L be the Place of the *Lizard*.

Draw LC = 2 Miles, and parallel to the EN E Line, and CD = .5 Miles; and parallel to it draw AB 'till it meet LC produced in B, then will AB be the Distance required, and the $\angle S A B$ the true Course.



TRIGONOMETRICALLY.

In the Triangle ASL there is given AS = 12 Miles, and SL = 51 Miles, and the Angle ASL = $118^{\circ} 7' \frac{1}{2} = 10 \frac{1}{2}$ Points = Distance between the NE by N and $W \frac{1}{2} N$, whence to find the $\angle S A L$ by *Case III.* of *Trigonometry*.

As the Sum of the Sides AS and	}	1,7993405
SL = 63 - - -		
To their Difference = 39 - - -	}	1,5910646
So is the Tangent of $\frac{1}{2}$ the $\angle s A$		
and L = $30^{\circ} 56'$ - - -	}	9,7776284

To the Tangent of $\frac{1}{2}$ their Difference = $20^{\circ} 21'$ - - -	}	9,5693525

Whence the Angle A will be found equal to $51^{\circ} 17'$, and consequently the direct Bearing of the Ship from the *Lizard*, is N $85^{\circ} 2'$ E or E by N $6^{\circ} 17'$ E, and for the Distance it will be,

As

CURRENT SAILING. 221

As the Sine of the $\angle A$ $51^{\circ} 17'$ - 9,8922329

To the Dist. from *Scilly* to the *Liz.* 51 1,7075702

So is the Sine of the $\angle$ at $S = 118^{\circ} 7' \frac{1}{2}$ 9,9454298

To the Distance between the Ship }
and *Lizard* 57,65 Miles - } 1,7607671

Again, In the Triangle $DL C$, are given the $\angle$ at $L = 7^{\circ} 32'$, the Distance between the $E N E$ and the $N 85^{\circ} 2'$, the Side $CL =$ the Current's Drift in an Hour, equal to 2 Miles; and the Side CD , the Ship's Distance, run in the same Time equal to 5 Miles, thence to find the Angle at D .

As the Distance run $CD = 5$ - 0,6989700

Is to Drift of Current $LC = 2$ Miles 0,3010300

So is Sine of Angle at $L = 17^{\circ} 32'$ 9,4789423

To the Sine of the $\angle D = 6^{\circ} 55'$ 9,0810023

Hence, because the Angle $CDL = BAL$, the Course the Ship must steer, is $S 88^{\circ} 3'$ East; and for the Distance it will be,

As the Sine of the $\angle C = 155^{\circ} 33'$ - 9,6168944

Is to the Distance between the Ship }
and *Lizard* 57,65 - } 1,7607671

So is Sine of Angle at $L = 17^{\circ} 32'$ 9,4789423

To Distance she must sail 41,96 Miles 1,6228150

So that the Ship in sailing 8 Hours $24'$ South $88^{\circ} 3'$, will arrive at the *Lizard*.



A
COMPENDIUM
 OF

*Sailing by Middle Latitude: Or, A
 Proportion drawn from the Middle Latitude,
 nearly agreeing with Mercator's Sailing.*

S E C T. X.

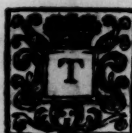
To find the Middle Latitude, always add the Latitude you are come from, to the Latitude you are come to, and Half the Sum is the Middle Latitude.

C A S E I.

Both Latitudes and Difference of Longitude of two Places given, to find the Course and Distance.

E X A M P L E.

A Ship sails from the Latitude of $45^{\circ} 30'$ North, between the North and the East, 'till she arrives in the Latitude of $50^{\circ} 10'$, and then finds her Difference of Longitude $8^{\circ} 16'$, I demand her Course and Distance?



HIS Way by Middle Latitude, depends upon this following Theorem.

As the Cosine of Middle Latitude,
 Is to the Tangent of the Course ;
 So is the Diff. of Lat. in Miles or Leagues,
 To the Diff. of Long. in Miles or Leagues.

A COMPENDIUM of, &c. 223

8° 16'

60

496 equal to the Difference of Longitude.

50° 10'

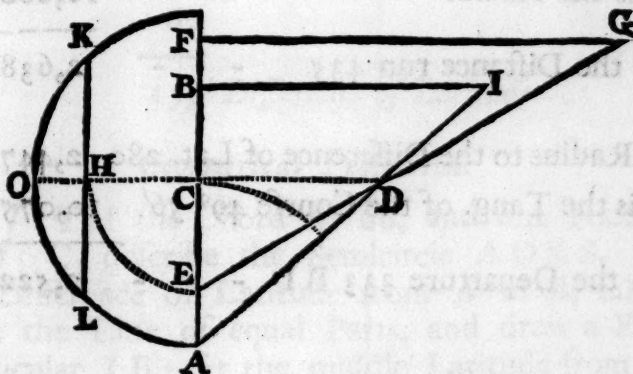
45 30

Difference of Lat. 4 40 equal to 280

The Sum 95 40

The Half 47 50 equal to Middle Lat.

To delineate this, with the Chord of 60°, and upon the Centre C, strike a Semicircle, and draw the Diameter A F, and draw O D at right Angles thereunto.



Set the Difference of Latitude from A to B, and draw B I perpendicular to A F.

Set the Middle Latitude from O to K and L, and draw K L to cut D O in H, make C E equal to C H, and make E F equal to A B, and draw F G parallel to B I, and set the Difference of Longitude given, from F to G, and draw the Line E G

224 A COMPENDIUM of

EG to cut OD in the Point D. Lay a Ruler upon A and the Point D, and draw the Line ADI to cut BI in I, so shall AI, being measured upon the same Scale of equal Parts that AB and FG were laid down by, shew the Distance required; and the Angle BAI is the Course, and BI the Departure.

As Difference of Lat. EF = AB 280 7,552842
Is to the Difference of Long. FG 496 2,695482
So is Cosine of the Middle Lat. = } 9,826930
47° 50' - - - - -

To the Tang. of the Course BAI 49° 56' 10,075254

As Cosine, Course 40° 4' AIB - - 9,808669

To the Difference of Latitude 280 2,447158

So is the Radius - - - 10,000000

To the Distance run 435 - - 2,638489

As Radius to the Difference of Lat. 280 2,447158

So is the Tang. of the Course 49° 56' 10,075254

To the Departure 333 BI - - 2,522412

Set the Difference of Latitude from A to B, and draw BI perpendicular to AB. Set the Middle Latitude from O to K and I, and draw KI to cut DO in H, make CE equal to KI, and make BE perpendicular to AB, and draw FG parallel to BI, and for the Difference of Longitude given, from F to G, and draw the Line BG.

CASE

C A S E II.

Both Latitudes and Course given, the Distance, Difference of Longitude, and Departure, requir'd.

E X A M P L E.

A Ship being in the Latitude of 30° North, sails N N W $\frac{3}{4}$ W, until she arrives in the Latitude of $32^{\circ} 29'$: I demand the Distance, Difference of Longitude, and Departure?

$$\begin{array}{rcl} \text{Latitude} & \left\{ \begin{array}{l} 32^{\circ} 29' \\ 30^{\circ} 00' \end{array} \right\} & \begin{array}{l} 62^{\circ} 39' \text{ The Sum.} \\ \hline \end{array} \end{array}$$

$$\begin{array}{rcl} \text{Difference} & 22^{\circ} 39' & 31^{\circ} 19' \text{ Middle Latitude.} \\ & 60 & \hline \end{array}$$

159 Difference of Latitude.

Geometrical Projection.

WITH the Chord of 60, and one Foot in C, describe the Semicircle A O K S, set the Difference of Latitude from A to B, taken from the Line of equal Parts, and draw a Perpendicular I B; set the middle Latitude from O to K and L taken from the Chords, and draw K L; set the Distance C H from C to E, and the Distance A B from E to F, and then draw the Perpendicular F G; on the Point A make the Angle of the Course N N W $\frac{3}{4}$ W, $31^{\circ} 18'$, 'till it cut B I in I and O C, continued in D; then draw D E 'till it cut F G in G, and the Projection is finished. The Parts are the same as in the last.

As C. of middle Latitude $31^{\circ} 19'$ - 0,068386

Is to the Difference of Latitude 159 2,201 397

So T. of the Course $30^{\circ} 56'$ - - - 9,777628

To the Difference of Longitude 111,5 2,047411

As C. of Course $59^{\circ} 4' \text{ A I B}$ - - 9,933369

Is to the Difference of Lat. 159 A B 2,201397

So is the Radius - - - 10,000000

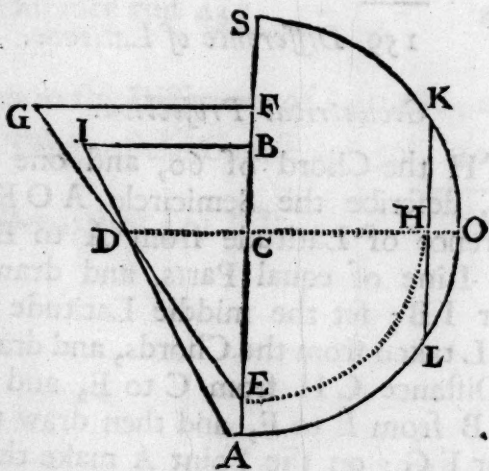
To the Distance 185,4 A I - - 2,268028

As Radius - - - - - 10,00000

To the Difference of Latitude 159 - 2,201397

So is T. of the Course $30^{\circ} 56'$ B A I 9,777628

To the Departure 95,28 B I - - - 1,979025



N. B. C stands for the Comp. Arith. of the true
Log. or the true Log. subtracted from 10,00000.

CASE

C A S E III.

Both Latitudes and Distance given, the Course; Difference of Longitude, and Departure, required.

E X A M P L E.

A Ship in the Latitude of $45^{\circ} 10'$ North, sails between the North and East until she arrives in the Latitude of $50^{\circ} 30'$, and then finds her Distance run 195 Leagues; I demand her Course, Difference of Longitude, and Departure?

$$\begin{array}{rcl}
 50^{\circ} 30' & \} & \text{Sum } 95^{\circ} 40' \\
 45^{\circ} 10' & \} & \\
 \hline
 & & 47^{\circ} 50' = \text{Midd. Latitude.} \\
 \text{Diff. of Lat. } 5^{\circ} 20' & = & 310
 \end{array}$$

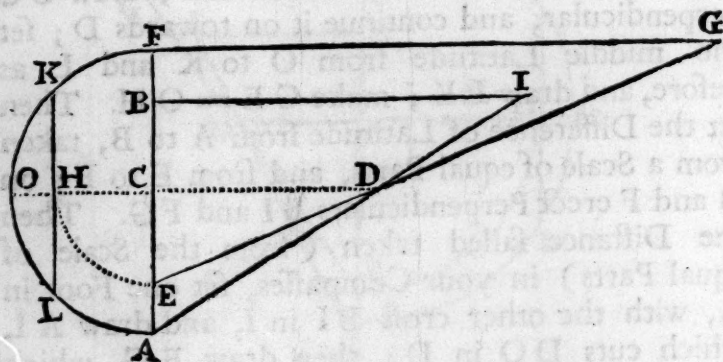
Geometrical Projection.

WITH the Chord of 60, and one Foot in O, draw the Semicircle A O F; draw O C perpendicular, and continue it on towards D; set the middle Latitude from O to K and L as before, and draw L K; make C E = O H. Then set the Difference of Latitude from A to B, taken from a Scale of equal Parts, and from E to F; on B and F erect Perpendiculars B I and F G. Then the Distance sailed taken (from the Scale of equal Parts) in your Compasses, set one Foot in A, with the other cross B I in I, and draw A I, which cuts D O in D; then draw E G, which finisheth the Projection, the Parts being the same as in the last.

As the Distance A I 585 Miles - - 2,76716
 Is to the Radius - - - 10,00000
 So is the Difference of Lat. A B 310 2,49136
 To the C. of the Course A I B 30° - 9,72420

So the Course is N E b E 1° 45' Easterly.

As C. of the middle Latitude 47° 50' 0,17307
 Is to the Difference of Latitude 310 2,49136
 So T. of the Course 58° - - - 10,20421
 To the Difference of Longitude 739 2,86864
 As Radius - - - - - 10,00000
 To the Diff. of Latitude 310 - - - 2,49136
 So T. of the Course 58° - - - 10,20421
 To the Departure 496,1 - - - 2,69557



C A S E IV.

Both Latitudes and Departure given, the Course, Distance and Difference of Longitude required.

E X A M P L E.

A Ship in Latitude $40^{\circ} 30'$ North, sails between the North and West until her Departure is 450 Miles, and then finds her Latitude $51^{\circ} 20'$; I demand her Course, Distance, and Difference of Longitude?

$$\begin{array}{r} 50^{\circ} 20' \\ 40 \quad 30 \end{array} \left. \vphantom{\begin{array}{r} 50^{\circ} 20' \\ 40 \quad 30 \end{array}} \right\} \text{Sum } 91^{\circ} 50'$$

$$10 \quad 50 = 650 \quad 45 \quad 55 \text{ Midd. Latitude.}$$

As the Difference of Latitude A B	650	2,81291
Is to the Departure B I	450 - - -	2,65321
So is the Radius	- - - -	10,00000
To T. of the Course	$34^{\circ} 42'$ - - -	9,84030

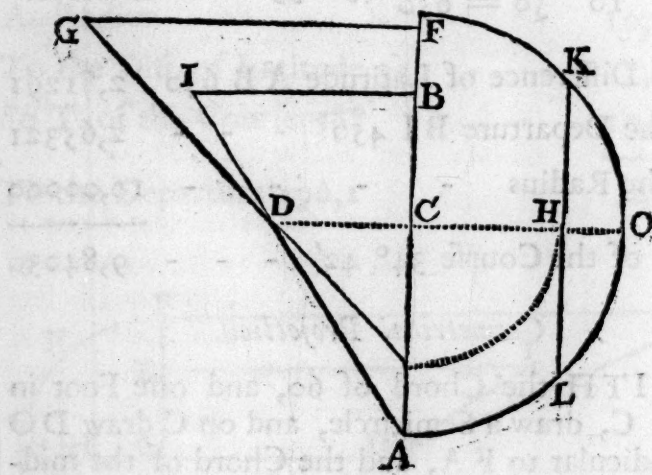
Geometrical Projection.

WITH the Chord of 60, and one Foot in C, draw a Semicircle, and on C draw DO perpendicular to FA, and the Chord of the middle Lat. set from O to K and L, and draw KL, make CE = CH; from the equal Parts set the Diff. of Lat. from A to B, and from A to F, and make IB and GF Perpendiculars to FA; from the equal Parts set the Departure from B to I, and draw IA, which cuts DO in D; thro' DE draw GD 'till it cut GF in G, which finisheth the Projection; the Parts are as before.

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As C of middle Latitude $45^{\circ} 55'$	-	0,15758
Is to T. of the Course $BAI\ 34^{\circ} 42'$	-	9,84030
So is the Difference of Latitude $AB\ 650$		<u>2,81291</u>
To the Difference of Long. $FG\ 646,8$		2,81079
As C. of the Course $55^{\circ} 18'$	- - -	9,91495
Is to the Difference of Latitude 650		2,81291
So is the Radius	- - - -	<u>10,00000</u>
To the Distance run $790,6$	- -	2,89796

The Course is NW *b* N. $57'$ Westerly.



CASE

C A S E V.

One Latitude, Course, and Distance given, to find the Difference of Latitude, Difference of Longitude and Departure.

E X A M P L E.

A Ship being in the Latitude of $39^{\circ} 40'$ North, sails N W $\frac{1}{2}$ W 54° Miles; I demand what Latitude she is now in, and her Difference of Longitude and Departure?

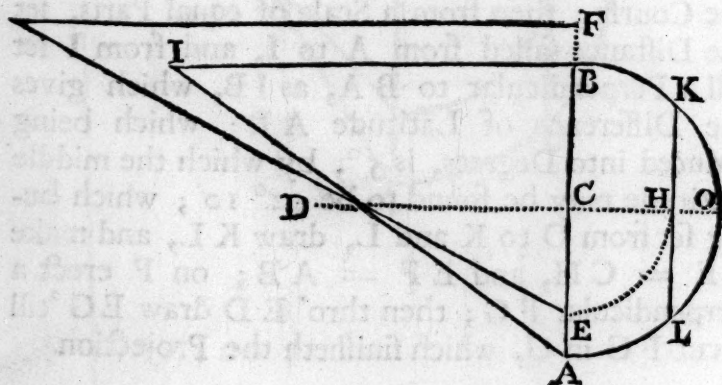
Note, You must find the Difference of Latitude before you can draw the Figure.

Geometrical Projection.

WITH the Chord of 60 , and one Foot in C, draw the Semicircle AOB, and at right Angles to the Diameter AB draw DO; on the Point A make the Angle IAB = $56^{\circ} 15'$ = the Course; then from a Scale of equal Parts, set the Distance sailed from A to I, and from I let fall a Perpendicular to BA, as IB, which gives the Difference of Latitude AB; which being reduced into Degrees, is 5° , by which the middle Latitude may be found to be $42^{\circ} 10'$; which being set from O to K and L, draw KL, and make CE = CH, and EF = AB; on F erect a Perpendicular FG; then thro' ED draw EG till it cut FG in G, which finisheth the Projection.

232 A COMPENDIUM of

As Radius to the Distance 540	- -	2,73239
So Comp. of Course $33^{\circ} 45'$	- -	<u>9,74474</u>
To the Difference of Latitude 300		2,47713
Lat. come from $39^{\circ} 40'$	} Sum of L. = $84^{\circ} 20'$	
Diff. of Lat. + $05^{\circ} 00'$		
		<u> </u>
Lat. come into $44^{\circ} 40'$	Half	<u>42 10</u>
As C. of middle Latitude $47^{\circ} 50'$	-	0,13007
Is to T. of the Course B A I $56^{\circ} 15'$		<u>10,17511</u>
So is the Difference of Latitude A B		<u>2,47713</u>
To the Difference of Long. 605,8 G F		2,78231
As Radius to the Distance A I 540		2,73239
So S. of the Course $56^{\circ} 15'$	- -	<u>9,91985</u>
To the Departure B I 499	- -	<u>2,65224</u>



CASE

Sailing by Middle Latitude. 233

C A S E VI.

One Latitude, Departure, and Course given, the Difference of Latitude, Difference of Longitude, and Distance required.

E X A M P L E.

A Ship from Lisbon, in Latitude $38^{\circ} 50'$ North, sails S W b W. until her Departure 560 Miles; I demand her Distance run, Difference of Longitude, and what Latitude she is in?

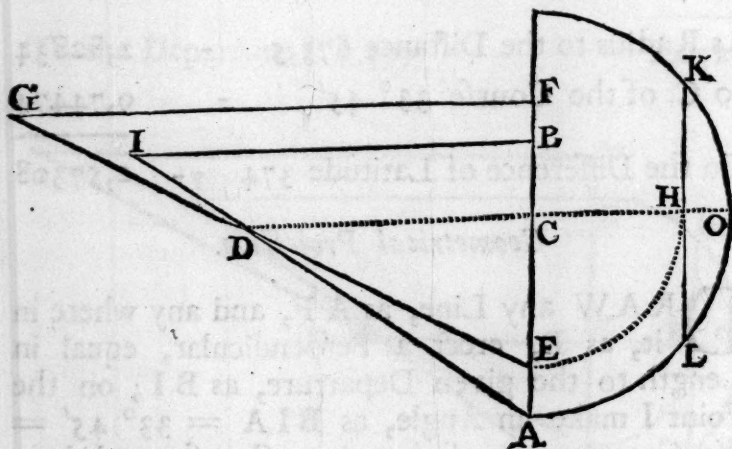
As S. of the Course $56^{\circ} 15'$	-	-	9,91985
To the Departure 560	-	-	2,74819
So is the Radius	-	-	10,00000
To the Distance 673,5	-	-	2,82834
As Radius to the Distance 673,5	-	-	2,82834
So C. of the Course $33^{\circ} 45'$	-	-	9,74474
To the Difference of Latitude 374	-	-	2,57308

Geometrical Projection.

DR A W any Line, as A F, and any where in it, as B, erect a Perpendicular, equal in Length to the given Departure, as B I; on the Point I make an Angle, as $B I A = 33^{\circ} 45' =$ the Complement of the given Course, and draw I A 'till it cuts F A in A; then, with the Chord of 60, and one Foot in A, and the other set in C, in the Line A F, with the Foot that was in A, draw the Semicircle A L K cross A F at right Angles in the Point C, with the Line D C O; so is
A B

Lat. come from	38°	50'	}	Sum of Lat.	71°	26'
Diff. of Lat. —	06	14				
Lat. come into	32	36	}	Midd. Lat.	35	43

As C. of the middle Lat. $54^{\circ} 17'$	-	0,09049
Is to T. of the Course $56^{\circ} 15'$	-	10,17511
So is the Difference of Latitude 374		<u>2,57308</u>
To the Difference of Longitude 689,7		2,83868



CASE

C A S E VII.

One Latitude, Distance, and Departure given, the Difference of Latitude, Difference of Longitude, and Course required.

E X A M P L E.

A Ship in the Latitude of $38^{\circ} 40'$ North, sails between the North and West 650 Miles, and then finds her Departure 546 Miles; I demand her Course, Difference of Latitude, and Difference of Longitude?

As the Distance 650	-	-	-	2,81291
Is to the Departure 546	-	-	-	2,37719
So is the Radius	-	-	-	10,00000
To S. of the Course $57^{\circ} 8'$	-	-	-	9,92428

Geometrical Projection.

DR A W any Line, as A B, and on B erect a Perpendicular B I, equal in Length to the given Departure; then, with the Distance sailed in your Compasses, and one Foot in I, with the other cross A B in A; then the Chord of 60 set from A to C, with the Radius A C draw the Semicircle A L K, and A B, the Difference of Latitude found, being converted into Degrees, the middle Latitude may be found from it to be $41^{\circ} 36'$, which set from O to L and K, and draw K L, make C E = C H, and E F = A B; on F erect a Perpendicular, as G F, at the Point C draw D O at right Angles to A F, and thro' the Point D draw A D, continued 'till it cuts F G in G, which finisheth the Projection.

As

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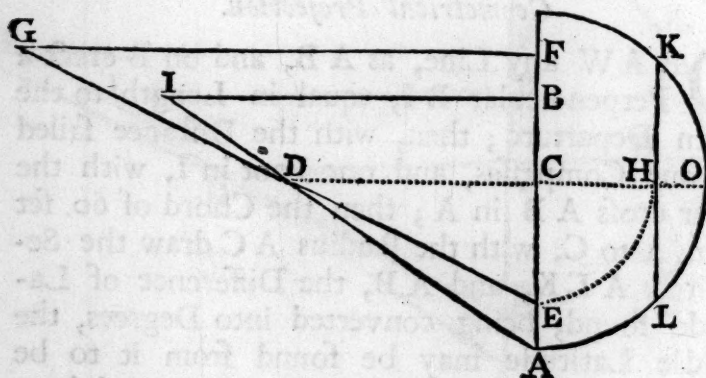
As T. of the Courſe $57^{\circ} 8'$ - - - 10,189697
 Is to the Departure 546 - - - 2,737193
 So is the Radius - - - 10,000000

To the Difference of Latitude 3528 2,547496

Lat. come from $38^{\circ} 40'$	Sum of Lat. $83^{\circ} 13'$
Diff. of Lat. $+ 5 53$	<u> </u>
Lat. come into 44 33	Middle Lat. $41 36$

As C. of middle Latitude $48^{\circ} 29'$ 0,126216
 Is to the T. of the Courſe $57^{\circ} 8'$ 10,189697
 So is the Difference of Latitude 352,8 2,547496
 To the Difference of Longitude 730,2 2,863409

The Courſe is N W *b* W $53'$ Weſterly.





A
COMPENDIUM
OF
MERCATOR SAILING.

S E C T. XI.



IT is very evident to any one who will but view a Terrestrial Globe, that all the Meridians do meet together in the North and South Poles, and that all the Parallels of Latitude do continually decrease according to their Distance from the Equinoctial; consequently that the Measure of a Degree of Longitude, is, in any of those Parallels, less than a Degree of Longitude in the Equinoctial, in such a Proportion as the Semidiameter of the Parallel, is to the Semidiameter or Radius of the Equinoctial; as may be thus proved.

Suppose the Arch PBD to be one Quarter of any Meridian, let P represent one of the Poles, and CD the Semidiameter of the Equinoctial, BD the Latitude of any Place, FB the Semidiameter of that Parallel.

Then

Then is BA the right Sine } of that Lat.
 And $FB (= CA)$ the Cofine }

Now it is well known that the Peripheries of all Circles are in a direct Proportion to their Diameters; consequently they are in a direct Proportion to their Semidiameters, *viz.* their Radius's: (*See the Figure in Page 244*). And,

As the Periphery of one Circle,
 Is to the Periphery of another;
 So is any Part of the first Circle's Periphery,
 To the like Part of the second Circle's Periphery.

Whence it follows, That,

As CD , the Radius of the Equinoctial,
 Is to any Difference of Longitude in Miles;
 So is FB the Cofine of any Latitude,
 To the Number of Miles in that Parallel, which
 are contained between the same two Meri-
 dians or Difference of Longitude.

As for Instance; Suppose it was required to find how many Miles in the Parallel of 50° of Latitude, will be equal to one Degree of Longitude, *viz.* 60 Miles in the Equinoctial, the Proportion as above, *viz.*

As Radius	- - - -	10,000,000
Is to the Difference of Long. 60 Miles		1,778,151.2
So is the Cofine of the Latitude 50°		9,808,067.5
To the Number of Miles requir'd	38,5	1,586,218.7

That is, $38\frac{1}{2}$ Miles in the Parallel of 50° of Latitude, will be equal to one Degree of Longitude, or 60 Miles in the Equinoctial.

From

From hence it is evident, that the Method of Plain Sailing being grounded upon a false Supposition, that the Degrees of Longitude are equal and alike in every Parallel of Latitude, to those of the Equinoctial, is very erroneous.

And therefore it hath been the great Industry of some of our Predecessors, to rectify those Errors, by contriving such a Projection of the Meridians and Parallels in *Plano* (*viz.* with right Lines) whereon all, or any Parts of the World, might be truly set down according to their respective Longitudes, Latitudes, Bearings, and Distances.

And, altho' such a Projection was hinted at by *Ptolomy* many hundred Years ago, yet that Hint was not pursued until one *Mercator*, a famous Geographer of the last Age, took it up, and published a Map of that Kind, whence it is called *Mercator's Chart*, or *Projection*.

But still there wanted a ready Way of describing that Projection, and a Demonstration of its Truth; both which, our ingenious Countryman Mr. *Edward Wright*, hath fully done in his excellent Treatise, intituled, *The Correction of Errors in Navigation*, Printed Anno 1599 (a Book well deserving the Perusal of all such as design to use the Sea) wherein he hath shewed how to enlarge the Degrees of the Meridian toward the Poles, in the same Proportion that the Parallels of Latitude do decrease towards them: And he was, undoubtedly, the first that ever made Tables of Meridional Parts for that Purpose, which he did by the continual Addition of the Secants of $1'$, $2'$, $3'$, $4'$, $5'$, &c. being by that Method computed exact enough for the Mariner's Use.

But, to compleat that Work, the learned Mathematician Dr. *Edmund Halley*, hath, in the *Philosophical Transactions*, N^o 219) given an easy Demon-

Demonstration of the Analogy of the Logarithmick Tangents to the Meridian Line, or Sum of the Secants, with various Methods of computing the same to the utmost Exactness; but for Brevity's sake, I shall omit that ingenious Discourse, and only insert a few Rules deduced from it, whereby the Table of Meridional Parts may either be easily made, or the Want thereof truly supplied by the Table of artificial Tangents.

I. To find the Meridional Parts to any given Latitude.

R U L E.

To the given Latitude add 90° , then find the Tangent of Half that Sum, from which subtract the Radius of the Table 10,000000, the Residue being divided by 1263,3, the Quotient will be the true Meridional Parts contained between the Equinoctial, and that Latitude, according to Mercator's Projection.

E X A M P L E I.

Suppose it were required to find the Meridional Parts contained between the Equinoctial and the Latitude $48^\circ 30'$.

Here $90^\circ + 48^\circ 30' = 138^\circ 30'$; and the Half of that is $69^\circ 15'$; the Tangent to which is 10,4215142; from which, if we subtract the Radius 10,0000000, the Remains is 4215142, which divided by 1263,3, the Quotient is 3336,6, the Meridional Parts answering the Latitude of $48^\circ 30'$ as was required.

EXAMPLE II.

Let it be required to find the Meridional Parts to the Latitude of $89^{\circ} 40'$.

Here $90^{\circ} + 89^{\circ} 40' = 179^{\circ} 40'$, the Half of which is $89^{\circ} 50'$; the Tangent of which is 12,5362727; which made less by the Radius 10,0000000, the Remainder is 25362727; which divided by 1263,3, the Quotient is 20075,6, the Meridional Parts answering $89^{\circ} 40'$, &c.

Proceeding in this Manner, it will not be difficult to calculate a Table of Meridional Parts to what Exactness you please; and from the Work of these two Examples, it will be easy to conceive the Reason of the two following Rules.

II. *To find the Meridional Parts contained between the Latitudes of any two Places that are in the same Hemisphere.*

R U L E.

To each of the given Latitudes add 90° , then find the Tangents of each of their half Sums (as above) the Difference of these two Tangents being divided by 1263,3, the Quotient will be the true Meridional Parts (or Miles) contained between those two Latitudes, according to Mercator's Projection.

E X A M P L E.

Let it be required to find the Meridional Parts between the Latitude of 50°, and 12° 14', both North.

$$\begin{array}{rcl}
 \text{Here } 90^\circ + 50^\circ = 140^\circ, \text{ its Half} & \} & 10,4389341 \\
 70^\circ \text{ Tangent} & - & - \\
 90^\circ + 12^\circ 14' = 102^\circ 14', \text{ its Half} & \} & 10,0934397 \\
 51^\circ 7' \text{ Tangent} & - & -
 \end{array}$$

$$\text{Difference of the Tangents} \quad - \quad - \quad - \quad 3454944$$

Which being divided by 1263,3, the Quotient is 2734,8, the Meridional Parts, or Miles, required.

III. *To find the Meridional Parts contained between the Latitudes of two Places that are in different Hemispheres.*

R U L E.

To each of the given Latitudes add 90°, and find the Tangent to each of their Half Sums as before, then add these two Tangents together, and their Sum made less by twice the Radius, viz. 20,0000000, the Remainder divided by 1263,3, the Quotient is the true Meridional Parts, or Miles, between the two Latitudes.

MERCATOR SAILING. 243

EXAMPLE.

Suppose it was required to find the Meridional Parts between the Latitude of $14^{\circ} 24'$ North, and $16^{\circ} 3'$ South Latitude?

$$\begin{array}{rcl} 90^{\circ} + 16^{\circ} 3' = 106^{\circ} 3', \text{ its Half} & \} & 10,1232799 \\ 53^{\circ} 1' 30'' \text{ Tangent} & - & - \end{array}$$

$$\begin{array}{rcl} 90^{\circ} + 14^{\circ} 24' = 104^{\circ} 24', \text{ its Half} & \} & 10,1103177 \\ 52^{\circ} 12' \text{ Tangent} & - & - \end{array}$$

$$\text{Sum of the Tangents} \quad - \quad - \quad 20,2335976$$

$$\text{Twice the Radius} \quad - \quad - \quad 20,0000000$$

$$\text{Their Remains} \quad - \quad - \quad 2335976$$

Which divided by 1263,3, the Quotient is 1849,1, the Meridional Parts contained between the two given Latitudes, according to *Mercator's* Projection.

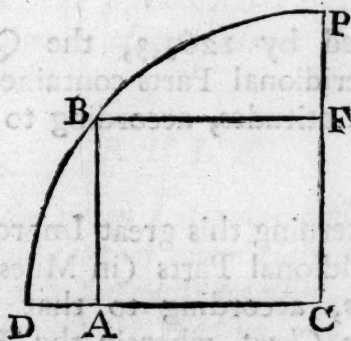
Thus far concerning this great Improvement of finding the Meridional Parts (in Miles) between any two Places, according to that Projection called *Mercator's Chart*, wherein the Degrees of the Meridian Line are enlarged towards the Poles, &c. as hath been said before.

But altho' the true Meridional Distance between any two Places, may be thus found by the Tables of Logarithmick Tangents only, and consequently the Want of a Table of Meridional Parts may, by this Means, be sufficiently supplied upon all Occasions; yet, because a Table of those Parts, if truly made, is very ready, and of good Use, either for graduating the Divisions of the Meridian Line in a *Mercator Chart*, or in calculating

244 A COMPENDIUM of, &c.

lating the usual Problems in Sailing, according to that Projection. I have therefore inserted such a Table, calculated, *de novo*, to every two Minutes so far as 80° of Latitude, which, I presume, is as near to either of the Poles, as Navigation is practicable.

Note, *Altho' the following Table be calculated to all the even Numbers of Minutes, viz. $2' 4' 6' 8'$, &c. yet the true Parts belonging to all the odd Minutes, viz. $1' 3' 5' 7'$, &c. may be found by Inspection only, viz. by taking the Half Sum of the two next even Numbers, and then adding it to the next less, &c. which is so easy to conceive, that it needs no Example.*



A T A-

A
T A B L E
O F
M E R I D I O N A L P A R T S,
O R
M I L E S.



L O N D O N:
Printed in the Year M.DCC.XXIX.

TABLE

MERIDIONAL PARTS

MILES



LONDON:

Printed in the Year M.DCC.XXIX.

A TABLE of, &c. 247

Min.	0° M. P.	1° M. P.	2° M. P.	3° M. P.	4° M. P.
0	0	60	120	180.1	240.2
2	2	62	122	182.1	242.2
4	4	64	124	184.1	244.2
6	6	66	126	186.1	246.2
8	8	68	128	188.1	248.2
10	10	70	130	190.1	250.2
12	12	72	132	192.1	252.2
14	14	74	134	194.1	254.2
16	16	76	136	196.1	256.2
18	18	78	138	198.1	258.2
20	20	80	140	200.1	260.3
22	22	82	142	202.1	262.3
24	24	84	144	204.1	264.3
26	26	86	146	206.1	266.3
28	28	88	148	208.1	268.3
30	30	90	150	210.1	270.3
32	32	92	152	212.1	272.3
34	34	94	154	214.1	274.3
36	36	96	156	216.1	276.3
38	38	98	158	218.1	278.3
40	40	100	160	220.1	280.3
42	42	102	162	222.1	282.3
44	44	104	164	224.1	284.3
46	46	106	166	226.1	286.3
48	48	108	168	228.1	288.3
50	50	110	170	230.1	290.3
52	52	112	172	232.1	292.4
54	54	114	174	234.1	294.4
56	56	116	176	236.1	296.4
58	58	118	178	238.1	298.4

Min.	5°	6°	7°	8°
	M. P.	M. P.	M. P.	M. P.
0	300.4	360.7	421.1	481.6
2	302.4	362.7	423.1	483.6
4	304.4	364.7	425.1	485.6
6	306.4	366.7	427.1	487.6
8	308.4	368.7	429.1	489.6
10	310.4	370.7	431.1	491.7
12	312.4	372.7	433.1	493.7
14	314.4	374.7	435.2	495.7
16	316.5	376.8	437.2	497.7
18	318.5	378.8	439.2	499.8
20	320.5	380.8	441.2	501.8
22	322.5	382.8	443.2	503.8
24	324.5	384.8	445.2	505.8
26	326.5	386.8	447.3	507.8
28	328.5	388.8	449.3	509.9
30	330.5	390.8	451.3	511.9
32	332.5	392.9	453.3	513.9
34	334.5	394.9	455.3	515.9
36	336.5	396.9	457.3	518.0
38	338.5	398.9	459.4	520.0
40	340.6	400.9	461.4	522.0
42	342.6	402.9	463.4	524.0
44	344.6	404.9	465.4	526.0
46	346.6	407.0	467.4	528.1
48	348.6	409.0	469.5	530.1
50	350.6	411.0	471.5	532.1
52	352.6	413.0	473.5	534.1
54	354.6	415.0	475.5	536.2
56	356.6	417.0	477.5	538.2
58	358.7	419.0	479.6	540.2

Meridional Parts, or Miles. 249

Min.	9°	10°	11°	12°
	M. P.	M. P.	M. P.	M. P.
0	542.2	603.1	664.1	725.3
2	544.3	605.1	666.1	727.4
4	546.3	607.1	668.1	729.4
6	548.3	609.2	670.2	731.5
8	550.3	611.2	672.2	733.5
10	552.4	613.2	674.3	735.6
12	554.4	615.3	676.3	737.6
14	556.4	617.3	678.3	739.7
16	558.4	619.3	680.4	741.7
18	560.5	621.3	682.4	743.8
20	562.5	623.4	684.5	745.8
22	564.5	625.4	686.5	747.9
24	566.6	627.4	688.5	749.9
26	568.6	629.5	690.6	752.0
28	570.6	631.5	692.6	754.0
30	572.6	633.5	694.7	756.1
32	574.7	635.6	696.7	758.1
34	576.7	637.6	698.7	760.2
36	578.7	639.6	700.8	762.2
38	580.8	641.7	702.8	764.3
40	582.8	643.7	704.9	766.3
42	584.8	645.7	706.9	768.4
44	586.8	647.7	708.9	770.4
46	588.9	649.8	711.0	772.5
48	590.9	651.8	713.0	774.5
50	592.9	653.9	715.1	776.6
52	595.0	655.9	717.1	778.6
54	597.0	657.9	719.2	780.7
56	599.0	660.0	721.2	782.7
58	601.0	662.0	723.3	784.8

Min.	13°	14°	15°	16°
	M. P.	M. P.	M. P.	M. P.
0	786.8	848.5	910.5	972.8
2	788.9	850.6	912.6	974.8
4	790.9	852.6	914.6	976.9
6	793.0	854.7	916.7	978.9
8	795.0	856.8	918.8	981.0
10	797.1	858.9	920.8	983.1
12	799.1	861.0	922.9	985.2
14	801.2	863.0	925.0	987.3
16	803.2	865.1	927.0	989.4
18	805.3	867.2	929.1	991.5
20	807.3	869.2	931.2	993.6
22	809.4	871.3	933.2	995.6
24	811.4	873.4	935.3	997.7
26	813.5	875.4	937.4	999.8
28	815.5	877.5	939.4	1001.9
30	817.6	879.6	941.5	1004.0
32	819.6	881.6	943.6	1006.1
34	821.7	883.7	945.6	1008.1
36	823.7	885.7	947.7	1010.2
38	825.8	887.8	949.8	1012.3
40	827.9	889.9	951.9	1014.4
42	829.9	892.0	953.9	1016.5
44	832.0	894.0	956.0	1018.6
46	834.0	896.1	958.1	1020.7
48	836.1	898.2	960.2	1022.7
50	838.2	900.2	962.3	1024.8
52	840.3	902.3	964.4	1026.9
54	842.3	904.3	966.5	1029.0
56	844.4	906.4	968.6	1031.1
58	846.5	908.4	970.7	1033.2

Meridional Parts, or Miles. 251

Min.	17° M. P.	18° M. P.	19° M. P.	20° M. P.
0	1035.3	1098.2	1161.5	1225.1
2	1037.4	1100.3	1163.6	1227.3
4	1039.5	1102.4	1165.7	1229.4
6	1041.6	1104.5	1167.8	1231.5
8	1043.7	1106.6	1170.0	1233.6
10	1045.8	1108.7	1172.1	1235.8
12	1047.9	1110.8	1174.2	1237.9
14	1049.9	1112.9	1176.3	1240.0
16	1052.0	1115.0	1178.4	1242.2
18	1054.1	1117.1	1180.5	1244.3
20	1056.2	1119.2	1182.7	1246.4
22	1058.3	1121.3	1184.8	1248.6
24	1060.4	1123.4	1186.9	1250.7
26	1062.5	1125.5	1189.0	1252.8
28	1064.6	1127.6	1191.1	1255.0
30	1066.7	1129.7	1193.2	1257.1
32	1068.8	1131.8	1195.4	1259.2
34	1070.9	1134.0	1197.5	1261.4
36	1073.0	1136.1	1199.6	1263.5
38	1075.1	1138.2	1201.7	1265.6
40	1077.2	1140.3	1203.9	1267.8
42	1079.3	1142.4	1206.0	1269.9
44	1081.4	1144.6	1208.1	1272.1
46	1083.5	1146.7	1210.2	1274.2
48	1085.6	1148.8	1212.4	1276.3
50	1087.7	1150.9	1214.5	1278.5
52	1089.8	1153.0	1216.6	1280.6
54	1091.9	1155.1	1218.7	1282.7
56	1094.0	1157.2	1220.9	1284.9
58	1096.1	1159.4	1223.0	1286.0

Min.	21°	22°	23°	24°
	M. P.	M. P.	M. P.	M. P.
0	1289,2	1353,7	1418,7	1484,1
2	1291,3	1355,8	1420,8	1486,3
4	1293,5	1358,0	1423,0	1488,4
6	1295,6	1360,1	1425,1	1490,6
8	1297,8	1362,3	1427,3	1492,8
10	1299,9	1364,4	1429,5	1495,0
12	1302,0	1366,6	1431,7	1497,2
14	1304,2	1368,7	1433,9	1499,4
16	1306,3	1370,9	1436,0	1501,6
18	1308,5	1373,1	1438,2	1503,8
20	1310,6	1375,3	1440,4	1506,0
22	1312,8	1377,4	1442,6	1508,2
24	1314,9	1379,6	1444,8	1510,4
26	1317,1	1381,8	1446,9	1512,6
28	1319,2	1383,9	1449,1	1514,8
30	1321,4	1386,1	1451,3	1517,0
32	1323,5	1388,3	1453,5	1519,2
34	1325,7	1390,4	1455,7	1521,4
36	1327,8	1392,6	1457,9	1523,5
38	1330,0	1394,8	1460,0	1525,8
40	1332,1	1396,9	1462,2	1528,0
42	1334,2	1399,1	1464,4	1530,2
44	1336,4	1401,3	1466,6	1532,4
46	1338,6	1403,4	1468,8	1534,6
48	1340,7	1405,6	1470,9	1536,8
50	1342,9	1407,8	1473,1	1539,0
52	1345,0	1509,9	1475,3	1541,3
54	1347,2	1412,1	1477,5	1543,4
56	1349,4	1414,3	1479,7	1545,6
58	1351,5	1416,5	1481,9	1547,8

Meridional Parts, *or* Miles. 253

Min.	25°	26°	27°	28°
	M. P.	M. P.	M. P.	M. P.
0	1550,0	1616,5	1683,6	1751,2
2	1552,2	1618,7	1685,8	1753,4
4	1554,4	1620,0	1688,0	1755,7
6	1556,6	1623,2	1690,3	1758,0
8	1558,8	1625,4	1692,5	1760,2
10	1561,0	1627,6	1694,8	1762,5
12	1563,2	1629,8	1697,0	1764,8
14	1565,4	1632,0	1699,3	1767,0
16	1567,6	1634,3	1701,5	1769,3
18	1569,8	1636,5	1703,8	1771,6
20	1572,1	1638,8	1706,0	1773,9
22	1574,3	1641,0	1708,3	1776,1
24	1576,5	1643,2	1710,5	1778,4
26	1578,7	1645,5	1712,8	1780,6
28	1580,9	1647,7	1715,0	1782,9
30	1583,2	1649,9	1717,3	1715,2
32	1585,4	1652,2	1719,5	1787,5
34	1587,6	1654,4	1721,8	1789,8
36	1589,8	1656,6	1724,0	1792,1
38	1592,0	1658,9	1726,3	1794,3
40	1594,3	1661,1	1728,6	1796,6
42	1596,5	1663,4	1730,8	1798,9
44	1598,7	1665,6	1733,1	1801,2
46	1600,9	1667,8	1735,3	1803,5
48	1603,1	1670,1	1737,6	1805,7
50	1605,4	1672,3	1739,9	1808,0
52	1607,6	1674,6	1742,1	1810,3
54	1609,8	1676,9	1744,4	1812,6
56	1612,0	1679,1	1746,6	1814,9
58	1614,3	1681,3	1748,9	1817,2

Min.	29°	30°	31°	32°
	M. P.	M. P.	M. P.	M. P.
0	1819,5	1888,4	1958,1	2028,4
2	1821,7	1890,7	1960,4	2030,7
4	1824,0	1893,0	1962,7	2033,1
6	1826,3	1895,3	1965,0	2035,5
8	1828,6	1897,6	1967,4	2037,8
10	1830,9	1899,9	1969,7	2040,2
12	1833,2	1902,3	1972,0	2042,6
14	1835,5	1904,6	1974,4	2044,9
16	1837,8	1906,9	1976,8	2047,3
18	1840,1	1909,2	1979,1	2049,7
20	1842,4	1911,5	1981,4	2052,0
22	1844,6	1913,8	1983,7	2054,4
24	1846,9	1916,2	1986,1	2056,8
26	1849,2	1918,5	1988,4	2059,1
28	1851,5	1920,8	1990,8	2061,5
30	1853,8	1923,1	1993,1	2063,9
32	1856,1	1925,4	1995,5	2066,2
34	1858,4	1927,8	1997,8	2068,6
36	1860,7	1930,1	2000,2	2071,0
38	1863,0	1932,4	2002,5	2073,4
40	1865,3	1934,7	2004,9	2075,7
42	1867,6	1937,1	2007,2	2078,1
44	1869,9	1939,4	2009,6	2080,5
46	1872,2	1941,7	2011,9	2082,9
48	1874,5	1944,0	2014,3	2085,3
50	1876,8	1946,4	2016,6	2087,7
52	1879,2	1948,7	2019,0	2090,1
54	1881,5	1951,0	2021,3	2092,5
56	1883,8	1953,4	2023,7	2094,9
58	1886,1	1955,7	2026,0	2097,3

Meridional Parts, *or* Miles. 255

Min.	33° M. P.	34° M. P.	35° M. P.	36° M. P.
0	2099,6	2171,5	2244,3	2318,0
2	2101,9	2173,9	2246,8	2320,5
4	2104,3	2176,3	2249,2	2323,0
6	2106,7	2178,7	2251,6	2325,4
8	2109,1	2181,2	2254,1	2327,9
10	2111,5	2183,6	2256,5	2330,4
12	2113,9	2186,0	2259,0	2332,9
14	2116,3	2188,4	2261,4	2335,3
16	2118,7	2190,8	2263,9	2337,8
18	2121,0	2193,3	2266,3	2340,3
20	2123,4	2195,7	2268,8	2342,8
22	2125,8	2198,1	2271,2	2345,3
24	2128,2	2200,5	2273,7	2347,8
26	2130,6	2203,0	2276,1	2350,2
28	2133,0	2205,4	2278,6	2352,7
30	2135,4	2207,8	2281,0	2355,2
32	2137,8	2210,2	2283,5	2357,7
34	2140,2	2212,7	2286,0	2360,2
36	2142,6	2215,1	2288,4	2362,7
38	2145,0	2217,5	2290,9	2365,2
40	2147,4	2219,9	2293,3	2367,7
42	2149,8	2222,4	2295,8	2370,2
44	2152,2	2224,8	2298,3	2372,7
46	2154,6	2227,2	2300,8	2375,2
48	2157,0	2229,7	2303,2	2378,7
50	2159,4	2232,1	2305,7	2381,0
52	2161,9	2234,6	2308,1	2383,5
54	2164,3	2237,0	2310,6	2386,0
56	2166,7	2239,4	2313,1	2388,4
58	2169,1	2241,9	2315,5	2390,9

Min.	37° M. P.	38° M. P.	39° M. P.	40° M. P.
0	2390,7	2468,3	2544,9	2622,7
2	2395,2	2470,8	2547,5	2625,3
4	2397,7	2473,4	2550,1	2627,9
6	2400,2	2475,9	2552,7	2630,5
8	2402,7	2478,5	2555,3	2633,2
10	2405,2	2481,0	2557,8	2635,8
12	2407,7	2483,5	2560,4	2638,4
14	2410,2	2486,1	2563,0	2641,0
16	2412,7	2488,6	2565,6	2643,6
18	2415,2	2491,2	2568,2	2646,3
20	2417,8	2493,7	2570,7	2648,9
22	2420,3	2496,3	2573,3	2651,5
24	2422,8	2498,8	2575,9	2654,1
26	2425,3	2501,4	2578,5	2656,8
28	2427,8	2503,9	2581,1	2659,4
30	2430,3	2506,5	2583,7	2662,0
32	2432,9	2509,0	2586,3	2664,6
34	2435,4	2511,6	2588,9	2667,3
36	2437,9	2514,2	2591,5	2669,9
38	2440,4	2516,7	2594,1	2672,5
40	2443,0	2519,3	2596,7	2674,1
42	2445,4	2521,8	2599,3	2677,8
44	2448,0	2524,4	2601,9	2680,5
46	2450,6	2527,0	2604,5	2683,1
48	2453,1	2529,5	2607,1	2685,7
50	2455,6	2532,1	2609,7	2688,4
52	2458,1	2534,7	2612,3	2691,0
54	2460,7	2537,2	2614,9	2693,7
56	2463,2	2539,8	2617,5	2696,3
58	2465,8	2542,4	2620,1	2699,0

Meridional Parts, or Miles. 257

Min.	41° M. P.	42° M. P.	43° M. P.	44° M. P.
0	2701,6	2781,7	2863,1	2945,9
2	2704,3	2784,4	2865,8	2948,6
4	2706,9	2787,1	2868,5	2951,4
6	2709,6	2789,8	2871,3	2954,2
8	2712,2	2792,5	2874,1	2957,0
10	2714,9	2795,1	2876,8	2959,8
12	2717,5	2797,9	2879,5	2962,5
14	2720,2	2800,6	2882,3	2965,3
16	2722,9	2803,3	2885,0	2968,1
18	2725,5	2806,0	2887,8	2970,9
20	2728,2	2808,7	2890,5	2973,7
22	2730,8	2811,4	2893,3	2976,5
24	2733,5	2814,1	2896,0	2979,3
26	2736,2	2816,8	2898,8	2982,1
28	2738,8	2819,5	2901,5	2984,9
30	2741,5	2822,3	2904,3	2987,7
32	2744,1	2825,0	2907,1	2990,5
34	2746,8	2827,7	2909,7	2993,3
36	2749,4	2830,4	2912,5	2996,1
38	2752,1	2833,1	2915,3	2998,9
40	2754,9	2835,8	2918,1	3001,8
42	2757,6	2838,6	2920,9	3004,6
44	2760,2	2841,3	2923,6	3007,4
46	2762,9	2844,0	2926,4	3010,2
48	2765,6	2846,7	2929,2	3013,0
50	2768,3	2849,5	2932,0	3015,8
52	2771,0	2852,2	2934,7	3018,7
54	2773,7	2854,9	2937,5	3021,5
56	2776,4	2857,7	2940,3	3024,3
58	2779,0	2860,5	2943,1	3027,1

Min.	45°	46°	47°	48°
	M. P.	M. P.	M. P.	M. P.
0	3030,0	3115,6	3202,8	3291,6
2	3032,8	3118,5	3205,7	3294,6
4	3035,6	3121,4	3208,6	3297,5
6	3038,4	3124,2	3211,6	3300,5
8	3041,3	3127,1	3214,5	3303,5
10	3044,1	3130,0	3217,4	3306,5
12	3047,0	3132,9	3220,4	3309,5
14	3049,8	3135,8	3223,3	3312,5
16	3052,6	3138,7	3226,3	3315,5
18	3055,5	3141,6	3229,2	3318,5
20	3058,3	3144,5	3232,2	3321,5
22	3061,2	3147,4	3235,1	3324,6
24	3064,0	3150,3	3238,1	3327,6
26	3066,9	3153,2	3241,0	3330,6
28	3069,7	3156,1	3244,0	3333,6
30	3072,6	3159,0	3246,9	3336,6
32	3075,4	3161,9	3249,9	3339,6
34	3078,3	3164,8	3252,9	3342,7
36	3081,1	3167,7	3255,8	3345,7
38	3084,0	3170,6	3258,8	3348,7
40	3086,9	3173,5	3261,8	3351,7
42	3089,7	3176,4	3264,7	3354,8
44	3092,6	3179,3	3267,7	3357,8
46	3095,5	3182,2	3270,7	3360,8
48	3098,3	3185,1	3273,7	3363,9
50	3101,2	3188,0	3276,6	3366,9
52	3104,1	3191,0	3279,6	3369,9
54	3107,0	3194,0	3282,6	3373,0
56	3109,8	3196,9	3285,6	3376,0
58	3112,7	3199,8	3288,6	3379,1

Meridional Parts, *or* Miles. 259

Min.	49° M. P.	50° M. P.	51° M. P.	52° M. P.
0	3382.1	3474.5	3568.8	3665.2
2	3385.2	3477.6	3572.0	3668.5
4	3388.2	3480.7	3575.2	3671.7
6	3391.3	3483.9	3578.4	3675.0
8	3394.3	3487.0	3581.6	3678.2
10	3397.4	3490.1	3584.8	3681.5
12	3400.4	3493.2	3588.0	3684.8
14	3403.5	3496.3	3591.1	3688.0
16	3406.6	3499.4	3594.3	3691.3
18	3409.6	3502.5	3597.5	3694.6
20	3412.7	3505.7	3600.7	3697.8
22	3415.8	3508.9	3603.9	3701.1
24	3418.8	3512.0	3607.1	3704.4
26	3421.9	3515.1	3610.3	3707.7
28	3425.0	3518.3	3613.6	3710.9
30	3428.1	3521.4	3616.8	3714.2
32	3431.2	3524.6	3620.0	3717.5
34	3434.2	3527.7	3623.2	3720.8
36	3437.3	3530.9	3626.4	3724.1
38	3440.4	3534.0	3629.6	3727.4
40	3443.5	3537.2	3632.9	3730.7
42	3446.6	3540.3	3636.1	3734.0
44	3449.7	3543.5	3639.3	3737.3
46	3452.8	3546.7	3642.5	3740.6
48	3455.9	3549.8	3645.8	3743.9
50	3459.0	3553.0	3649.0	3747.2
52	3462.1	3556.1	3652.3	3750.5
54	3465.2	3559.3	3655.5	3753.8
56	3468.3	3562.5	3658.7	3757.2
58	3471.4	3565.7	3662.0	3760.5

Min.	53°	54°	55°	56°
	M. P.	M. P.	M. P.	M. P.
0	3763.8	3864.7	3968.0	4073.9
2	3767.1	3868.1	3971.5	4077.5
4	3770.4	3871.5	3975.0	4081.1
6	3773.8	3874.9	3978.5	4084.7
8	3777.1	3878.3	3982.0	4088.3
10	3780.4	3881.7	3983.5	4091.9
12	3783.8	3885.1	3989.0	4095.5
14	3787.1	3888.6	3992.5	4099.1
16	3790.5	3892.0	3996.0	4102.7
18	3793.8	3895.4	3999.5	4106.3
20	3797.2	3898.8	4003.0	4109.9
22	3800.5	3902.3	4006.5	4113.5
24	3803.9	3905.7	4010.0	4117.1
26	3807.2	3909.1	4013.6	4120.7
28	3810.6	3912.6	4017.1	4124.3
30	3813.9	3916.0	4020.6	4127.9
32	3817.3	3919.5	4024.2	4131.6
34	3820.7	3922.9	4027.7	4135.2
36	3824.0	3926.4	4031.2	4138.8
38	3827.4	3929.8	4034.8	4142.5
40	3830.8	3933.3	4038.3	4146.
42	3834.2	3936.7	4041.9	4149.
44	3837.5	3940.2	4045.4	4153.
46	3840.9	3943.7	4049.0	4157.
48	3844.3	3947.1	4052.5	4160.
50	3847.7	3950.6	4056.1	4164.3
52	3851.1	3954.1	4059.7	4168.0
54	3854.5	3957.6	4063.2	4171.7
56	3857.9	3961.0	4066.8	4175.3
58	3861.3	3964.5	4070.4	4179.0

Meridional Parts, or Miles. 261

Min.	57°	58°	59°	60°
	M. P.	M. P.	M. P.	M. P.
0	4182.7	4294.3	4409.2	4527.4
2	4186.3	4298.1	4413.1	4531.4
4	4190.0	4301.9	4417.0	4535.4
6	4193.7	4305.7	4420.8	4539.4
8	4197.4	4309.5	4424.7	4543.4
10	4201.1	4313.2	4428.6	4547.5
12	4204.7	4317.0	4432.5	4551.5
14	4208.4	4320.8	4436.4	4555.5
16	4212.1	4324.6	4440.4	4559.5
18	4215.8	4328.4	4444.3	4563.6
20	4219.5	4332.2	4448.2	4567.6
22	4223.2	4336.1	4452.1	4571.6
24	4227.0	4339.9	4456.0	4575.7
26	4230.7	4343.7	4460.0	4579.7
28	4234.4	4347.5	4463.9	4583.8
30	4238.1	4351.3	4467.8	4587.8
32	4241.8	4355.2	4471.8	4591.9
34	4245.6	4359.0	4475.7	4596.0
36	4248.3	4362.8	4479.7	4600.1
38	4253.0	4366.7	4483.6	4604.1
40	4256.8	4370.5	4487.6	4608.2
42	4260.5	4374.4	4491.6	4612.3
44	4264.3	4378.2	4495.7	4616.4
46	4268.0	4382.1	4499.7	4620.5
48	4271.8	4385.9	4503.7	4624.6
50	4275.5	4389.8	4507.5	4628.7
52	4279.3	4393.7	4511.4	4632.8
54	4283.0	4397.5	4515.4	4636.9
56	4286.8	4401.4	4519.4	4641.0
58	4290.6	4405.3	4523.4	4645.1

Min.	61°	62°	63°	64°
	M. P.	M. P.	M. P.	M. P.
0	4649,3	4775,0	4905,0	5039,5
2	4653,4	4779,3	4909,4	5044,0
4	4657,5	4783,5	4913,8	5048,6
6	4661,7	4787,8	4918,2	5053,2
8	4665,8	4792,1	4922,6	5057,7
10	4669,9	4796,4	4927,1	5062,3
12	4674,1	4800,7	4931,5	5066,9
14	4678,2	4804,9	4935,9	5071,5
16	4682,4	4809,2	4940,4	5076,1
18	4686,6	4813,5	4944,8	5080,7
20	4690,7	4817,8	4949,3	5085,3
22	4694,9	4822,2	4953,7	5090,0
24	4699,1	4826,5	4958,2	5094,6
26	4703,2	4830,8	4962,7	5099,2
28	4707,4	4835,1	4967,1	5103,0
30	4711,6	4839,4	4971,6	5108,5
32	4715,8	4843,8	4976,1	5113,1
34	4720,0	4848,1	4980,6	5117,8
36	4725,2	4852,5	4985,1	5122,5
38	4728,4	4856,8	4989,6	5127,1
40	4732,6	4861,2	4994,1	5131,8
42	4736,9	4865,5	4998,6	5136,5
44	4741,1	4869,9	5003,1	5141,2
46	4745,3	4874,3	5007,6	5145,9
48	4749,5	4878,6	5012,2	5150,6
50	4753,8	4883,0	5016,7	5155,3
52	4758,0	4887,4	5021,2	5160,9
54	4762,3	4891,8	5025,8	5164,7
56	4766,5	4896,2	5030,3	5169,4
58	4770,8	4900,6	5034,9	5174,1

Meridional Parts, or Miles. 263

Min.	65° M. P.	66° M. P.	67° M. P.	68° M. P.
0	5178,8	5323,6	5474,0	5630,9
2	5183,6	5328,5	5479,2	5636,2
4	5188,3	5333,4	5484,3	5641,5
6	5193,1	5338,3	5489,4	5646,9
8	5197,8	5343,3	5494,6	5652,3
10	5202,6	5348,2	5499,7	5657,6
12	5207,3	5353,2	5504,9	5663,0
14	5212,1	5358,1	5510,0	5668,4
16	5216,9	5363,1	5515,2	5673,8
18	5221,7	5368,1	5520,4	5679,2
20	5226,5	5373,0	5525,6	5684,6
22	5231,3	5378,0	5530,8	5690,0
24	5236,1	5383,0	5536,0	5695,5
26	5240,9	5388,0	5541,2	5700,9
28	5245,7	5393,0	5546,4	5706,3
30	5250,5	5398,0	5551,6	5711,8
32	5255,3	5403,0	5556,8	5717,3
34	5260,1	5408,1	5562,1	5722,7
36	5265,0	5413,1	5567,3	5728,2
38	5269,8	5418,1	5572,6	5733,7
40	5274,7	5423,2	5577,8	5739,2
42	5278,5	5428,2	5583,1	5744,7
44	5284,4	5433,3	5588,4	5750,2
46	5289,3	5438,4	5593,7	5755,7
48	5294,2	5443,5	5599,0	5761,3
50	5299,0	5448,5	5604,3	5766,8
52	5303,9	5453,6	5609,6	5772,3
54	5308,8	5458,7	5614,9	5777,9
56	5313,7	5463,8	5620,2	5783,5
58	5318,6	5468,9	5625,5	5789,0

Min.	69°	70°	71°	72°
	M. P.	M. P.	M. P.	M. P.
0	5794.6	5966.0	6145.7	6334.9
2	5800.2	5971.8	6151.8	6341.4
4	5805.8	5977.7	6158.0	6347.8
6	5811.4	5983.5	6164.2	6354.3
8	5817.0	5989.4	6170.4	6360.8
10	5822.6	5995.3	6176.6	6367.4
12	5828.2	6001.2	6182.8	6373.9
14	5833.9	6007.1	6189.0	6380.9
16	5839.5	6013.0	6195.2	6387.0
18	5845.1	6019.0	6201.4	6393.6
20	5850.8	6024.9	6207.6	6400.2
22	5856.5	6030.8	6213.9	6406.8
24	5862.2	6036.8	6210.2	6413.4
26	5867.9	6042.7	6226.5	6420.0
28	5873.5	6048.7	6232.6	6426.6
30	5879.3	6054.7	6239.0	6433.2
32	5885.0	6060.7	6245.3	6439.9
34	5890.7	6066.7	6251.6	6446.6
36	5896.4	6072.7	6258.0	6453.3
38	5902.1	6078.8	6264.4	6460.0
40	5907.9	6084.8	6270.7	6466.7
42	5913.7	6090.8	6277.1	6473.4
44	5919.5	6096.9	6283.5	6480.1
46	5925.2	6103.0	6289.8	6486.8
48	5931.0	6109.1	6296.2	6493.6
50	5936.8	6115.1	6302.6	6500.4
52	5942.6	6121.2	6309.0	6507.2
54	5948.4	6127.3	6315.5	6514.0
56	5954.3	6133.4	6322.0	6520.8
58	5960.1	6139.5	6328.5	6527.6

Meridional Parts, or Miles. 265

Min.	73° M. P.	74° M. P.	75° M. P.	76° M. P.
0	6534,4	6745,7	6970,3	7210,0
2	6541,3	6753,0	6978,0	7218,3
4	6548,2	6760,3	6985,8	7226,6
6	6555,0	6767,6	6993,6	7234,9
8	6561,9	6774,9	7001,4	7243,2
10	6568,8	6782,2	7009,2	7251,6
12	6575,7	6789,5	7017,0	7260,0
14	6582,6	6796,8	7024,8	7268,4
16	6589,5	6804,2	7032,6	7276,8
18	6596,4	6811,6	7040,5	7285,2
20	6603,4	6819,0	7048,4	7293,6
22	6610,4	6826,4	7056,3	7302,1
24	6617,4	6833,8	7064,2	7310,6
26	6624,4	6841,2	7072,1	7319,1
28	6631,4	6848,7	7080,1	7327,7
30	6638,5	6856,2	7088,1	7336,2
32	6645,5	6863,7	7096,1	7344,8
34	6652,6	6871,2	7104,1	7353,4
36	6659,7	6878,7	7112,2	7362,0
38	6666,8	6886,2	7120,2	7370,7
40	6673,9	6893,8	7128,3	7379,4
42	6681,0	6901,4	7136,4	7388,1
44	6688,1	6909,0	7144,5	7396,8
46	6695,2	6916,6	7152,6	7405,5
48	6702,4	6924,2	7160,8	7414,2
50	6709,6	6931,8	7169,0	7423,0
52	6716,8	6939,5	7177,2	7431,8
54	6724,0	6947,2	7185,4	7440,6
56	6731,2	6954,9	7193,6	7449,4
58	6738,5	6962,6	7201,8	7458,3

266 *A TABLE of, &c.*

Min.	77°	78°	79°	80°
	M. P.	M. P.	M. P.	M. P.
0	7467.2	7744.6	8045.7	8375.3
2	7476.1	7754.2	8056.2	8386.8
4	7485.0	7763.8	8066.7	8398.3
6	7493.9	7773.5	8077.3	8409.9
8	7502.9	7783.2	8087.9	8411.6
10	7511.9	7792.9	8098.5	8433.3
12	7520.9	7802.7	8109.1	8445.0
14	7529.9	7812.5	8119.8	8456.8
16	7539.0	7822.3	8130.5	8468.6
18	7548.1	7832.2	8141.3	8480.4
20	7557.2	7842.0	8152.1	8492.3
22	7566.3	7851.9	8162.9	8504.2
24	7575.5	7861.8	8173.7	8516.2
26	7584.7	7871.8	8184.6	8528.2
28	7593.9	7881.8	8195.5	8540.2
30	7603.1	7891.8	8206.5	8552.3
32	7612.3	7901.9	8217.5	8564.4
34	7621.6	7911.9	8228.5	8576.6
36	7630.9	7922.1	8239.6	8588.9
38	7640.5	7932.2	8250.7	8601.1
40	7649.6	7942.4	8261.8	8613.4
42	7659.0	7952.6	8273.0	8625.8
44	7668.4	7962.8	8284.2	8638.2
46	7677.8	7973.1	8295.5	8650.7
48	7687.3	7983.4	8306.8	8663.2
50	7696.8	7993.7	8318.1	8675.7
52	7706.3	8004.0	8329.4	8688.3
54	7715.8	8014.4	8340.8	8701.0
56	7725.4	8024.8	8352.3	8713.7
58	7735.0	8035.2	8363.8	8726.4

By

By the Help of this Table of Meridional Parts, or Miles, the enlarged Distances on the Meridian, between any two given Latitudes, may speedily be found, which admits of three Cases.

C A S E I.

If one of the proposed Places be under the Equinoctial, and the other have either North or South Latitude, the Meridional Parts found against the Latitude, will be the Distance required.

E X A M P L E.

Suppose it be required to find the Meridional Parts between the Equinoctial and the Latitude of $50^{\circ} 31'$, in the Column under 50° , and against $30'$, stand 3521,4, which are the Meridional Parts required.

C A S E II.

If the two proposed Places are both in one Hemisphere, viz. either both North, or both South, then the Difference of the Meridional Parts found against each Latitude, will be the Meridional Distance required.

E X A M P L E.

Let it be required to find the Meridional Parts between the Latitude of $25^{\circ} 38'$, and the Latitude of $51^{\circ} 48'$, both North.

Under 51, and against 48, is 3645,8 } Subtr.
And under 25, and against 38, is 1592,0 }

The Meridional Distance is - 2053,8 as was required.

C A S E

C A S E III.

If the two proposed Places are in contrary Hemispheres, viz. if one of them have North, and the other South Latitude, then the Sum of the Meridional Parts found against each Latitude, will be the Meridional Distance required.

E X A M P L E.

Let it be required to find the Meridional Distance between the Latitude of $26^{\circ} 15'$ North, and $14^{\circ} 52'$ South Latitude.

Under 26, and against 15, is 1633,2

And under 14, and against 52 is 902,3

The Meridional Distance is 2535,5 as was required.

The Use of these Meridional Distances will appear in the Calculation of the following Problems, for finding the true Course and Distance from one Place to another; and altho' they are called the Meridional Distance between one Latitude and another, yet they are not the true Distances which a Ship must run in sailing directly under the Meridian, from one Latitude to another; that Distance being only the Difference of Latitudes reduced into Miles, as hath already been explained.

For a farther Construction of *Mercator's Chart*; as also for the most expeditious Way of measuring the Course and Distance thereby, we recommend to the Learner those Charts now ready to be published by Mr. *Hafelden*, Teacher of the Mathematicks; in which his most ingenious Method of measuring the Distance with the same Ease as on a plain Chart, will appear a great Improvement to every Navigator.

C A S E

CASE I.

The Latitudes of two Places, and their Difference of Longitude, being given, to find the true Course, and direct Distance between them.

EXAMPLE.

Admit a Ship sails from the Lizard, Lat. $50^{\circ} 10'$ North, to the Island of Bermudas, whose Latitude is $32^{\circ} 25'$ North, what will the direct Course and Distance be, their Difference of Longitude being 56° Westerly?

$$\begin{array}{r} \text{Latitudes} \quad \left\{ \begin{array}{l} 50^{\circ} \quad 10' \\ 32 \quad 25 \end{array} \right. \\ \hline \end{array}$$

$$\begin{array}{r} \text{Difference} \quad 17 \quad 45 \\ \hline \quad \quad \quad 60 \end{array}$$

$$\text{Miles} \quad 1065 = \text{proper Difference of Lat.}$$

$$\begin{array}{r} \text{Its Meridional Parts} \quad 3490,1 \\ \text{Its Meridional Parts} \quad 2057,9 \end{array} \quad \left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} \text{Subtract} \quad 56 \\ \hline \quad \quad \quad 60 \end{array}$$

$$\text{Meridional Distance} \quad 1432,2 \quad \text{Diff. of Long.} \quad 3360$$

GEOMETRICALLY.

FROM C, representing the *Lizard*, draw EC = 1432,2 Miles (from a Scale of equal Parts) = Meridional Difference of Latitude; on E erect a Perpendicular, and from a Scale of equal Parts, set off 3360 Miles, the Difference of Longitude from E to Q; then draw QC, which compleats the Triangle.

From

From the same Scale of equal Parts, set off 1065 Miles, the proper Difference of Latitude from C to A; draw AB parallel to EQ 'till it cut CQ in B, then is your Projection finished.

Here C represents the *Lizard*, Q the Island of *Bermudas*, the $\angle C$ the Angle of the Ship's Course, AC the true Difference of Latitude, EC is the Meridional Difference of Latitude enlarged, EQ the true Difference of Longitude in Miles, AB the Departure, and BC the true Distance of the *Lizard* from *Bermudas*.

By the Logarithms.

In the Triangle CEQ, there is given EC and EQ to find the $\angle C$; and in the Triangle CAB, there is given AC, and the $\angle C$, to find CB.

First, To find the Course.

As EC = 1432,2 = Difference of	}	3,1560035
Latitude enlarged - - -		
Is to the Radius 90° - - -		10,0000000
So is EQ = 3360, the Difference	}	3,5263393
of Longitude - - -		
To the Tangent of $\angle C = 66^\circ 55'$	}	10,3703358
the Ship's Course SW <i>b</i> W +		
10° 40' W - - -		

Then, for the Distance,

As $\angle B$ the Cosine of Course = 23,5	9,5933631
Is to the proper Difference of Lat. }	3,0273496
AC = 1065 Miles - - -	
So is the Radius - - -	10,0000000
To BC = 2716 Miles the Dist. requir'd	3,4339865
	<i>By</i>

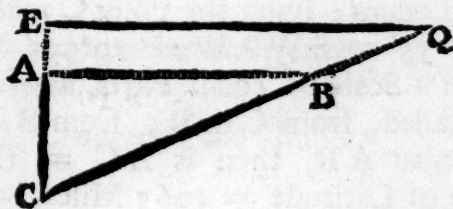
MERCATOR SAILING. 271

By the Gunter Scale.

The Distance between 1432,2, and 3360 on the Line of Numbers, applied to the Line of Tangents, will reach from 45° , to $66^{\circ} 15'$: And the Distance between $23^{\circ} 5'$, and 90° on the Sines, will reach from 1065 on the Line of Numbers, to 2716, the Distance required.

By the Sliding Scale.

Place 1432,2 on the Line of Numbers, against 45° on the Tangents, then will 3360 on the Numbers, be against $66,55$ on the Tangents. Again, Place $23^{\circ} 5'$ on the Sines, against 1065 on the Numbers, and then 90 on the Sines, will be against 2716 on the Numbers.



CASE

CASE II.

One Latitude, with the Course and Distance, being given, to find the other Latitude, and the Difference of Longitudes.

EXAMPLE.

Admit a Ship sails from the Lizard, Lat. $50^{\circ} 10'$ North, upon a SW b W $+ 10^{\circ} 40'$ Westerly Course, until she hath sailed 2716 Miles, what Latitude will she be in, and what will the Difference of Longitude be?

GEOMETRICALLY.

DR A W any Line, as EC; let C represent the *Lizard*; upon the Point C make an Angle $= 66^{\circ} 55' = \text{SW b W} + 10^{\circ} 40'$ Westerly; then from a Scale of equal Parts, lay 2716, the Distance sailed, from C to B; from B let fall the Perpendicular AB, then is AC = the proper Difference of Latitude $= 1065$ Miles, which converted into Degrees, is $17^{\circ} 45'$; then from the

First Lat.	$50^{\circ} 10'$	Its Merid. Parts	3490,1
Subtract	<u>17 45</u>	Its Merid. Parts	<u>2057,9</u>
New Lat.	32 25	Diff. of L. enlarg'd	1432,2 = EC.

Then from a Scale of equal Parts, set off 1432,2, from C to E, and draw EQ parallel to AB till it cut CB prolonged in Q, which compleats the Projection, the Parts being the same as in the preceding Case.

By

MERCATOR SAILING. 273

By the Logarithms.

As the Radius 90°	- - -	10,0000000
Is to CB = 2716 the Dist. sailed		3,4339631
So is the $\angle ABC = 23^\circ 5'$ the	} 9,5933631	
Cofine of the Course - - -		
To the proper Difference of Latitude AC = 1065 Miles	- }	3,0273262

And 1065 Miles converted into Degrees, as in the Projection, gives EC = 1432,2.

Then, for the Difference of Longitude.

As Radius 90°	- - -	10,0000000
Is to EC = 1432,2 the Difference	} 3,1560035	
of Latitudes enlarged - - -		
So is the $\angle C 66^\circ 55'$ the Ship's	} 10,3703358	
Course - - -		
To EQ = 3360 Miles, the Difference of Longitude	- }	3,5263393

Which being reduced into Degrees, will be 56° for the true Difference of Longitude towards the West.

T

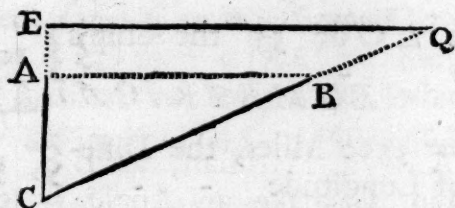
By

By the Gunter Scale.

The Distance between 90 , and $23^{\circ} 5'$ on the Line of Sines, will reach from 2716 on the Line of Numbers, to $1065 = AC$; and then for the second Proportion, the Distance between 90° , and $66^{\circ} 55'$ on the Line of Sines, will reach from $1432,2$ on the Line of Numbers, to 3360 , the Difference of Longitude required.

By the Sliding Scale.

Place 90 on the Line of Sines, against 2716 on the Line of Numbers, and $23^{\circ} 5'$ will be against 1065 on the Line of Numbers. Again, Place 90 on the Sines, against $1432,2$ on the Numbers, and then $66^{\circ} 55'$ on the Sines, will be against 3360 on the Numbers, the Difference of Longitude required.



CASE

CASE III.

The Latitudes of two Places, and the Course between them, being given, to find their Distance and Difference of Longitude.

EXAMPLE.

Suppose a Ship sail from the Latitude of $32^{\circ} 25'$ North, a NEbE Course $10^{\circ} 40'$ more Easterly, 'till she come into the Latitude of $50^{\circ} 10'$, what is her Distance sailed, and how much hath she altered her Longitude?

Lat.	{ $50^{\circ} 10'$	Merid. Parts of which are	3490,1
	{ $32\ 25$	Ibid. - - -	2057,9
	17 45	Dist. of the Lat. enlarged	1432,2
	60		

1065 *Proper Difference of Latitude.*

GEOMETRICALLY.

DRAW a Meridian, as EC, and on the Point C make an Angle = $66^{\circ} 55'$, the Complement of the Ship's Course; from a Scale of equal Parts, take 1432,2, the Distance of the Latitudes enlarged, and set it from C to E; and from the same Scale take 1065, the proper Difference of Latitude, and set it from C to A.

Then on the Points A and E erect Perpendiculars 'till they cut CQ in Q and B, which finishes the Projection.

Here Q represents the Place sailed from, and C the Place sailed to, and the rest of the Parts, as in Case I.

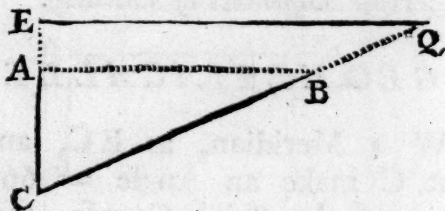
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By the Logarithms.

As the $\angle ABC = 23^\circ 5'$ the An-	}	9,5933631
gle of the Ship's Course -		
Is to $AC = 1065$ the proper Diff.	}	3,0273496
of Latitude - - -		
So is the Radius 90° - - -		10,0000000
To $CB = 2716$ Miles, the true Dist.		3,4339865

Then, the $\angle EQC = \angle ABC$, because AB parallel to EQ : Therefore,

As the $\angle ABC = 23^\circ 5'$ the An-	}	9,593363
gle of the Ship's Course -		
Is to $EC = 1432,2$ the Lat. enlarged		3,156003
So is $\angle ECQ = 66^\circ 55'$ the Com-	}	9,963757
plement of the Course -		
To $EQ = 3360$ Miles Diff. of Long.		3,526397



By the Gunter Scale.

The Distance between $23^\circ 5'$, and 90 on the Line of Sines, will reach from 1065 on the Line of Numbers, to $2716 = CB$, the Distance; and then the Distance between $23^\circ 5'$, and $66^\circ 55'$ on the Line of Sines, will reach from $1432,2$ on the Line of Numbers, to $3360 = EQ$, the Difference of Longitude.

By

MERCATOR SAILING. 277

By the Sliding Scale.

Place the Sine of $23^{\circ} 5'$ on the Sines, against 1065 on the Numbers, and 90 on the Sines, will be against 2716 on the Numbers: And, again, If $23^{\circ} 5'$ on the Sines, be placed against 1432,2 on the Numbers, then will $66^{\circ} 55'$ on the Sines, be against 3360 on the Numbers.

CASE IV.

The Latitude of two Places, and their Distance, being given, to find the Course, and their Difference of Longitude.

EXAMPLE.

Suppose a Ship sails from the Latitude of $50^{\circ} 10'$ North, between the South and West 2716 Miles, and then is found to be in the Latitude $32^{\circ} 25'$ North, 'tis required to find what Course she hath sailed upon, and how many Degrees she hath altered her Longitude?

Lat.	{	$50^{\circ} 10'$	Merid. Parts to which are	3490,1
		$32^{\circ} 25'$	D ^o .	2057,9

Diff.	17 45	Dist. of the Lat. enlarged	1432,2 = EC.
	60		

Miles 1065

GEOMETRICALLY.

DRAW a Meridian, as AC, and from a Scale of equal Parts, set off 1065, the Difference of Latitude from C to A; on A erect a Perpendicular, of any convenient Length, from

T 3

the

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the same Scale of equal Parts, with 2716 = the Distance sailed, putting one Foot of the Compasses in C, with the other cross the Perpendicular on A in B, then draw A B; from the same Scale set off 1432,2 (the Latitude enlarged) from C to E, draw E Q parallel to A B 'till it cut C B prolonged in Q, which finisheth the Projection.

The Parts are the same as in the last Case, only here C is the Angle of the Ship's Course.

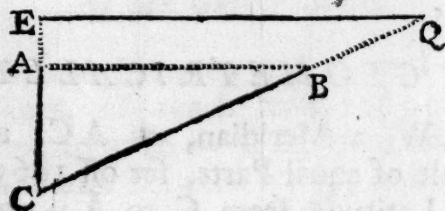
By the Logarithms.

As CB, the Distance sailed = 2716	3,4339865
Is to Radius 90° - - -	10,0000000
So is A C = 1065, the Diff. of Lat.	<u>3,0273496</u>

To $\angle ABC = 23^\circ 5'$ the Comp.	} 9,5933631
of the Course, whence the Courses	
SW b W + 10° 40' Westerly	

Then,

As $\angle EQC = 23^\circ 5'$ the Com-	} 9,5933631
plement of the Course - - -	
Is to EC = 1432,2, the Distance	} 3,1560035
of the Latitudes enlarged - - -	
So is $\angle ECQ = 66^\circ 55'$ Ship's Course	<u>9,9637574</u>
To EQ = 3360 = Diff. of Longitude	3,5263978



By

By the Gunter Scale.

The Distance between 2716, and 1065 on the Line of Numbers, will reach from 90° on the Sines, to $23^\circ 5'$; and the Distance between $23^\circ 5'$ and $66^\circ 55'$ on the Sines, will reach from 1432,2 on the Line of Numbers, to 3360.

By the Sliding Scale.

Place 2716 on the Line of Numbers, against 90 on the Line of Sines, and then 1065 on the Line of Numbers, will be against $23^\circ 5'$ on the Sines, which is the Complement of the Course: And, if you place $23^\circ 5'$ on the Sines, against 1432,2 on the Numbers, then will $66^\circ 55'$ be against 3360 on the Line of Numbers.

Thus you have, I presume, a plain Account of the Method of resolving the most useful Questions in sailing between the Meridian and either the East or West Points, according to the true Chart. There are several other Questions usually inserted in Books of Navigation: As,

1st. *Having one Latitude, the Course and Difference of Longitude given, to find the other Latitude and Distance?*

2dly, *Having both Latitudes and Departure given, to find the Course, the Distance sailed, and Difference of Longitude?*

3dly, *Having one Latitude, the Course and Departure given, to find the other Latitude, the Distance and Difference of Longitude?*

The Calculation and Projection of these Questions, is easily deduced from what has been already done, and therefore, for Brevity's Sake, and because they are not of any great Use in Practice, I omit that Work, and proceed to two useful Cases in sailing under any Parallel of Latitude, usually called, *Parallel Sailing by Mercator's Chart*.

PARALLEL SAILING.

IF a Ship sail directly under the Equinoctial, either East or West, the Distance sailed (in Sea Miles) being converted into Degrees, will be the true Difference of Longitude: And on the contrary, if the Difference of Longitude between any two Places be reduced into Minutes (*viz.* Sea Miles) they will be the true Distance that must be sailed from one Place to the other.

But if a Ship sail directly under, or in any Parallel of Latitude, then the Distance sailed, will be proportionably less than the Difference of Longitude, according to the Distance of that Parallel from the Equator, as hath been already proved in the Beginning of this Section, from whence the Proportions for resolving the following Cases are deduced.

CASE

CASE V.

The Difference of Longitude between any two Places that are both in one known Parallel of Latitude being given, to find their true Distance in Sea Miles.

EXAMPLE.

Suppose a Ship was to sail from Brest in France, to Consumption-Bay in Newfoundland, which is due West from Brest, both of them in the Parallel of Latitude $48^{\circ} 22'$ North, and their Difference of Longitude $48^{\circ} 46'$, I demand how many Sea Miles she must sail to arrive at the said Bay?

$$\begin{array}{r} 48^{\circ} 46' \\ 60 \\ \hline \end{array}$$

Difference of Long. 2926

GEOMETRICAL T.

WITH the Sine of 90° , and one Foot in C, make the Arch E Q, and with the Sine of $41^{\circ} 38'$ (the Complement of Latitude) and one Foot in P, draw the Arch A B; then from a Scale of equal Parts set 2926 (the Difference of Longitude in Miles) from E to Q, and draw the Lines A P, A Q, E Q, and A B, and your Projection is finished.

Here C represents the Pole, the Arch E Q the Equinoctial, and the Arch A B the Parallel of Latitude for $48^{\circ} 22'$ North, the right Line A Q is the Difference of Longitude in Miles, and the Points A and B *Brest* and *Consumption-Bay*, and the right Line A B the Distance between *Brest* and *Consumption-Bay*.

By

By the Logarithms.

Here are given AE the Radius, and AP the Complement of Latitude, and EQ the Difference of Longitude; therefore,

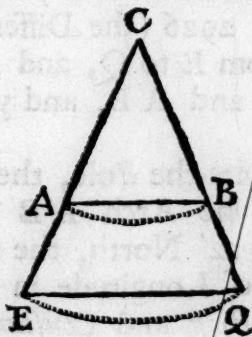
As the Radius $90^\circ = AE$	- -	10,0000000
Is to $EQ = 2926 =$ Difference of	}	3,4662743
Longitude in Miles - -		
So is the Sine of $PA = 41^\circ 38'$		9,8224042
To $AB = 1944$ Miles the Dist. requir'd		<u>3,2886785</u>

By Gunter's Scale.

The Distance between 90° , and $41^\circ 38'$ on the Line of Sines, applied to the Line of Numbers, will reach from 2926, to 1944, the Distance required.

By the Sliding Scale.

Place 90 on the Sines, against 2926 on the Numbers, and then $41^\circ 38'$ will be against 1944 on the Numbers.



CASE

CASE VI.

The Distance between any two Places that are both in one known Parallel of Latitude, being given, to find their Difference of Longitude.

EXAMPLE.

Suppose a Ship sail 1944 Miles, either directly East, or West, in the Parallel of $48^{\circ} 22'$ North, or South Latitude, what will the Difference of Longitude be?

THE geometrical Construction differs from the preceding only in this; that the Difference of Longitude in that, is laid off from E to Q, and in this, the Distance is laid off from A to B; in all respects else it is the same.

And the Proportion for Calculation, comes from the same Proposition with the preceding Case; that is,

As AC = $41^{\circ} 38'$ the Cosine of Lat.	9,8224042
Is to AB = 1944 the Dist. sailed	3,2886785
So is CE = 90° Radius - -	10,0000000

To EQ = 2926 the Difference of }	3,4662743
Longitude required - - }	

This is the common Method of solving these Cases, and is undoubtedly true, but, for Variety sake, I shall here give a Method of solving them by the Table of Meridional Parts; thus,

RULE.

R U L E.

To the given Latitude add 30', and subtract 30' from it, and find the Meridional Parts to that Sum and Difference; and then for the Solution of Case V. the Proportion is,

*As the Difference of those Meridional Parts,
Is to 60 Miles;
So is the Difference of Longitude in Miles,
To the Distance required.*

But for the Solution of Case VI. the Proportion is,

*As 60 Miles,
Is to the Difference of the Meridional Parts;
So is the given parallel Distance in Miles,
To the Difference of Longitude in Miles.*

In the Cases V. and VI.

$$\begin{array}{rcl}
 48^{\circ} 22' + 30' = 48^{\circ} 52' & \text{Mer. Parts} & 3369,9 \\
 48 - 22 - 30 = 47 \quad 52 & \text{Ibid.} & - \quad 3279,6 \\
 \hline
 & & \text{Diff. } 90,3
 \end{array}$$

N. B. 1^o = 60 Miles.

Then for the Cases V. and VI.

$$\begin{array}{l}
 90,3 : 60 :: 2926 : 1944 \\
 60 : 90,3 :: 1944 : 2926
 \end{array}
 \left. \vphantom{\begin{array}{l} 90,3 : 60 \\ 60 : 90,3 \end{array}} \right\} \text{the same as above.}$$

I do not remember that I ever met with these Proportions for finding of the parallel Distances, &c. performed by the Meridional Parts in any Treatise of Navigation. The Proportions from
whence

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whence I deduced them, and by which they shall be demonstrated, you will find in the next Section.

Note, *That there are several other Questions in Parallel Sailing: As,*

- 1st, *If the Distance sailed in one Parallel of Latitude, and the Difference of Longitude, were given, to find what Parallel of Latitude?*
2. *If two Ships, either under the Equinoctial, or any other given Parallel of Latitude, being any known Distance asunder, should both sail any equal Distance given, either directly North, or directly South thence, to find how far they will then be asunder?*

These, and such-like Questions, may be easily resolved, if what hath been already said be well considered; but they are usually proposed rather to exercise the Learner, than for any great Use they are of in Practice; and for that Reason I shall omit setting down the Work of them, and conclude this Section with a Solution of the same Traverse, as was proposed at the latter End of *Plain Sailing*.

The EXAMPLE is this,

Suppose a Ship was to sail from the Lizard, Latitude $50^{\circ} 10'$ to Bermudas, Latitude $32^{\circ} 25'$, both North, whose Difference of Longitude is $56^{\circ} = 3360$ Miles towards the West, and being at Sea, is forced upon the following Cases: First, WSW 100 Miles, then SSE 78 Miles, then due West 95 Miles, NW by W $\frac{1}{4}$ West 67 Miles, South 58 Miles, and SW by S $\frac{3}{4}$ W 90 Miles,

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Miles, 'tis required to find the Latitude the Ship is in, and the Difference of Longitude, and her direct Course and Distance from the last Place she is in, to Bermudas?

By these several Courses and Distances, there must be found their respective Differences of Latitude and Longitude (as *Case II.* of this Section) for these Courses that are between the East and West Points; and by *Case VI.* when the Course is due East, or due West, that so the new Latitude the Ship is in at the Beginning of every new Course, may be known, or otherwise, the Difference of Longitudes cannot be truly found, because by these two Latitudes, *viz.* the last, and the new Latitude, the Meridional Parts must be obtained.

The several Parts of this Traverse being truly compared according to their Data's, and their respective Answers disposed of, each under its proper Title, the Work may stand as in the following TABLE.

Courses	Points	Dist. failed	Diff. of Lat.			Diff. of Long.	
			North	South		East	West
o	o	o	o	o	50,10	o	o
W. S. W.	6	100	- -	38,2	49,32	- -	143,4
S. S. E.	2	78	- -	72,1	48,20	45,4	- -
W.	8	95	- -	- -	48,20	- -	142,9
NW $\frac{1}{4}$ W	5 $\frac{1}{4}$	67	34,4	- -	48,54	- -	85,9
S.	o	58	- -	58,0	47,56	- -	- -
SW $\frac{1}{2}$ S $\frac{1}{2}$ W	3 $\frac{1}{2}$	90	- -	69,6	46,47	- -	83,6

45,4 | 456,7

45,4

411,3

Hence

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Hence it appears, that at the End of this Traverse the Ship is in the Latitude of $46^{\circ} 47'$ North, and that she hath altered her Longitude from the Meridian of the *Lizard*, $6^{\circ} 51' 18''$ West; that is $456,7 - 45,4 = 411,3$ Miles, which being converted into Degrees, is $6^{\circ} 51'$, rejecting the Seconds.

Then, to find the Distance of the Ship from the Island of *Bermudas*, and what Course she must steer to arrive there. See *Case I.* of this *Sett.*

$$\text{Latitudes } \begin{cases} 46^{\circ} & 47' \\ 32 & 25 \end{cases}$$

$$\text{Difference } 14 \quad 22 = 8602 \text{ Miles.}$$

$$\begin{array}{r} \text{Meridional Parts of which are} \quad 3183,7 \\ \text{Ibid.} \quad - \quad - \quad - \quad - \quad 2057,9 \\ \hline \end{array}$$

$$\text{Difference of Latitude enlarged } 1125,8$$

And from the Difference of Longitude between the *Lizard* and *Bermudas*, viz. 56° , subtract the Difference of Longitude gained by Traverse, and the Remainder is $49^{\circ} 9' = 2949$ Miles, the Difference of Longitude between the Ship's Place and *Bermudas*.

Then, to find the Course, *per Case I.* of this *Section.*

$$\begin{array}{r} \text{As the Difference of Latitude in-} \\ \text{larged} = 1125,8 \text{ Miles} \quad - \quad - \quad \} \quad 3,0514228 \\ \text{Is to the Radius} \quad - \quad - \quad - \quad 10,0000000 \\ \text{So is the Diff. of Long. } 2949 \text{ Miles} \quad 3,4696749 \\ \hline \text{To the Tang. of the Course } 69^{\circ} 6' \quad 10,4182521 \end{array}$$

Then,

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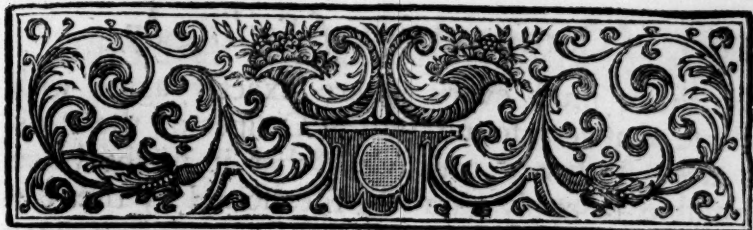
Then, for the Distance,

As $20^{\circ} 54'$ the Cofine of the Courfe	9,5523494
Is to proper Diff. of Lat. = 862 Miles	2,9355073
So is the Radius - - -	10,0000000
To the Dist. required = 2416,3 Miles	3,3831579

From hence it appears, that the Course the Ship must steer from the Place she is in, to the Island of *Bermudas*, must be $WSW + 1^{\circ} 36' W$ and the Distance she must sail is 2416 Miles, and not 3132 Miles, as it comes to by *Plain Sailing*.



A COM-



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COMPENDIUM
O F

*Sailing by the Mean Parallel Difference
of Longitude.*

S E C T. XII.



THE Method which I shall here advance for the resolving nautical Problems, and call by the Name of *Sailing by the mean parallel Difference of Longitude*, is performed by the Help of a Table of parallel Miles (or Parts of a Mile) instead of a Table of Meridional Parts: The Reason thereof is briefly laid down in the two following Sections.

S E C T. I.

Of MERCATOR'S PROJECTION.

WHEN I first considered the Application or Use of Meridional Parts or Miles, in finding parallel Distances, &c. as in Cases V. and VI. in *Mercator Sailing*; and that the Answers found
U by

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by them, must every where exactly agree with those Answers found by the Cosines of the Latitudes; I thence concluded, that it might be as possible to apply parallel Miles found by the Cosines, to the Solution of Questions in sailing upon any assigned Course, as to apply the meridional Miles, to the finding the parallel Distances; and, in order to that, I considered the Nature of *Mercator's Projection* (or Chart) made by the Help of meridional Miles, and the Projection made by the Help of the parallel Miles; And first of *Mercator's*.

In *Mercator's Projection* the Meridians remain parallel, or equally distant one from another, and are crossed by the Parallels of Latitude at right Angles, and the Degrees of Longitude, are every where equal, as in the plain Chart, only the Distances of these Parallels to gradually increase as they approach the Poles, according to their respective Distances from the Equinoctial.

That is, the Degrees of Latitude from the Equator, are to a Degree of Latitude or Longitude, equal to 60' or Miles, in the following Proportion, *viz.*

As the Cosine of the Latitude,

Is to the Radius;

So is a true Degree of Latitude = 60 Miles,

To the Measure of an enlarged Degree in the Meridian at that Latitude.

As for Instance; Suppose it was required to find the Measures of the enlarged Degree between the Parallel of 49° and 50° of Latitude: First, the Middle of that Degree is $49^{\circ} 30'$, and the Proportion as above.

As

the Mean Parallel Diff. of Long. 291

As the Cosine of $49^{\circ} 30'$	-	-	9,8125444
Is to the Radius	-	-	10,0000000
So is a true Degree = 60 Miles	-		1,7781512

To the enlarged Degree = 92,39 Miles 1,9656068

That is 92,39 Miles is the Measure of a Degree between the Parallel of 49° and 50° of Latitude; being the same as is given by the meridional Parts, or Miles: Thus,

Lat.	$\left\{ \begin{array}{l} 50^{\circ} \\ 49 \end{array} \right.$	Its Merid. Parts	$\left\{ \begin{array}{l} 3474,5 \\ 3382,1 \end{array} \right.$	Subtract.
		Ibid.	-	

Hence that Deg. of Lat. enlarg'd 92,4 = same as before.

Then if the enlarged Degrees of Latitude thus found (either by the Proportion, or by the Table of meridional Parts) be truly protracted, or set off, upon the two outside Meridians, which limit the Breadth of the Projection, according to their respective Distances from the Equinoctial, which must be done by the Help of such a Scale of equal Parts, as 60 thereof are equal to a Degree of Longitude in that Projection (which are every where equal to one another) then if the Parallels of Latitude be drawn thro' these Degrees (or any assign'd Number of them) at right Angles to those two Meridians, and the rest of the Meridians drawn parallel to them, that Projection will be a true *Mercator Chart*, representing so much of the Globe in *Plano*, as it was designed for.

As may be seen from the following *Mercator Chart*, made to represent the Meridians and Parallels of the Globe, from 36 to 50° of Latitude, and to 10° Difference of Longitude, which are mark'd at the Top and Bottom of the Projection.

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	1	2	3	4	5	6	7	8	9	10	
50											50
49											49
48											48
47											47
46											46
45											45
44											44
43											43
42											42
41											41
40											40
39											39
38											38
37											37
36											36
	1	2	3	4	5	6	7	8	9	10	

S E C T. II.

Of the PARALLEL PROJECTION.

IN the Projection made by the Help of the Parallel Miles, the Meridians are not parallel, or equally distant one from another (as they are in *Mercator's Chart*) but the Approach nigher to one another, as the Approach, the Poles, and the Latitude, do continually decrease according to their respective Distances from the Equinoctial; that is, the Degrees of Longitude in the Parallels remote from the Equinoctial, are to a true Degree of Longitude = 60', or Miles, in this Proportion, *viz.*

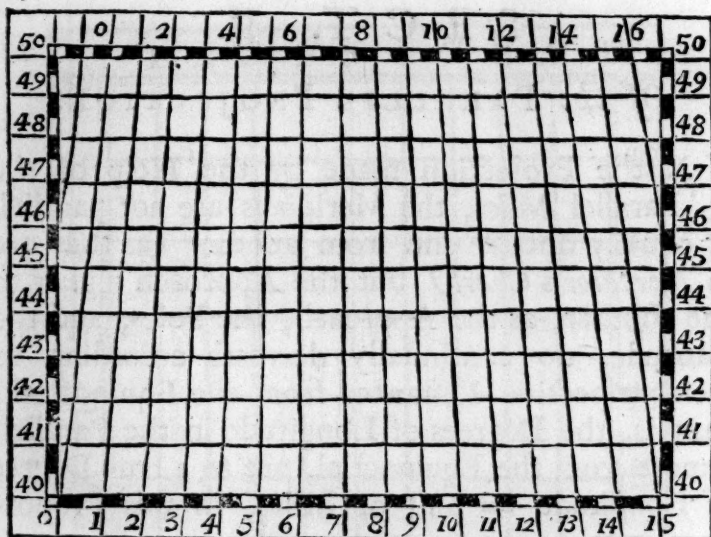
As the Radius,

Is to the Cosine of the Latitude;

So is a true Degree of Longitude = 60',

To the Measure of a Degree in that Parallel of Latitude.

Now if these parallel, or contracted, Degrees found by this Proportion, be truly protracted, or set off, in their respective Parallels of Latitude, or if they are only set off in those two Parallels at the Top and Bottom, which limit the Length of the Projection, by the Help of such a Scale of equal Parts, as 60 of them make one Degree of Latitude (which, in this Projection, remains every where equal to one another); then if the Meridians be drawn thro' these Degrees (or any assigned Number of them) and the rest of the Parallels of Latitude be drawn thro' their respective Degrees (as in the following Figure) that Projection, or Chart, will as truly represent so much of the Globe in *Plano* as it was designed for, as that of *Mercator's*.



For both the Proportions from whence they are derived, are equally true, consequently the Projections made by them, must also be as equally true; and, to me, the last Method seems to be the most natural or agreeable with the Globe itself; and that it is so received and approved of, doth plainly appear from the Number of Maps that are every where made by it; to instance only a few, the Maps of all the particular Countries of the World, described in the geographical Part of Sir *Jonas Moore's* System of the Mathematicks (*Vide* Vol. II.) and the new corrected Map of *France*, made by Mr. *Picard*, and Mr. *Delasbire* (*Vide Philosophical Transactions*, Number 226, &c.) are all delineated according to this Projection.

Now here perhaps it may be objected, that altho' this Projection be in itself so true, as that the Situation of Places may very well be represented upon it, with respect to their Latitudes and Longitudes, yet their Bearings, or Rhumbs, and
Distances

the Mean Parallel Diff. of Long. 295

Distances of those Places one from another, cannot be truly found by it, because the Meridians and Parallels of Latitude do not cut or cross one another at right Angles, as they do on *Mercator's Chart*.

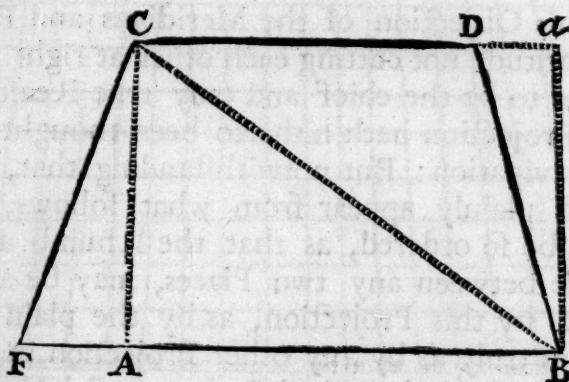
This Objection of the Meridians and Parallels of Latitude not cutting each other at right Angles, I take to be the chief and only true Reason why this Projection hath hitherto been thought useleſs in Navigation: But notwithstanding that, I hope it will plainly appear from what follows, that it may be ſo ordered, as that the Rhumb and Diſtance between any two Places, may be as eaſily found by this Projection, as by the plain Chart, and as truly as by any other Projection in *Plano*; and that by only reducing the parallel Differences of Longitude between any two propoſed Places, to a Mean, which may be thus done.

If it be required to find the Courſe and Diſtance between the two Places represented by the Points C and B, let fall a Perpendicular from the Point C, upon the Parallel FB, *viz.* CA, and it will be the Difference of the two Latitudes (according to the Projection above); join the Points C and B with a right Line, and it will form the right-angled Triangle CAB, wherein the Side CA is the Difference of Latitude; the $\angle$ at C, *viz.* $\angle ACB$, will be the Courſe from C to B, the Hypothenuſe CB is the Diſtance between C, B, and the Side $AB = (\frac{1}{2} FB + \frac{1}{2} CD)$ will be the mean parallel Difference of Longitude between the two Places C and B.

But if it be required to find the Courſe from B to C, then continue the Parallel CD to *a*, making *aB* perpendicular to CD continued, then will $aB = CA$, be the Difference of Latitudes, $Ca = AB$, will be the mean Difference of Longi-

296 *A Compendium of Sailing, &c.*

tudes as before, and the $\angle aBC$ will be the Course from B to C, their Distance BC being the same as above.



These Parts of right-angled Triangle CAB (or CaB) being thus considered, it will be very easy to apply them to the Solution of all useful Questions in sailing, and their Answers will be almost as easily found, as those in *Plain Sailing*, and very near the Truth to those of *Mercator's*; as will be manifest by the Calculations of the following Cases.

Now, in order to facilitate the Work of finding (AB , or Ca) the mean Difference of Longitude between any two Places (whose Latitudes and Difference of Longitudes are given) upon which the Solution of Cases, in sailing by this Method, do chiefly depend, I have here inserted a Table of the semi, or half parallel Parts of one Mile's Difference of Longitude, in their respective Degrees and Minutes of Latitude.

A
T A B L E

OF THE
PARALLEL PARTS

In Half a MILE's

Differ. *of* Longitude.



L O N D O N:

Printed in the Year M.DCC.XXIX.

T A B L E

OF THE

PARALLEL PARTS

In Fish & Marine

Director of Longitude



Printed in the Year M.DCC.XXIX.

A Table of the Parallel Parts, &c. 299

Note, The Parallel Parts belonging to the first three Degrees of Latitude, do differ so little from those in the Equinoctial, that it may suffice to set them down only to the whole Degrees: Thus,

In the Equinoctial they are 0,5000, viz. Half a Mile; and in 1° of Latitude, they are 0,4999; in 2° they are 0,4997; in 3° they are 0,4993 Parts of a Mile, &c. as follows.

Min.	4°	5°	6°	7°	8°	9°	10°
	P. P.	P. P.	P. P.	P. P.	P. P.	P. P.	P. P.
0	0,4988	0,4981	0,4973	0,4963	0,4951	0,4938	0,4924
10	0,4987	0,4980	0,4971	0,4961	0,4949	0,4936	0,4922
20	0,4986	0,4978	0,4969	0,4959	0,4947	0,4934	0,4920
30	0,4984	0,4977	0,4968	0,4957	0,4945	0,4932	0,4917
40	0,4983	0,4975	0,4966	0,4955	0,4943	0,4929	0,4914
50	0,4982	0,4974	0,4964	0,4953	0,4941	0,4926	0,4911
Min.	11°	12°	13°	14°	15°	16°	17°
	P. P.	P. P.	P. P.	P. P.	P. P.	P. P.	P. P.
0	0,4908	0,4891	0,4872	0,4851	0,4830	0,4806	0,4781
4	0,4907	0,4889	0,4871	0,4850	0,4828	0,4805	0,4780
8	0,4906	0,4888	0,4869	0,4849	0,4827	0,4803	0,4778
12	0,4905	0,4887	0,4868	0,4847	0,4825	0,4801	0,4776
16	0,4904	0,4886	0,4866	0,4846	0,4824	0,4800	0,4775
20	0,4903	0,4885	0,4865	0,4844	0,4822	0,4798	0,4773
24	0,4901	0,4883	0,4864	0,4843	0,4822	0,4796	0,4771
28	0,4900	0,4882	0,4862	0,4841	0,4819	0,4795	0,4769
32	0,4899	0,4881	0,4861	0,4840	0,4817	0,4793	0,4768
36	0,4898	0,4880	0,4860	0,4838	0,4816	0,4792	0,4766
40	0,4897	0,4878	0,4858	0,4837	0,4814	0,4790	0,4764
44	0,4895	0,4877	0,4857	0,4836	0,4813	0,4788	0,4762
48	0,4894	0,4876	0,4856	0,4834	0,4811	0,4786	0,4761
52	0,4893	0,4874	0,4854	0,4833	0,4809	0,4785	0,4759
56	0,4892	0,4873	0,4853	0,4831	0,4808	0,4783	0,4757

300 *A Table of the Parallel Parts in*

Min.	18°	19°	20°	21°
	P. P.	P. P.	P. P.	P. P.
0	0.4755	0.4727	0.4698	0.4668
2	0.4754	0.4727	0.4697	0.4667
4	0.4753	0.4726	0.4696	0.4666
6	0.4752	0.4725	0.4695	0.4665
8	0.4752	0.4724	0.4694	0.4664
10	0.4751	0.4723	0.4693	0.4663
12	0.4750	0.4722	0.4692	0.4662
14	0.4749	0.4721	0.4691	0.4661
16	0.4748	0.4720	0.4690	0.4660
18	0.4747	0.4719	0.4689	0.4659
20	0.4746	0.4718	0.4688	0.4658
22	0.4745	0.4717	0.4687	0.4657
24	0.4744	0.4716	0.4686	0.4656
26	0.4744	0.4715	0.4685	0.4655
28	0.4743	0.4714	0.4684	0.4653
30	0.4742	0.4713	0.4683	0.4652
32	0.4741	0.4712	0.4682	0.4651
34	0.4740	0.4711	0.4681	0.4650
36	0.4739	0.4710	0.4680	0.4649
38	0.4738	0.4709	0.4679	0.4648
40	0.4737	0.4708	0.4678	0.4647
42	0.4736	0.4707	0.4677	0.4646
44	0.4735	0.4706	0.4676	0.4645
46	0.4734	0.4705	0.4675	0.4644
48	0.4733	0.4704	0.4674	0.4643
50	0.4732	0.4703	0.4673	0.4642
52	0.4731	0.4702	0.4672	0.4641
54	0.4730	0.4701	0.4671	0.4640
56	0.4729	0.4700	0.4670	0.4639
58	0.4728	0.4699	0.4699	0.4638

Half a Mile's Diff. of Long. 301

Min.	22°	23°	24°	25°
	P. P.	P. P.	P. P.	P. P.
0	0.4636	0.4602	0.4568	0.4531
2	0.4635	0.4601	0.4567	0.4530
4	0.4634	0.4600	0.4566	0.4529
6	0.4633	0.4599	0.4564	0.4528
8	0.4632	0.4598	0.4563	0.4527
10	0.4631	0.4597	0.4562	0.4526
12	0.4630	0.4596	0.4561	0.4524
14	0.4629	0.4595	0.4560	0.4523
16	0.4628	0.4594	0.4559	0.4521
18	0.4627	0.4592	0.4557	0.4520
20	0.4626	0.4591	0.4556	0.4519
22	0.4625	0.4590	0.4555	0.4518
24	0.4623	0.4589	0.4553	0.4517
26	0.4622	0.4588	0.4552	0.4516
28	0.4621	0.4587	0.4551	0.4515
30	0.4620	0.4585	0.4550	0.4513
32	0.4619	0.4584	0.4549	0.4512
34	0.4618	0.4583	0.4548	0.4511
36	0.4617	0.4582	0.4546	0.4509
38	0.4616	0.4581	0.4545	0.4508
40	0.4615	0.4580	0.4544	0.4507
42	0.4614	0.4578	0.4542	0.4505
44	0.4612	0.4577	0.4541	0.4504
46	0.4611	0.4576	0.4540	0.4503
48	0.4609	0.4575	0.4539	0.4501
50	0.4608	0.4574	0.4538	0.4500
52	0.4607	0.4573	0.4537	0.4499
54	0.4606	0.4571	0.4535	0.4498
56	0.4605	0.4570	0.4534	0.4497
58	0.4604	0.4569	0.4533	0.4496

302 *A Table of the Parallel Parts in*

Min.	26°	27°	28°	29°
	P. P.	P. P.	P. P.	P. P.
0	0.4494	0.4455	0.4415	0.4373
2	0.4493	0.4454	0.4414	0.4372
4	0.4492	0.4453	0.4413	0.4371
6	0.4490	0.4451	0.4411	0.4369
8	0.4489	0.4440	0.4410	0.4368
10	0.4488	0.4449	0.4408	0.4367
12	0.4486	0.4447	0.4406	0.4365
14	0.4485	0.4446	0.4405	0.4364
16	0.4484	0.4445	0.4403	0.4362
18	0.4482	0.4444	0.4402	0.4361
20	0.4481	0.4442	0.4401	0.4359
22	0.4480	0.4441	0.4399	0.4358
24	0.4478	0.4439	0.4398	0.4356
26	0.4477	0.4438	0.4397	0.4355
28	0.4476	0.4437	0.4396	0.4354
30	0.4475	0.4435	0.4394	0.4352
32	0.4474	0.4434	0.4393	0.4351
34	0.4473	0.4433	0.4392	0.4349
36	0.4471	0.4431	0.4390	0.4347
38	0.4470	0.4430	0.4389	0.4346
40	0.4469	0.4429	0.4388	0.4345
42	0.4467	0.4427	0.4386	0.4343
44	0.4466	0.4426	0.4385	0.4342
46	0.4465	0.4425	0.4384	0.4341
48	0.4463	0.4423	0.4382	0.4339
50	0.4462	0.4422	0.4381	0.4338
52	0.4461	0.4421	0.4379	0.4337
54	0.4459	0.4419	0.4377	0.4335
56	0.4458	0.4418	0.4376	0.4334
58	0.4457	0.4417	0.4375	0.4332

Half a Mile's Diff. of Long. 303

Min.	30°	31°	32°	33°
	P. P.	P. P.	P. P.	P. P.
0	0,4330	0,4286	0,4240	0,4193
2	0,4329	0,4285	0,4239	0,4192
4	0,4327	0,4283	0,4237	0,4190
6	0,4326	0,4282	0,4235	0,4189
8	0,4324	0,4280	0,4234	0,4187
10	0,4323	0,4279	0,4233	0,4186
12	0,4321	0,4277	0,4231	0,4184
14	0,4320	0,4276	0,4229	0,4183
16	0,4318	0,4274	0,4228	0,4181
18	0,4317	0,4273	0,4227	0,4179
20	0,4315	0,4271	0,4225	0,4177
22	0,4314	0,4270	0,4224	0,4176
24	0,4312	0,4268	0,4222	0,4174
26	0,4311	0,4267	0,4221	0,4173
28	0,4310	0,4265	0,4219	0,4171
30	0,4308	0,4264	0,4217	0,4169
32	0,4307	0,4262	0,4215	0,4168
34	0,4306	0,4261	0,4214	0,4167
36	0,4304	0,4259	0,4212	0,4165
38	0,4303	0,4258	0,4211	0,4164
40	0,4301	0,4256	0,4209	0,4162
42	0,4300	0,4254	0,4208	0,4160
44	0,4298	0,4252	0,4206	0,4158
46	0,4297	0,4251	0,4205	0,4157
48	0,4295	0,4249	0,4203	0,4155
50	0,4294	0,4248	0,4202	0,4154
52	0,4292	0,4246	0,4200	0,4152
54	0,4291	0,4245	0,4198	0,4150
56	0,4289	0,4243	0,4196	0,4148
58	0,4288	0,4242	0,4195	0,4147

304 *A Table of the Parallel Parts in*

Min.	34° P. P.	35° P. P.	36° P. P.	37° P. P.
0	0.4145	0.4096	0.4045	0.3993
2	0.4144	0.4094	0.4044	0.3992
4	0.4142	0.4092	0.4042	0.3990
6	0.4141	0.4091	0.4040	0.3988
8	0.4139	0.4089	0.4038	0.3986
10	0.4137	0.4087	0.4037	0.3985
12	0.4135	0.4085	0.4035	0.3983
14	0.4134	0.4084	0.4033	0.3981
16	0.4132	0.4082	0.4031	0.3979
18	0.4131	0.4081	0.4029	0.3978
20	0.4129	0.4079	0.4028	0.3976
22	0.4128	0.4078	0.4026	0.3974
24	0.4126	0.4076	0.4024	0.3972
26	0.4124	0.4074	0.4023	0.3970
28	0.4122	0.4072	0.4021	0.3968
30	0.4121	0.4071	0.4019	0.3967
32	0.4119	0.4069	0.4017	0.3965
34	0.4118	0.4067	0.4016	0.3963
36	0.4116	0.4065	0.4014	0.3961
38	0.4116	0.4064	0.4013	0.3960
40	0.4112	0.4062	0.4011	0.3958
42	0.4111	0.4061	0.4009	0.3956
44	0.4109	0.4059	0.4007	0.3954
46	0.4108	0.4057	0.4006	0.3953
48	0.4106	0.4055	0.4004	0.3951
50	0.4108	0.4054	0.4002	0.3949
52	0.4102	0.4052	0.4000	0.3947
54	0.4101	0.4050	0.3999	0.3946
56	0.4099	0.4048	0.3997	0.3944
58	0.4098	0.4047		0.3942

Half a Mile's Diff. of Long. 305

Min.	38°	39°	40°	41°
	P. P.	P. P.	P. P.	P. P.
0	0.3940	0.3886	0.3830	0.3773
2	0.3938	0.3884	0.3828	0.3771
4	0.3936	0.3882	0.3826	0.3770
6	0.3935	0.3880	0.3824	0.3768
8	0.3933	0.3878	0.3823	0.3766
10	0.3931	0.3877	0.3821	0.3764
12	0.3929	0.3875	0.3819	0.3762
14	0.3928	0.3873	0.3817	0.3760
16	0.3926	0.3871	0.3815	0.3758
18	0.3924	0.3869	0.3813	0.3756
20	0.3922	0.3867	0.3811	0.3754
22	0.3920	0.3866	0.3810	0.3752
24	0.3918	0.3864	0.3808	0.3750
26	0.3917	0.3862	0.3806	0.3749
28	0.3915	0.3860	0.3804	0.3747
30	0.3913	0.3858	0.3802	0.3735
32	0.3911	0.3856	0.3800	0.3733
34	0.3909	0.3854	0.3798	0.3731
36	0.3908	0.3852	0.3796	0.3729
38	0.3906	0.3850	0.3794	0.3727
40	0.3904	0.3849	0.3792	0.3735
42	0.3902	0.3847	0.3790	0.3733
44	0.3900	0.3845	0.3789	0.3731
46	0.3899	0.3843	0.3787	0.3729
48	0.3897	0.3841	0.3785	0.3727
50	0.3895	0.3839	0.3783	0.3725
52	0.3893	0.3838	0.3781	0.3723
54	0.3891	0.3836	0.3779	0.3721
56	0.3889	0.3834	0.3777	0.3719
58	0.3887	0.3832	0.3775	0.3717

306 *A Table of the Parallel Parts in*

Min.	42°	43°	44°	45°
	P. P.	P. P.	P. P.	P. P.
0	0.3716	0.3657	0.3597	0.3536
2	0.3714	0.3655	0.3595	0.3534
4	0.3712	0.3653	0.3593	0.3532
6	0.3710	0.3651	0.3591	0.3530
8	0.3708	0.3649	0.3589	0.3528
10	0.3706	0.3647	0.3587	0.3526
12	0.3704	0.3645	0.3585	0.3524
14	0.3702	0.3643	0.3583	0.3522
16	0.3700	0.3641	0.3581	0.3520
18	0.3698	0.3639	0.3579	0.3518
20	0.3696	0.3637	0.3576	0.3516
22	0.3694	0.3635	0.3574	0.3514
24	0.3692	0.3633	0.3572	0.3512
26	0.3690	0.3631	0.3570	0.3510
28	0.3688	0.3629	0.3568	0.3508
30	0.3686	0.3627	0.3566	0.3506
32	0.3684	0.3625	0.3564	0.3503
34	0.3682	0.3623	0.3562	0.3501
36	0.3680	0.3621	0.3560	0.3499
38	0.3678	0.3619	0.3558	0.3497
40	0.3676	0.3617	0.3556	0.3495
42	0.3674	0.3615	0.3554	0.3493
44	0.3672	0.3613	0.3552	0.3491
46	0.3670	0.3611	0.3550	0.3489
48	0.3668	0.3609	0.3548	0.3487
50	0.3666	0.3607	0.3546	0.3485
52	0.3665	0.3605	0.3544	0.3483
54	0.3663	0.3603	0.3542	0.3481
56	0.3661	0.3601	0.3540	0.3478
58	0.3659	0.3599	0.3538	0.3476

Half a Mile's Diff. of Long. 307

Min.	46°	47°	48°	49°
	P. P.	P. P.	P. P.	P. P.
0	0,3473	0,3410	0,3346	0,3280
2	0,3471	0,3408	0,3344	0,3278
4	0,3469	0,3406	0,3342	0,3276
6	0,3467	0,3404	0,3340	0,3274
8	0,3465	0,3402	0,3337	0,3272
10	0,3463	0,3399	0,3335	0,3269
12	0,3461	0,3397	0,3333	0,3267
14	0,3459	0,3395	0,3331	0,3265
16	0,3457	0,3393	0,3328	0,3263
18	0,3455	0,3391	0,3326	0,3261
20	0,3453	0,3389	0,3324	0,3258
22	0,3450	0,3387	0,3322	0,3256
24	0,3448	0,3385	0,3320	0,3254
26	0,3446	0,3382	0,3318	0,3252
28	0,3444	0,3380	0,3315	0,3249
30	0,3442	0,3378	0,3313	0,3247
32	0,3440	0,3376	0,3311	0,2245
34	0,3438	0,3374	0,3309	0,3243
36	0,3435	0,3372	0,3307	0,3241
38	0,3433	0,3369	0,3305	0,3239
40	0,3431	0,3367	0,3302	0,3237
42	0,3429	0,3365	0,3300	0,3235
44	0,3427	0,3363	0,3298	0,3232
46	0,3425	0,3361	0,3296	0,3230
48	0,3423	0,3358	0,3294	0,3227
50	0,3421	0,3357	0,3291	0,3225
52	0,3418	0,3355	0,3289	0,3223
54	0,3416	0,3352	0,3287	0,3221
56	0,3414	0,3350	0,3285	0,3218
58	0,3412	0,3348	0,3283	0,3216

308 *A Table of the Parallel Parts in*

Min.	50°	51°	52°	53°
	P. P.	P. P.	P. P.	P. P.
0	0,3214	0,3147	0,3078	0,3009
2	0,3212	0,3145	0,3076	0,3007
4	0,3210	0,3142	0,3074	0,3004
6	0,3207	0,3140	0,3072	0,3002
8	0,3205	0,3137	0,3069	0,3000
10	0,3203	0,3135	0,3067	0,2998
12	0,3201	0,3133	0,3064	0,2995
14	0,3198	0,3131	0,3062	0,2993
16	0,3196	0,3128	0,3060	0,2990
18	0,3194	0,3126	0,3058	0,2988
20	0,3192	0,3124	0,3056	0,2986
22	0,3198	0,3122	0,3054	0,2984
24	0,3187	0,3120	0,3052	0,2981
26	0,3185	0,3118	0,3049	0,2978
28	0,3183	0,3115	0,3046	0,2976
30	0,3181	0,3113	0,3044	0,2974
32	0,3178	0,3110	0,3042	0,2972
34	0,3176	0,3108	0,3040	0,2970
36	0,3174	0,3106	0,3037	0,2967
38	0,3172	0,3104	0,3035	0,2965
40	0,3169	0,3101	0,3032	0,2962
42	0,3167	0,3099	0,3030	0,2960
44	0,3165	0,3097	0,3028	0,2958
46	0,3163	0,3095	0,3026	0,2956
48	0,3160	0,3093	0,3024	0,2954
50	0,3158	0,3090	0,3021	0,2951
52	0,3156	0,3087	0,3018	0,2948
54	0,3154	0,3085	0,3016	0,2946
56	0,3151	0,3083	0,3014	0,2944
58	0,3149	0,3081	0,3012	0,2942

Half a Mile's Diff. of Long. 309

Min.	54°	55°	56°	57°
	P. P.	P. P.	P. P.	P. P.
0	0,2939	0,2868	0,2796	0,2723
2	0,2937	0,2866	0,2794	0,2721
4	0,2934	0,2863	0,2791	0,2718
6	0,2932	0,2861	0,2788	0,2716
8	0,2929	0,2858	0,2786	0,2713
10	0,2927	0,2856	0,2784	0,2711
12	0,2925	0,2853	0,2781	0,2708
14	0,2923	0,2851	0,2779	0,2706
16	0,2920	0,2849	0,2776	0,2704
18	0,2917	0,2847	0,2774	0,2702
20	0,2915	0,2844	0,2772	0,2699
22	0,2913	0,2841	0,2770	0,2697
24	0,2911	0,2839	0,2767	0,2695
26	0,2908	0,2837	0,2765	0,2692
28	0,2906	0,2834	0,2762	0,2689
30	0,2904	0,2832	0,2760	0,2687
32	0,2901	0,2830	0,2757	0,2685
34	0,2898	0,2828	0,2755	0,2682
36	0,2896	0,2826	0,2752	0,2679
38	0,2894	0,2823	0,2750	0,2677
40	0,2892	0,2820	0,2747	0,2674
42	0,2890	0,2818	0,2745	0,2672
44	0,2887	0,2815	0,2743	0,2669
46	0,2885	0,2813	0,2741	0,2667
48	0,2882	0,2810	0,2738	0,2664
50	0,2880	0,2808	0,2736	0,2662
52	0,2877	0,2806	0,2733	0,2659
54	0,2875	0,2804	0,2731	0,2657
56	0,2873	0,2801	0,2728	0,2654
58	0,2871	0,2799	0,2726	0,2652

310 *A Table of the Parallel Parts in*

Min.	58°	59°	60°	61°
	P. P.	P. P.	P. P.	P. P.
0	0.2650	0.2575	0.2590	0.2424
2	0.2648	0.2573	0.2498	0.2422
4	0.2645	0.2571	0.2495	0.2419
6	0.2643	0.2568	0.2493	0.2417
8	0.2640	0.2565	0.2490	0.2414
10	0.2638	0.2563	0.2488	0.2412
12	0.2635	0.2560	0.2485	0.2409
14	0.2633	0.2558	0.2483	0.2407
16	0.2630	0.2555	0.2480	0.2404
18	0.2628	0.2553	0.2478	0.2401
20	0.2625	0.2550	0.2475	0.2398
22	0.2623	0.2548	0.2473	0.2396
24	0.2620	0.2545	0.2471	0.2393
26	0.2618	0.2543	0.2468	0.2391
28	0.2615	0.2540	0.2465	0.2388
30	0.2613	0.2538	0.2462	0.2386
32	0.2610	0.2535	0.2459	0.2383
34	0.2608	0.2533	0.2457	0.2381
36	0.2605	0.2530	0.2454	0.2378
38	0.2603	0.2528	0.2452	0.2376
40	0.2600	0.2525	0.2449	0.2373
42	0.2598	0.2523	0.2447	0.2371
44	0.2595	0.2520	0.2444	0.2368
46	0.2593	0.2518	0.2442	0.2366
48	0.2590	0.2515	0.2439	0.2363
50	0.2588	0.2513	0.2437	0.2361
52	0.2585	0.2510	0.2434	0.2358
54	0.2583	0.2508	0.2432	0.2355
56	0.2580	0.2505	0.2429	0.2352
58	0.2578	0.2503	0.2427	0.2349

Half a Mile's Diff. of Long. 311

Min.	62°	63°	64°	65°
	P. P.	P. P.	P. P.	P. P.
0	0,2347	0,2270	0,2192	0,2113
2	0,2345	0,2268	0,2190	0,2110
4	0,2342	0,2265	0,2187	0,2108
6	0,2340	0,2262	0,2184	0,2104
8	0,2337	0,2259	0,2181	0,2102
10	0,2335	0,2256	0,2178	0,2099
12	0,2332	0,2254	0,2176	0,2097
14	0,2330	0,2252	0,2173	0,2095
16	0,2327	0,2249	0,2171	0,2092
18	0,2325	0,2247	0,2168	0,2089
20	0,2322	0,2244	0,2166	0,2087
22	0,2319	0,2242	0,2163	0,2084
24	0,2316	0,2239	0,2160	0,2081
26	0,2313	0,2237	0,2157	0,2078
28	0,2311	0,2234	0,2155	0,2076
30	0,2308	0,2231	0,2153	0,2073
32	0,2306	0,2228	0,2150	0,2071
34	0,2304	0,2225	0,2147	0,2068
36	0,2301	0,2223	0,2145	0,2065
38	0,2308	0,2221	0,2142	0,2062
40	0,2295	0,2218	0,2139	0,2060
42	0,2292	0,2215	0,2136	0,2057
44	0,2290	0,2213	0,2134	0,2055
46	0,2288	0,2210	0,2132	0,2052
48	0,2285	0,2201	0,2129	0,2050
50	0,2282	0,2204	0,2126	0,2047
52	0,2280	0,2202	0,2124	0,2044
54	0,2278	0,2200	0,2121	0,2041
56	0,2275	0,2197	0,2118	0,2039
58	0,2273	0,2195	0,2115	0,2036

312 *A Table of the Parallel Parts in*

Min.	66°	67°	68°	69°
	P. P.	P. P.	P. P.	P. P.
0	0.2034	0.1954	0.1873	0.1792
2	0.2031	0.1951	0.1870	0.1789
4	0.2028	0.1948	0.1868	0.1786
6	0.2025	0.1945	0.1865	0.1783
8	0.2023	0.1943	0.1862	0.1781
10	0.2020	0.1940	0.1859	0.1778
12	0.2018	0.1937	0.1857	0.1775
14	0.2015	0.1935	0.1854	0.1772
16	0.2012	0.1932	0.1851	0.1770
18	0.2009	0.1930	0.1848	0.1767
20	0.2007	0.1927	0.1846	0.1765
22	0.2004	0.1924	0.1843	0.1762
24	0.2002	0.1921	0.1841	0.1759
26	0.1999	0.1918	0.1838	0.1756
28	0.1996	0.1916	0.1835	0.1754
30	0.1993	0.1913	0.1832	0.1751
32	0.1991	0.1911	0.1830	0.1748
34	0.1988	0.1908	0.1827	0.1745
36	0.1986	0.1905	0.1824	0.1743
38	0.1983	0.1902	0.1821	0.1740
40	0.1980	0.1900	0.1819	0.1737
42	0.1977	0.1897	0.1816	0.1734
44	0.1975	0.1894	0.1813	0.1732
46	0.1972	0.1891	0.1810	0.1729
48	0.1969	0.1889	0.1808	0.1726
50	0.1966	0.1886	0.1805	0.1723
52	0.1964	0.1884	0.1803	0.1721
54	0.1951	0.1881	0.1800	0.1718
56	0.1959	0.1878	0.1797	0.1715
58	0.1956	0.1875	0.1795	0.1712

Half a Mile's Diff. of Long. 313

Min.	70°	71°	72°	73°
	P. P.	P. P.	P. P.	P. P.
0	0.1710	0.1628	0.1545	0.1462
2	0.1707	0.1625	0.1542	0.1459
4	0.1705	0.1622	0.1540	0.1456
6	0.1702	0.1619	0.1537	0.1453
8	0.1699	0.1617	0.1534	0.1451
10	0.1696	0.1614	0.1531	0.1448
12	0.1694	0.1611	0.1528	0.1445
14	0.1691	0.1608	0.1526	0.1442
16	0.1688	0.1606	0.1523	0.1439
18	0.1685	0.1603	0.1520	0.1437
20	0.1683	0.1600	0.1517	0.1434
22	0.1680	0.1597	0.1515	0.1431
24	0.1677	0.1595	0.1512	0.1428
26	0.1674	0.1592	0.1509	0.1426
28	0.1672	0.1589	0.1506	0.1423
30	0.1669	0.1586	0.1503	0.1420
32	0.1666	0.1584	0.1501	0.1417
34	0.1663	0.1581	0.1498	0.1414
36	0.1661	0.1578	0.1495	0.1412
38	0.1658	0.1575	0.1492	0.1409
40	0.1655	0.1573	0.1490	0.1406
42	0.1652	0.1570	0.1487	0.1403
44	0.1650	0.1567	0.1484	0.1400
46	0.1647	0.1564	0.1481	0.1398
48	0.1644	0.1562	0.1478	0.1395
50	0.1641	0.1559	0.1476	0.1392
52	0.1639	0.1556	0.1473	0.1389
54	0.1636	0.1553	0.1470	0.1386
56	0.1633	0.1550	0.1467	0.1384
58	0.1630	0.1547	0.1465	0.1381

314 *A Table of the Parallel Parts in*

Min.	74°	75°	76°	77°
	P. P.	P. P.	P. P.	P. P.
0	0.1378	0.1294	0.1210	0.1125
2	0.1375	0.1291	0.1207	0.1122
4	0.1372	0.1288	0.1204	0.1119
6	0.1370	0.1286	0.1201	0.1116
8	0.1367	0.1283	0.1198	0.1113
10	0.1364	0.1280	0.1195	0.1111
12	0.1361	0.1277	0.1193	0.1108
14	0.1359	0.1274	0.1190	0.1105
16	0.1356	0.1272	0.1187	0.1102
18	0.1353	0.1269	0.1184	0.1099
20	0.1350	0.1266	0.1181	0.1096
22	0.1347	0.1263	0.1178	0.1093
24	0.1345	0.1260	0.1176	0.1091
26	0.1342	0.1257	0.1173	0.1088
28	0.1339	0.1255	0.1170	0.1085
30	0.1336	0.1252	0.1167	0.1082
32	0.1333	0.1249	0.1164	0.1079
34	0.1330	0.1246	0.1161	0.1076
36	0.1328	0.1243	0.1158	0.1074
38	0.1325	0.1241	0.1156	0.1071
40	0.1322	0.1238	0.1153	0.1068
42	0.1319	0.1235	0.1150	0.1065
44	0.1316	0.1232	0.1147	0.1062
46	0.1314	0.1229	0.1144	0.1059
48	0.1311	0.1226	0.1142	0.1056
50	0.1308	0.1224	0.1139	0.1054
52	0.1305	0.1221	0.1136	0.1051
54	0.1302	0.1218	0.1133	0.1048
56	0.1300	0.1215	0.1130	0.1045
58	0.1297	0.1212	0.1127	0.1042

Half a Mile's Diff. of Long. 115

Min.	78°	79°	80°
	P. P.	P. P.	P. P.
0	0.1039	0.0954	0.0868
2	0.1037	0.0951	0.0865
4	0.1034	0.0945	0.0862
6	0.1031	0.0948	0.0860
8	0.1028	0.0943	0.0857
10	0.1025	0.0940	0.0854
12	0.1022	0.0937	0.0851
14	0.1020	0.0934	0.0848
16	0.1017	0.0931	0.0845
18	0.1014	0.0928	0.0842
20	0.1011	0.0925	0.0839
22	0.1008	0.0923	0.0837
24	0.1005	0.0920	0.0834
26	0.1002	0.0917	0.0831
28	0.1000	0.0914	0.0828
30	0.0997	0.0911	0.0825
32	0.0994	0.0908	0.0822
34	0.0991	0.0905	0.0819
36	0.0988	0.0903	0.0817
38	0.0985	0.0900	0.0814
40	0.0982	0.0897	0.0811
42	0.0980	0.0894	0.0808
44	0.0977	0.0891	0.0805
46	0.0974	0.0888	0.0802
48	0.0971	0.0885	0.0799
50	0.0968	0.0882	0.0796
52	0.0965	0.0880	0.0794
54	0.0963	0.0877	0.0791
56	0.0960	0.0874	0.0788
58	0.0957	0.0871	0.0785

316 *A Compendium of Sailing by*

Now, by the Help of this Table, the mean parallel Difference of Longitude between any two Places, may be found thus,

Having the Latitudes of the two Places, and the Differences of their Longitudes given, then, whether the Latitudes are both North, or both South, or one of them North, and the other South? Add their respective parallel Parts found in the Table, into one Sum, and multiply that Sum with the given Difference of Longitude in Miles, the Product will be the mean parallel Distance in Miles, between the Meridians of those two Places, as the last Figure will shew.

EXAMPLE.

Suppose two Places, one in $24^{\circ} 30'$, the other in $39^{\circ} 26'$ of Latitude, and let their Difference of Longitude be $41^{\circ} 22' = 2482$ Miles; Then,

$$\begin{array}{rcl} \text{Lat. } \left\{ \begin{array}{l} 24^{\circ} 30' \\ 39 \quad 26 \end{array} \right. & \begin{array}{l} \text{Its parallel Parts } 0,4550 \\ \text{Ibid.} \quad - \quad - \quad - \quad 0,3862 \end{array} & \left. \vphantom{\begin{array}{l} 24^{\circ} 30' \\ 39 \quad 26 \end{array}} \right\} \text{Add} \\ & & \hline & & 0,8412 \end{array}$$

And ,8412 multiply'd by 2482 = 2087,8584 Miles the mean parallel Difference of Longitude, or Distance between the Meridians of these two Places: Or this may be performed by the Table of Logarithms.

Sum

the Mean Parallel Diff. of Long. 317

Sum of the parallel Parts is ,8412, }
 the Logarithm is - - - } 9,9248933

The Difference of Longitude is }
 2482 Miles, the Logarithm is } 3,3948018

The parallel Distance is 2087,85, }
 &c. the Logarithm of which is } 3,3197011

The Use of these mean parallel Differences of Longitude, and of their Logarithms found as above, will fuller appear in the Calculations of the following Problems, in Sailing according to this Method.



THE



THE
CALCULATION
OF
NAUTICAL PROBLEMS
BY
PARALLEL PARTS.

S E C T. XIII.



NOTE, If two Places lie under the Equinoctial, their Position is due East and West, and their Difference of Longitude reduced into Miles, is their true Distance in Miles ; or, if two Places lie under the same Meridian, their Position lies North and South, and their Difference of Latitude converted into Miles, shews their Distance ; and so far this Method of Sailing is the same with *Plain* and *Merca-*
tor Sailing.

PROB.

P R O B. I.

The Latitudes of two Places, and their Difference of Longitude, being given, to find their Course and Distance.

E X A M P L E.

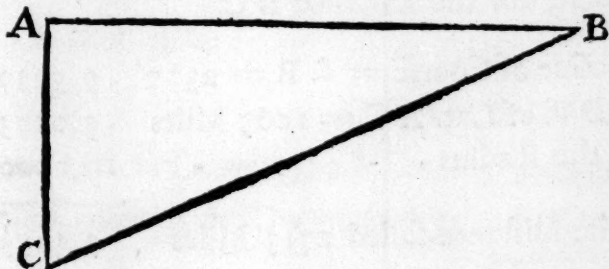
Admit a Ship sail from the Lizard, Latitude $50^{\circ} 10'$ North, to the Island of Bermudas, whose Latitude is $32^{\circ} 25'$ North, and its Difference of Longitude is supposed to be $56^{\circ} = 3360$ Miles West of the Lizard, what will the Course and Distance be?

$$\begin{array}{rcl} \text{Lat. } \left\{ \begin{array}{l} 50^{\circ} 10' \\ 32 \quad 25 \end{array} \right. & \begin{array}{l} \text{Parallel Parts } 0,3203 \\ \text{Ibid.} \quad - \quad 0,4222 \end{array} \\ \hline & & \end{array}$$

$$\begin{array}{rcl} \text{Diff. } \begin{array}{l} 17 \quad 45 \\ 60 \end{array} & \begin{array}{l} \text{Their Sum } 0,7425 \\ \text{Diff. of Long. } 3360 \end{array} & \left. \vphantom{\begin{array}{l} 17 \quad 45 \\ 60 \end{array}} \right\} \text{Mult.} \\ \hline & & \end{array}$$

Miles $1065 = AC$ Product $2494,8 \text{ M.} = AB$

HENCE, in the right angled Triangle CAB , there is given the Difference of Latitude $AC = 1065$ Miles, and the mean parallel Difference of Longitude $AB = 2494,8$ Miles, to find the $\angle C$ and BC , as in *Plain Sailing*; thus,



As

320 *The Calculation of*

As $AC = 1065$ Miles, Diff. of Lat. 3,0273496
 Is to the Radius - - - 10,0000000
 So is $AB = 2494,8$ Miles, the mean }
 Difference of Longitude - - } 3,3099773

 To the Tang. of the Course $66^\circ 53'$ }
 = $SW b W + 10^\circ 38'$ Westerly } 10,3696277

Or the Course may be otherways found without multiplying the Sum of the parallel Parts, with the Difference of Longitude, &c. as above ; thus,

R U L E.

If the Logarithm of the Sum of the parallel Parts, the Logarithm of the Difference of Longitude, and the arithmetical Complement of the Logarithm of the Difference of Latitude, be all three added together, and 10 be subtracted from their Sum, the Remainder will be the Tangent of the Course ; as in the last Example.

Sum of the parallel Parts is 0,7425 L. 9,8706965
 Difference of Longitude is 3360 Log. 3,5263393
 Diff. of Lat. 1065 Arith. Comp. - 6,9726504

 T. of $66^\circ 53'$ Ang. of Course as above 10,3696862

Then, for the Distance BC .

As Cosine of Course = $\angle B = 23^\circ 7'$ 9,5939555
 Is to Diff. of Lat. $AC = 1065$ Miles 3,0273496
 So is the Radius - - - 10,0000000

 To the Distance sailed 2713 Miles 3,4333941

P R O B.

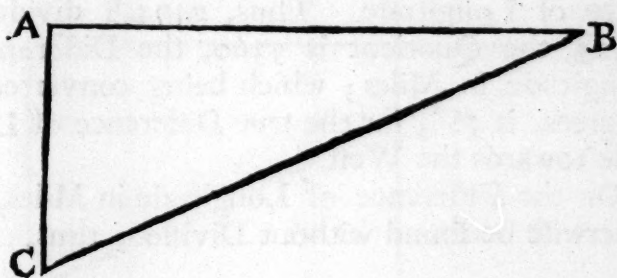
P R O B. II.

One Latitude, with the Course and Distance, being given, to find the other Latitude, and the Difference of Longitude.

E X A M P L E.

Suppose a Ship sail from the Latitude of $50^{\circ} 10'$ North, upon a S W by W $10^{\circ} 38'$ Westerly Course, until she hath sailed 2713 Miles, what Latitude will she be in, and what will the Difference of Longitude be?

IN this Example there is given S W by W $10^{\circ} 38'$ Westerly, which is $66^{\circ} 53' = \angle C$, and the Distance sailed 2713 Miles = CB, to find A B the mean parallel Difference of Longitude, and A C the Difference of Latitudes.



To find A C.

As Radius	- - - -	10,0000000
Is to the Dist. sail'd = 2713 Miles		3,4333941
So is the Cosine of the Course $66^{\circ} 53'$		9,5939555
To the Diff. of Lat. = 1065 Miles		<u>3,0273496</u>

Y

Which

Which being converted into Degrees is $17^{\circ} 45'$:
Then from the

1st Lat. $50^{\circ} 10'$	Parallel Parts	0,3204	} Add
Subtract <u>17 45</u>	Ibid. - -	<u>0,4222</u>	
New L. 32 25	Their Sum is	0,7425	to be reserved.

Then,

As the Radius - - -	10,0000000
Is to the Dist. fail'd = 2713 Miles	3,4333941
So is the Sine of the Course $66^{\circ} 53'$	<u>9,9636496</u>
To the mean Diff. of Long. = 2494,8	3,3970437

Which being divided by the reserved Sum of the parallel Parts, the Quotient will be the Difference of Longitude. Thus, 2494,8 divided by ,7425, the Quotient is 3360, the Difference of Longitude in Miles; which being converted into Degrees, is 56° , for the true Difference of Longitude towards the West.

Or the Difference of Longitude in Miles, may otherwise be found without Division, thus,

From the Logarithm last found -	3,3970437
Take the Log. of the Sum of parallel Parts, viz. of 0,7425, the Logarithm of which is - -	9,8706965

There will remain the Logarithm of the Difference of Longitude, viz. 3360 as above - - -	3,5263472
--	-----------

P R O B.

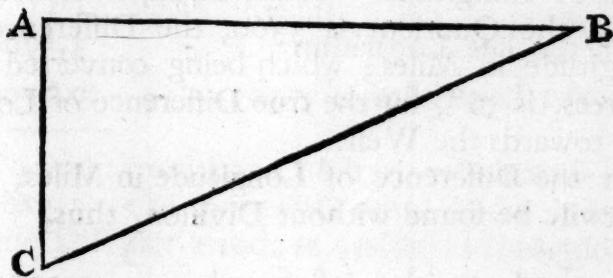
P R O B. III.

The Latitudes of two Places, and the Course between them, being given, to find their Distance and Difference of Longitude.

E X A M P L E.

Suppose a Ship sail from the Latitude of $32^{\circ} 25' N.$ upon a NE $\frac{1}{2}$ E Course $10^{\circ} 38'$ more Easterly, until she come to the Latitude of $50^{\circ} 10'$, what Distance hath she sailed, and how many Degrees hath she altered her Longitude?

IN this Example there is given the NE $\frac{1}{2}$ E + $10^{\circ} 38'$ Point = $66^{\circ} 53' = \angle C$, and the Difference of the Latitudes, viz. $32^{\circ} 25'$ taken from $50^{\circ} 10'$, the Remains is $17^{\circ} 45' = 1065$ Miles = AC; to find AB and CB.



To find BC.

As the Cosine of the Course $66^{\circ} 53'$	9,5939555
Is to the Diff. of Latitudes 1065	3,0273496
So is the Radius - - -	10,0000000
To the Distance sailed 2713 Miles	3,4333941

Next, To find A B, the mean Difference of Longitude.

Lat.	{	50° 10'	Parallel Parts	0,3203	}	Add
		32 25	Ibid. - -	0,4222		
			Their Sum	0,7425		

As the Radius	-	-	-	10,0000000
Is to the Difference of Latitude 1065				3,0273496
So is the T. of the Course 66° 53'				10,3696277
				3,3969773

To the mean Diff. of Long. = 2494,8 M. 3,3969773

Which being divided by 0,7425, the Sum of the parallel Parts, the Quotient will be the Difference of Longitude, *viz.* 3360 Miles = 56°, the Difference of Longitude towards the East: Or,

From the last Logarithm	-	3,3969773
Take the Logarithm of ,7425	-	9,8706965
		3,5262808

The Remainder is the Logarithm }
 3360, which is the Difference of }
 Longitude in Miles, as above }

PROB.

P R O B. IV.

The Latitudes of two Places, and their Distance, being given, to find their Course and Difference of Longitude.

E X A M P L E.

Suppose a Ship sail from the Latitude of $50^{\circ} 10' N.$ between the South and the West 2713 Miles, and then is found by Observation, to be in the Latitude of $32^{\circ} 25' North$, 'tis required to find what Course the Ship hath sailed upon, and how many Degrees she hath altered her Longitude?

HERE is given AC, and BC = 2713 Miles, to find AB, and the $\angle C$, viz.

$$\begin{array}{rcl} \text{Lat.} & \left\{ \begin{array}{ll} 50^{\circ} 10' & \text{Parallel Parts } 0,3203 \\ 32 & 25 \quad \text{Ibid.} \quad - \quad - \quad 0,4222 \end{array} \right\} & \text{Add} \\ & & \hline & & 0,4222 \end{array}$$

$$AC = 17 \quad 45 = 1065 \qquad 7425 \text{ Sum.}$$

To find the $\angle C$.

$$\text{As the Distance sailed} = 2713 \text{ Miles} \quad 3,4334498$$

$$\text{Is to the Radius} \quad - \quad - \quad - \quad 10,0000000$$

$$\text{So is the Diff. of Lat.} = 1065 \text{ Miles} \quad 3,0273496$$

$$\text{To the Cosine of the Course} = 66^{\circ} 53' \quad 9,5938998$$

Which is SWbW + $10^{\circ} 38' W.$ Then,

As the Radius - - - 10,0000000
 Is to the Distance failed = 2713 Miles 3,4334498
 So is the Sine of the Course = $66^{\circ} 53'$ 9,9636496
 To the mean Diff. of Long. 2494,9 3,3969774

Or thus,

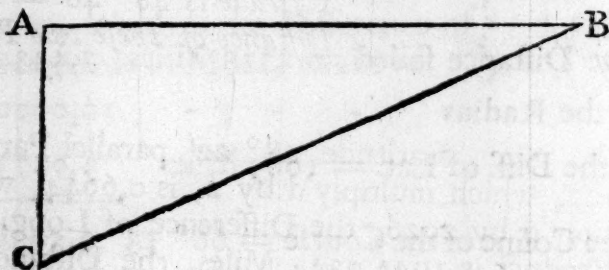
The Dist. BC = 2713 }
 D. of Lat. AC = 1065 } Add and subtract.

Their Sum 3778 Its Log. 3,5772620
 Their Diff. 1648 Ibid. 3,2169572

6,7942192 Sum

Whence the mean Diff. = 2495 }
 Log. of which is - - - } 3,3971096 = $\frac{1}{2}$ S.

From whence the true Difference of Longitude in Miles, &c. may easily be obtained, either by Division, or by the Logarithms, as in the 2d and 3d Problems foregoing.



PROB.

P R O B. V.

The Difference of Longitude between any two Places that are both in any known Parallel of Latitude, being given, to find their true Distance in Sea Miles.

THE Solution of this Problem is easily performed by the Table of parallel Parts, and the Reason thereof is as easily conceived, if it be considered that those Numbers are only the Parts of a half Mile's Difference of Longitude in each of their respective Latitudes. Whence it follows, That,

If the Difference of Longitude in Miles between any two Places, both being in one Parallel of Latitude, be multiply'd by the Double of the parallel Parts belonging to that Latitude, the Product will be the true Distance in Miles, as in the following

E X A M P L E.

Suppose two Places lie both in the Parallel of $48^{\circ} 22'$ (either North, or South) Latitude, and their Difference of Longitude is $48^{\circ} 46' = 2926$ Miles, what is the Distance of these two Places asunder?

The given Latitude $48^{\circ} 22'$ parallel Parts = 0,3322, which multiply'd by 2, is 0,6644, which multiply'd by 2926, the Difference of Longitude, the Product is 1944,0344 Miles, the Distance required.

Or otherways, by the Logarithms without Multiplication; thus,

Y 4

Double

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Double of parallel Parts is 0,6644 Log. 9,8224296

Diff. of Long. in Miles is 2926 Log. 3,4662743

Hence the Dist. is 1944 Miles, as above 3,2887039

P R O B. VI.

The Distance between any two Places that are both in any known Parallel of Latitude, being given, to find their true Difference of Longitude.

THIS Problem is only the Converse of the last, and is therefore to be answered by the converse Operation; thus, Divide the Distance given, by the Double of the parallel Parts belonging to the given Latitude, and the Quotient will be the Difference of Longitude in Miles, &c. as in the following

E X A M P L E.

Suppose a Ship sail 1944 Miles (either directly East, or directly West) in the Parallel of $48^{\circ} 22'$ Lat. what is her Difference of Longitude?

Here the given Latitude $48^{\circ} 22'$, parallel Parts 0,3322, the Double of which is 6644, and 1944, the given Distance, divided by 6644, the Quotient is 2926, the Difference of Longitude in Miles.

Or otherwise, by the Logarithms without Division; thus,

From

Nautical Prob. by Parallel Parts. 329

From the Log. of the Dist. 1944 Miles	3,2886963
Take the Log. of double Parts 0,6644	9,8224296
The Remains will be the Log. of 2926	3,4662667

Which being converted into Degrees, will be $48^{\circ} 46'$, the true Difference of Longitude, as was required.

P R O B. VII.

If the Distance sailed in any Parallel of Latitude, and the Difference of Longitude were given, to find the Parallel of Latitude.

FROM the Work above, it will be easy to perceive, that if the given Distance be divided by the Difference of Longitude in Miles, Half that Quotient will be the parallel Parts, which being found in the Table, will shew, or assign the Latitude sought, &c. for the rest.

Thus you have the Method of computing a Ship's Motion upon any single Course (having the usual Datas) by the Help of the Table of parallel Parts; and that nothing may be wanting to render it as plain and easy to be understood as possibly I can, I shall here give a Solution at large, to the same Traverse that hath been already proposed in *Plain and Mercator Sailing*, viz.

Suppose a Ship is to sail from the Lizard, Latitude $50^{\circ} 10'$ to Bermudas, Latitude $32^{\circ} 25'$, both North, whose Difference of Longitude is $56^{\circ} = 3360$ Miles West of the Lizard, and being put to Sea, sails upon the following Courses, viz. WSW 100 Miles, SSE 78 Miles, W 95 Miles, NW

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NW $\frac{1}{4}$ W 67 Miles, S 58 Miles, and then
 SW $\frac{3}{4}$ W 90 Miles; what Latitude will the
 Ship be in, and how much will she have altered
 her Longitude; and what will the direct Course
 and Distance be from the Place at the End of this
 Traverse, to Bermudas?

What Angles these several Courses make with
 the Meridian, is set down in the Traverse Tables
 in *Plain and Mercator Sailing*; and the Differ-
 ence of Latitudes and Longitude upon the first
 Course, may be found by *Prob. II.* of this *Section*.
 Thus,

As Radius	- - - - -	10,0000000
Is to the Distance sailed 100 Miles	-	2,0000000
So is the Cosine of the Course $67^{\circ} 30'$		9,5828397
To the Diff. of Latitude 38,2 Miles		1,5828397

Then $50^{\circ} 10' - 9^{\circ} 38' = 49^{\circ} 32'$, the Ship's
 Latitude.

Lat.	{	50° 10'	Parallel Parts	0,3203	}	Add
		49 32	Ibid. - -	0,3245		
				0,6448		

Its Logarithm is 9,8094250

Then as the Radius	- - -	10,0000000
Is to the Distance sailed = 100 Miles		2,0000000
So is the Sine of the Course $67^{\circ} 30'$		9,9656153
To the mean Diff. of Long. = 92,38		1,9656153

Which

Nautical Prob. by Parallel Parts. 331

Which being divided by the Sum of the parallel Parts, the Quotient is 143,3 Miles, the true Difference of Longitude.

Or the Difference of Longitude may more readily be found by this Proportion.

As the arithmetical Complement of }
 the Sum of the parallel Parts, } 0,1905750
viz. 0,6448 - - -

Is to the Distance sailed = 100 Miles 2,0000000

So is the Sine of the Course $67^{\circ} 30'$ 9,9656153

To the Diff. of Long. = 143,3 Miles 2,1561903

II. Upon the 2d Course.

As Radius - - - - 10,0000000

Is to the Distance = 78 Miles - 1,8920946

So is the Cosine of the Course $22^{\circ} 30'$ 9,9656153

To the Diff. of Lat. = 72,1 Miles 1,8577099

Hence the new Latitude the Ship is in, will be $48^{\circ} 20'$.

Lat. $\left\{ \begin{array}{l} 49^{\circ} 32' = P. ,3245 \\ 48 \quad 20 \quad \text{Ibid.} ,3324 \end{array} \right\}$ Add, and then,

As Sum of these Parts ,6569 Ar. Comp. 0,1825007

Is to the Distance sailed 78 Miles 1,8920946

So is the Sine of the Course $22^{\circ} 30'$ 9,5828397

To the Diff. of Longitude 45,4 Miles 1,6574350

III. To

III. To find the Difference of Longitude upon the 3d Course, *viz.* due West in the Parallel of $48^{\circ} 20'$ Latitude, whose parallel Parts are ,3324, which doubled is ,6648 : Then,

From the Log. of Dist. fail'd 95 Miles	1,9777236
Take the Log. of double Parts ,6648	9,8226910
The Difference of Long. 142,9 Miles	2,1550326

IV. Upon the 4th Course.

As Radius - - - - -	10,0000000
Is to the Distance failed = 67 Miles	1,8260748
So is the Cosine of the Course $59^{\circ} 4'$	9,7109972
To the Diff. of Lat. 34,4 Miles North	1,5370720

Which being added to the last Latitude, the Sum is $48^{\circ} 54'$, a new Latitude.

Lat. $\left\{ \begin{array}{l} 48^{\circ} 20' = P. ,3324 \\ 48 \quad 54 \quad \text{Ibid.} ,3287 \end{array} \right\}$	Add, and then,
--	----------------

As Sum of these Parts ,6611 Ar. Comp.	0,1797628
Is to the Distance failed = 67 Miles	1,8260748
So is the Sine of the Course = $59^{\circ} 4'$	9,9333688
To the Diff. of Long. = 86,9 Miles	1,9392064

V. The 5th Course being directly South, it only alters the Latitude just so much as the Distance failed, *viz.* 58 Miles, or Minutes, and therefore the new Lat. the Ship is in is $47^{\circ} 56'$.

VI. Upon

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VI. Upon the 6th and last Course.

As the Radius - - - - 10,000000
 Is to the Distance failed = 90 Miles 1,9542425
 So is the Cosine of the Course $39^{\circ} 22'$ 9,8882372
 To the Diff. of Lat. 69,6 Miles - 1,8424797

Hence the new Latitude will be $46^{\circ} 47'$.

Lat. $\left\{ \begin{array}{l} 47^{\circ} 56' = P. ,3350 \\ 46 \quad 47 \quad Ib. ,3424 \end{array} \right\}$ Add, and then,

As Sum of these Parts ,6774 Ar. Comp. 0,1691548
 Is to the Distance failed 90 Miles 1,9542425
 So is the Sine of the Course $39^{\circ} 22'$ 9,8022816
 To the Diff. of Longitude 84,2 Miles 1,9256789

Now, if the Answers thus found to the several Parts of this Traverse, be collected and truly disposed of, each under its proper Head, the Work will stand as in this TABLE.

Courses	Points	Diff. failed	Diff. of Lat.		Lat. the Ship's in	Diff. of Long.	
		M.	North	South		East	West
W. S. W.	6	100	- -	38,2	49,32	- -	143,3
S. S. E.	2	78	- -	72,1	48,20	45,4	- -
W.	8	95	- -	- -	48,20	- -	142,9
NW $\frac{1}{4}$ W	$5 \frac{1}{4}$	67	34,4	- -	48,54	- -	86,6
S.	0	58	- -	58,0	47,56	- -	- -
SW $\frac{1}{2}$ S	$3 \frac{1}{2}$	90	- -	69,6	46,47	- -	84,2

Sum - 45,4 | 457,0

From
I

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From the Work of this Traverse, it appears, that the Ship is got into the Latitude of $46^{\circ} 47'$ North, and hath altered her Longitude $6^{\circ} 51' 36''$ Westward from the Meridian of the *Lizard* (that is, from 457 take 45,4, and the Remains is 411,6 Miles = $6^{\circ} 51' 36''$, which differs but $18''$ from what was found by *Mercator Sailing*.

Then, to find what Course and Distance the Ship must steer to arrive at *Bermudas*, work as in *Prob. I.* of this *Section*.

Lat.	$\begin{cases} 46^{\circ} & 47' \\ 32 & 25 \end{cases}$	Parallel Parts	$\begin{cases} 3424 \\ 4222 \end{cases}$	} Add
		Ibid. - -		
Diff.	14 22 = 862 Miles		57646	Sum.

And from the Difference of Longitude of the *Lizard*, and the Island of *Bermudas*, viz. 56° subtract the Difference of Longitude gained by the Traverse $6^{\circ} 51'$, and the Remains $49^{\circ} 9' = 2949$ Miles for the Diff. of Longitude between the Place the Ship is in and *Bermudas*: Then,

As the Diff. of Lat. 862 Miles	Comp. Ar.	7,0644927
Is to the Diff. of Longitude 2949 Miles		3,4696748
So is the Sum of the = P. 57646	Log.	9,8834343
To the Tangent of the Course $69^{\circ} 5'$		10,4176018

Next, To find the Distance.

As the Cosine of the Course $69^{\circ} 5'$		9,5526801
Is to the Diff. of Latitude 862 Miles		2,9355073
So is the Radius - - -		10,0000000
To the Distance required 2414,5 Miles		3,3828272

Hence

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Hence the Course that the Ship must steer from the Place she is in, to the Island of *Bermudas*, must be $WSW + 1^{\circ} 35'$ to the West, and the Distance she must sail to arrive there, is 2414,5 Miles, which is only 1 Mile $\frac{3}{4}$ less than it was found by *Mercator Sailing*.

Now, altho' the Work of this Traverse, and the rest of the Calculation in this *Section*, if compared with their respective Cases in *Mercator Sailing*, do sufficiently shew, that the Difference between the Answers found by parallel Parts, and those found by meridional Parts, is very inconsiderable, yet it may not be amiss to give the Reader some farther Satisfaction therein; and, in order to that, I shall here propose to find the Course and Distance between two Places that are very remote from each other, and solve the Questions at large, according to all the Methods already taught; and likewise the true Distance by great Circle Sailing, that so it may appear how the Truth of this Method may be depended on.

Q U E S T I O N.

Suppose a Ship was to sail from Longnass, which is upon the East Coast of Iceland, in the Latitude $66^{\circ} 26'$ North, to the River of Amazons, which is under the Equinoctial, or on the Coast of Brazil, in America, whose Difference of Longitude from Longnass, we will suppose to be 36° Westward, what Course and Distance must she sail?

In this Question there is given the Difference of Latitudes, $66^{\circ} 26' = 3986$ Miles, and the Difference of Longitude $36^{\circ} = 2160$ Miles, to find the Course and Distance.

1. By

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1st. By Plain Sailing.

As the Diff. of Latitudes 3986 Miles	3,6005373
Is to the Radius - - - -	10,0000000
So is the Diff. of Long. 2160 Miles	3,3344537
To T. of the $\angle$ of the Course $28^{\circ} 27'$	9,7339164

Then,

As the Cosine of the Course $28^{\circ} 27'$	9,9441041
Is to the Diff. of Lat. 3986 Miles	3,6005373
So is the Radius - - - -	10,0000000
To the Distance required 4533 Miles	3,6564332

2dly. By Mercator Sailing.

Lat. $\left\{ \begin{array}{l} 66^{\circ} 26' \\ 00 \quad 00 \end{array} \right.$ M. P. 5388 } Or Lat. enlarg'd.
 Ibid. 0000 }

As the Diff. of Lats. enlarg'd 5388 Miles	3,7314276
Is to the Radius - - - -	10,0000000
So is the Diff. of Long. 2160 Miles	3,3344537
To T. of Ang. of Ship's Course $21^{\circ} 15'$	3,6329138

Then,

As the Cosine of the Course $21^{\circ} 15'$	9,9676235
Is to true Diff. of Lat. 3986 Miles	3,6005373
So is the Radius - - - -	10,0000000
To the Distance 4294,5 Miles -	3,6329138

3dly. By

Nautical Prob. by Parallel Parts. 337

3. By Middle Latitude.

The middle Difference of Latitude is $33^{\circ} 13'$:
Then,

As Diff. of Lat. 3986 Miles	- - -	3,6005373
Is to Cosine of the midd. Lat. $33^{\circ} 13'$		9,9225205
So is the Diff. of Long. 2160 Miles		3,3344537
To the Tang. of the Course $24^{\circ} 23'$		9,6564369

Then,

As the Cosine of the Course $24^{\circ} 23'$		9,9594248
Is to the Diff. of Lat. 3986 Miles		3,6005373
So is the Radius - - - -		10,0000000
To the Dist. required 4376,3 Miles		3,6411125

4. By the Arch of a great Circle.

It is supposed that the Learner does not yet understand *Spherical Trigonometry*; for which Reason, it would be altogether inconsistent to place a Work here to which the Learner is an imagined Stranger, and I shall therefore content myself with setting down the Distance as it may be found by a spherical Calculation, which the Learner may go thro' when he is acquainted with the Solution of spherical Triangles, and will find it to be $71^{\circ} 8' = 4268$ Miles, being the true Distance between *Longness* and the River *Amazon*s.

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5. *To find the Course and Distance by the Method of Parallel Parts.*

Latitudes	{	66° 26'	Parallel Parts	0,1999
		00 00	D°. - -	0,5000
Their Sum				6999

Then,

As Diff. of Lat. 3986 Miles	- - -	3,6005373
Is to the Diff. of Long. 2160 Miles		3,3344537
So is the Sum of the P. P. 0,6999 Log.		9,8450360
To the Tang. of the Course 20° 47'		9,5789524

Then,

As the Cosine of the Course 20° 47'		9,9707786
Is to the Diff. of Lat. 3986 Miles		3,6005373
So is the Radius - - -		10,0000000
To the Dist. required 4263,4 Miles		3,6297587

Let us now collect these several Distances and compare them together, that we may see which of them comes nearest to that found by the Arch of a great Circle, which is undoubtedly the true Distance between the aforesaid Places.

The Distance by the Arch is 4268 Miles: By *Plain Sailing*, 4533 Miles, which is 265 Miles too much: By *Mercator Sailing* 4294 Miles, and is 26 Miles too much: By *Middle Latitude* 4376 Miles, being 108 Miles too much: By *Parallel Parts*

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Parts 4263 Miles, being only 5 Miles too little, which is a very small Difference in so large a Distance, and is an Error on the safest Side.

In this last Example the Difference of Latitudes between the two proposed Places is 3986 Miles, which is almost double to 2160, the Difference of Longitude; and in the first Example of parallel Parts, the Difference of Longitude is 3360 Miles, which is more than triple the 1065, the Difference of Latitude; and yet in both the Examples, the Distance found by the Help of the parallel Parts, differs but a very little from the true Distance, which is a sufficient Argument to warrant the Practice of this Method.

However, as a farther Confirmation of the Truth thereof, I shall here propose a third Example, wherein the Difference of Latitudes, and that of Longitudes, are nearer equal to each other.

Suppose it was required to find the Distance between the Lizard, Latitude $50^{\circ} 10'$ North, and the Island of Barbadoes, in the Latitude of $13^{\circ} 10'$ North, whose Difference of Longitude is $52^{\circ} 58'$ West of the Lizard?

The Distances of these two Places being truly found, according to all the aforefaid Methods of Sailing, will be as follows, viz.

The Distance by *Great Circle Sailing* 3396 Miles, being the true Distance: By *Middle Latitude* 3499 Miles, which is 103 Miles too much: By *Mercator Sailing* 3433 Miles, and is 37 Miles too much: By *Parallel Parts* 3393 Miles, being 3 Miles too little.

And now, upon the Whole, I hope the Reader hath received ample Satisfaction how far (or with what Safety) this Method of Sailing which I have here proposed, may be relied on in Practice: And, that I may adventure (without Presumption) to say of it as Mr. *Edward Wright*, the Author of *Mercator's Sailing*, and Mr. *Norwood*, who understood it very well, did both say of *Mercator's Chart* (and consequently of that Method of Sailing) *viz.* That it agrees with the Globe itself, without any sensible or considerable Error.

Note, *By the Globe itself, is here meant, sailing by the Arch of a great Circle.*

And as to the Easiness of the Operations performed by the Help of these parallel Parts, I believe the Reader may judge of that, by what he hath seen done in this Section, which may also be as easily performed with Scale and Compasses, either by projecting, or calculating: But, above all (take my Advice in this) what Method soever you chuse, stick close to the Doctrine of plain Triangles, *viz.* To Calculations performed by the Canon of Sines, &c. as in this Treatise, which is, above all other Ways, the most exact, and, with a little Practice, will be found as easy and expeditious, as any other Way whatsoever.

I shall only beg Leave to add, That if the ingenious Mariner will but give himself the Trouble to consider a little of what hath been said concerning the Nature of this Projection, or Chart, made by parallel Parts, and the Explanation of the Figure annex'd to the Chart, he must needs understand how to protract a Ship's Traverse upon it, without any farther Directions,
by

by having the usual Datas given, and consequently be able to keep a Reckoning of any Voyage at Sea, upon such a Chart; and, I am very sure he will find it much easier to lay down and measure his Courses and Distances upon such a Chart, than upon that of *Mercator's*, which, it is well known, cannot be done without some Difficulty: Not that I reflect upon *Mercator's Projection*, which has so justly found a general Reception among the most skilful Part of the World, and is, in my Opinion, the best, before this Time, published, notwithstanding what a certain Gentleman hath advanced in Praise of *Middle Latitude Sailing*, viz. *That it will serve, without any considerable Error, in Runnings of 150 Leagues between the Equator and the parallel Parts of 30°; of 100 Leagues between that and 60°, and of 50, as far as we have any Occasion, as some ingenious Men have very well observed.* 'Tis possible that it may serve in short Voyages; and so (several Authors tell us *Plain Sailing* will do, without any great Error, in such Places as are near to the Equinoctial; also in many other Places, for short Voyages; and even for long ones, provided that a Man was sure to return the same Way he went, or near the same; or, as a late Author says, by breaking a long Voyage into many short ones, a Voyage may be pretty well perform'd by *Plain Sailing*, especially if it be made near the same Meridian.

These, and the like Cautions, which direct when and where, or how far, to use either *Plain Sailing*, or that by the *Middle Latitude*, do, to me, seem only to conceal or cover the Imperfections of those Methods; for whatsoever Method will not hold pretty near to the Truth (viz. to

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that by the Arch of a great Circle) in long Distances, is proportionally false in short ones; and altho' the Errors are not so evident in short Voyages, yet the Aggregate, or Sum of all these Errors, will amount to that of a long Voyage; and, therefore, it is my Opinion, that neither *Plain Sailing*, nor that by the *Middle Latitude*, ought to be depended upon in any Voyage.





A
COMPENDIUM
OF THE
VARIATION *of the* COMPASS.

S E C T. XIV.

I.  HE Compass is an Instrument that no Navigator ought to be ignorant of, either as to its Use or Properties, which are, indeed, very wonderful.

II. The Use of the Compass is to direct the skilful Mariner thro' the great Deep by a certain Road, so as to attain the desired Port, according as the Wind permits, and is known to almost every Sailor.

III. It was many Years after the Use of the Magnet was first discovered, that it was known there was any Variation in the Needle; but for Years it was believed, that the N. and S. Points of the Compass, pointed to the N. and S. Points

of the Horizon: But that has long since been proved to be a Mistake, and the Method of Sailing rendered in a manner almost certain, which it was not before; for it is evident, that if the Instrument by which the Navigator directed his Course (was believed to be true) and was really false, that Direction must of Necessity be false also: But altho' the Compass is false, yet if we know to what Degree it is so, then it is in effect the same thing as if it was really true; and therefore it is a Matter of the greatest Importance to the skilful Seaman, to know how much the North or South Point of the Compass, differs from the North and South Points of the Horizon; which Difference is called the Variation, and the Variation of the Compass being not the same in all Places, nor even always the same in the same Place, it is absolutely necessary to attain it by some Observation or another, and none has been found better, than by comparing the magnetical Amplitude, with the true Amplitude; or, the magnetical Azimuth, with the true Azimuth.

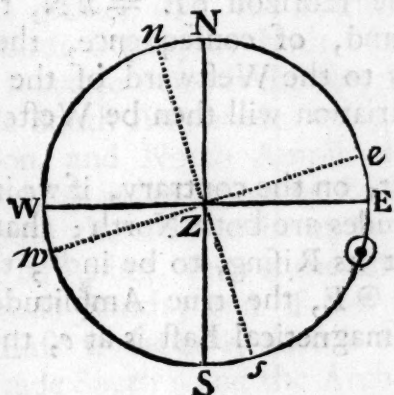
IV. The true Amplitude is found by spherical Triangles, and the magnetical Amplitude is found by an Observation of the Rising or Setting of Sun or Star, and is what is so easy to be obtained, that it would be altogether needless to say any thing of it in this Place, and shall therefore proceed to find the Variation from the Comparison of these two Amplitudes, as briefly (tho' plainly) as possible.

V. In the annex'd Scheme let W N E S represent the Horizon, and the Points W, N, E, S, the West, North, East, and South Points; and by Observation, suppose it is found, that the Points *w, n, e, s*, are the magnetical West, North, East,

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East, and South Points thereof, then is the Arch n N (or, which is the same thing, the $\angle n z N$) the Variation of the Compass required.

Fig. I.



So that the Variation is that Angle made between the true Meridian of the Place, and the magnetical Meridian, whose angular Point is in the Zenith; or, which is the same thing, it is the Measure of the Arch $n N = s S$ of the Horizon, contained between the aforefaid Meridians, turned into Degrees and Minutes.

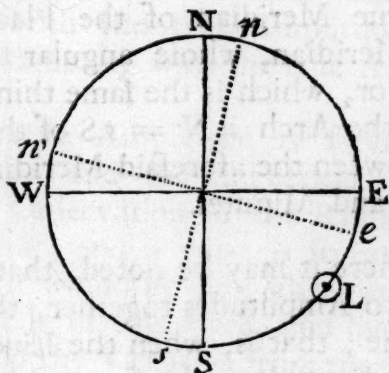
VI. And here it may be noted, that if in comparing the two Amplitudes together, they happen to be the same; that is, when the Line $w e$ is the same with $W E$, and $n s$ the same with $N S$, then there is no Variation; but if they differ, and are the same Way, *viz.* either both North, or both South, then their Difference is the Variation; which, if it be at Sun-rising, and the Amplitudes are both North, and the magnetical exceeds the true, the Variation is East; but, if they are both South, the Variation is West, as will be shewn hereafter.

For,

For, in the preceding Figure, if $\odot$ represent the Sun at its Rising, then is $E \odot$ the true Amplitude; and let us imagine magnetical East to be in e , then is $e \odot$ the magnetical Amplitude; and it is evident, the magnetical Amplitude, made less by the true Amplitude (both being South) is the Arch of the Horizon $eE = nN$, the Variation required, and, of consequence, the magnetical North, lies to the Westward of the true North, and the Variation will then be Westerly.

VII. But, on the contrary, if we imagine, that the Amplitudes are both North; that is, suppose the Sun, at its Rising, to be in $\odot$, then it is evident, that $\odot E$, the true Amplitude, is North; and if the magnetical East is at e , then is $\odot e$ the

Fig. II.



magnetical Amplitude North; and if we subtract the Arch $\odot E$, the true Amplitude, from the Arch $\odot e$, the magnetical Amplitude, the Remainder is the Arch $Ee = Nn$, the Variation, which, in this Case, is Easterly; for it is evident, from the Nature of the Thing, that the Point n , the magnetical North, will be to the Eastward of the

the true North, because the magnetical East is, by Supposition, to the Southward of the true East, and, of consequence, the magnetical Amplitude must be the greatest.

VIII. But if it should so happen, that the true Amplitude is greater than the magnetical Amplitude, and both South, or both North, it will then be the Reverse of both the foregoing Examples, and the South Amplitudes will give an Easterly Variation, and North Amplitudes, a Westerly Variation, as in the following Examples.

For, suppose the Sun to be at L (in the last Scheme) at its Rising, then is the Arch LE the true Amplitude South; and if we imagine the magnetical East to be at *e*, then is $\odot e$ the magnetic Amplitude South; and the Arch $\odot e$ subtracted from the Arch LE, the Remainder is the Arch $Ee = Nn$, the Variation required, which, in this Case, will be Easterly, as may be easily seen by the Figure.

IX. But if we suppose the Sun to be at L (*Fig. I.*) then is the Arch LE the true Amplitude North; and if by Observation it is found that the magnetic East is at *e*, then is Le the magnetical Amplitude North; and the Arch Le , subtracted from the Arch LE, the Remainder is the Arch $eE = Nn$, the Variation required, which is Westerly, and is the Reverse of the first Example, as the last was of the second.

X. If these four Examples be understood, it will be easy to deduce the following Observations about the Amplitudes at Setting, *viz.* That if the magnetical Amplitude exceeds the true Amplitude, and both South, then the Variation will be Easterly; but if they are both North, the Variation

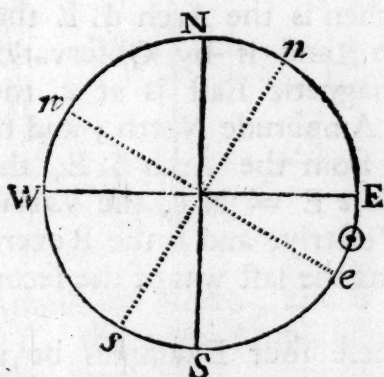
tion is Westerly: And if the true Amplitude exceeds the magnetic, and both North, at Setting, then will the Variation be East; but if they are both South, the Variation will be West.

These Observations seem to be such necessary Consequences from the foregoing Examples, that they need no Examples to explain them; for they are what every ingenious Reader would have made himself, the Reasons for them being so obvious.

XI. If, in comparing the Amplitudes together, it is found that they differ in Position, that is, one is South, and the other North, then their Sum will be the Variation; which, if it be at Rising, and the magnetical Amplitude be North, the Variation is East; but if the magnetical Amplitude be South, the Variation is then West.

If in the annex'd Figure ☉ be the Place of the Sun at Rising, then is ☉ E the true Amplitude

Fig. III.



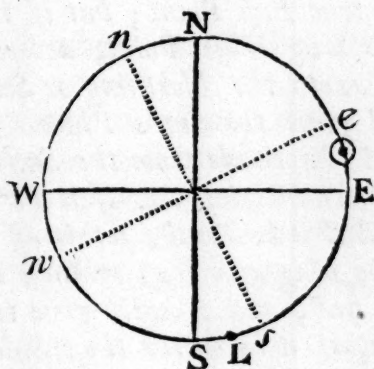
South: And if, upon Observation, it should so happen that *e* is the magnetical East, it is evident that ☉, the magnetical Amplitude, is North; and

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and that the Arch $E \odot$, the true Amplitude, added to $\odot e$, the magnetic Amplitude, gives the Arch $E e = N n$, the Variation requir'd, which, in this Case, is East.

But if (as in *Fig. IV.*) the Sun is suppos'd in $\odot$, then is $E \odot$ the true Amplitude North; and if, by Observation, it is found that the magnetic East is in e , then is $e \odot$ the magnetical Amplitude South; both which added together, is the Arch $E e = N n$, the Variation required, which, in this Case, will be Westerly.

Fig. IV.



From what has been said in the two last Examples, it will be easy to conceive, that if the Observation be taken at the Setting of the Sun, and by comparing the Amplitudes together, they are found to differ, and the Magnetical be South, then is the Variation Easterly; and if the Magnetical be North, then is the Variation Westerly. These being only the Reverse of the two former Examples, need none, as being very easy in themselves.

By

By comparing these Examples together, it will be easy to raise the following short Rule, *viz.*

If the Amplitudes, either at Rising or Setting, are the same Way; that is, both North, or both South, then your Difference is the Variation; but if they are the contrary; that is, one North, or one South, then their Sum will be the Variation.

And, to find whether it be East or West, it may be done by the following Method.

Set off the true Amplitude, if it be in the Morning, from the true East Point; but if in the Evening, from the true West Point, according as the Case directs, as to the Northing or Southing of it, and that will give the Sun's Place. Then set off the magnetical Amplitude from the Sun's Place, the contrary Way to its Title, viz. If it be North set it off South; and if it be South, set it off North, and the Point is the Magnetic East or West Point; from which set off 90° , and it will give the Magnetic North; which, if it be on the West Side of the true North, the Variation is Westerly; but if on the East Side, the Variation is Easterly.

Upon the due Consideration of what has been said concerning Amplitude, we may easily perceive, that the same will hold good when we take the magnetic and true Azimuth, and compare them together,

For (Fig. I.) if we suppose the Sun to be in $\odot$, then is the true Azimuth $\odot s$; and if we imagine that the magnetical South is at s , then the magnetical Azimuth, taken from the true Azimuth, gives the Arch $sS = nN$, the Variation, which, in
this

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this Case, is evident from the Figure to be Westerly, the same as in the first Example of the Amplitudes.

Again (*Fig. II.*) If we imagine the Sun to be at $\odot$, then $N \odot$ is the true Azimuth; and if by Observation it is found that n is the Magnetical North, then is $n \odot$ the magnetical Azimuth, which being taking one from the other, the Remainder must be the Arch of the Horizon $N n$, the Variation required.

For Variety's Sake, we will take one more Example, *viz.*

(*Fig. IV.*) where let L be the Sun's Place, and, of consequence, SL the true Azimuth East. If by Observation it is found that s is the Magnetical North, it is evident that the Arch SL , the true Azimuth, added to Ls , the magnetical Azimuth West is the Variation required, and in this Case will be Westerly.

These Things are so plain in themselves, that I think there need no more Examples, because it will be repeating, in effect, what has been already observed, over again; upon which Account, I believe the ingenious Mariner will, if he understands what has been said upon this Head, be very well able to go on with any other Varieties, either of Amplitude, or Azimuth: But forasmuch as what has been said, has not been reduced to Practice, I shall, before I conclude this Head, give two or three Examples in Numbers, which will set the Matter in a clearer Light.

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EXAMPLE I.

Suppose by Calculation the true Amplitude is found to be $12^{\circ} 14' = \text{Arch } E \odot$, and by Observation the magnetical Amplitude is $38^{\circ} 16'$, both South $= \text{Arch } e \odot$ (Fig. I.) what is the Variation?

From the magnetical Amplitude - $38^{\circ} 16'$

Subtract the Amplitude - $12 \quad 14$

The Rem. is the Variation $= E e = 26 \quad 02 \text{ W.}$

EXAMPLE II.

Suppose by Calculation the true Amplitude is $15^{\circ} 12'$ North $= \odot E$, and by Observation it is found that $e \odot$ (Fig. IV.) is the magnetical Amplitude South $12^{\circ} 14' = \odot e$, what is the Variation?

To the true Amplitude. - - - $15^{\circ} 12'$

Add the magnetic Amplitude - - $12 \quad 14$

The Sum is the Variation $= \text{Arche } e E = 27 \quad 26 \text{ W.}$

EXAMPLE III.

Suppose by Calculation the Sun's Azimuth is found to be $14^{\circ} 16'$ West $= \text{Arch } SL$ (Fig. IV.) and the magnetic Amplitude is West $= \text{Arch } Ls = 12^{\circ} 12'$, what is the Variation?

The true Azimuth - - - $14^{\circ} 16'$

Added to the magnetic Azimuth - $12 \quad 12$

The Sum is the Variation $= \text{Arch } Ss = 26 \quad 28 \text{ W.}$

In

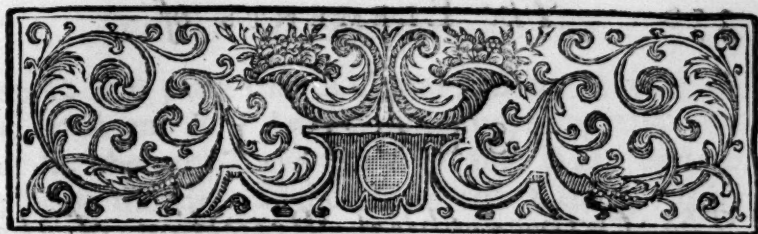
the Variation of the Compass. 353

In these Examples I have used the Azimuth in the Forenoon, but the Thing will hold the same if it were in the Afternoon; and through the whole I have considered the Sun only as the Object of Observation; not that the Observations of these Kinds are confined to it only; but any of the fixed Stars will very well do, because their Places are easily obtain'd, and it is necessary, if possible, that Observations of both should be taken; and, in Voyages, the oftner it is done, it is so much the better, for the Correction of the Variation is a Thing of the utmost Consequence to the Navigator, for Reasons already mentioned.



In this example I have used the Ashmolean
in the Museum, but the thing will hold the
same if it were in the Ashmolean; and though
the whole I have considered as but only as the
Object of Observation; not that the Observa-
tions of such kind are confined to it only;
but any of the kind may well be well
done, because their places are easily observed, and
it is necessary, it possible, that Observations
of both should be taken; and in Voyage, the
other it is done, it is in much the better, for the
Collection of the Ashmolean is a fine one of the
small Collection to the Navigator, for Res-
ons already mentioned.





THE
ELEMENTS
OF
SPHERICAL TRIGONOMETRY.

PART II.



S plain Trigonometry teacheth us to measure the Sides, and Angles of a right lined Triangle ; so *Spherical Trigonometry* teacheth to measure the Sides and Angles of Triangles, whose Sides are Parts of Circles ; which the Sides of all Triangles are, that can be, described upon a Sphere or Globe. But because a Sphere or Globe is not only cumbersome in its self, but in many Cases not very true, because of the Errors of the Mechanism, it will be necessary, before we come to Calculation, to teach, how any Circle or Circles of the Sphere may be Geometrically transferred or represented, upon a Plane in any given Position, which is a Branch of Perspective, and is principally of two Kinds, and are called *Orthographical Projection*, and *Stereographical Projection* ; of which, each in its proper Place.

S E C T. I.

The Orthographical Projection of the Sphere.

D E F I N I T I O N I.

THE Orthographical Projection of the Sphere, is the transferring the Representation of the Sphere, seen at an infinite Distance, together with its Circles, into a Plane, by Rays perpendicular to that Plane; the Foundation of which depends upon the following Supposition, *viz.*

That the Eye is at an infinite Distance both from the Object, and from the Plane upon which the Projection is made.

Which being granted, the following Proposition will appear to be plainly demonstrated.

P R O P. I.

The Rays by which the Eye sees any Object at an infinite Distance, differ infinitely little from parallel Rays.

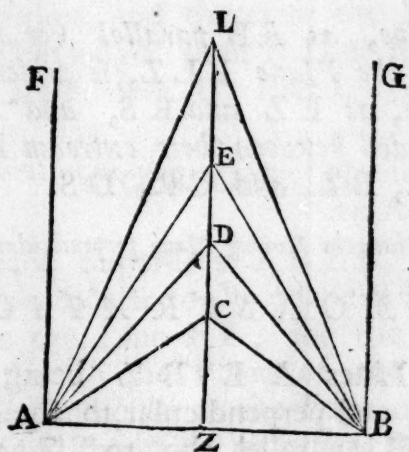
D E M O N S T R A T I O N

MAKE LZ perpendicular to AB, and in the Middle of it, draw FA and GB parallel to LZ; then draw AC, BC, AD, BD, AE, BE, AL, BL. Now suppposing the Eye at

at C, it is plain that AC and BC are Rays from C to A and B. And again, supposing the Eye at D, then the Rays are AD and BD; and so placing the Eye farther from the Center, as at E and L, the Rays become EA and LA, &c. The Eye being placed still farther, the Rays become longer; and as the Length of the Rays increase, the Angle at the Object increaseth; for the Angle DAZ is greater than the $\angle CAZ$, by *Prob. X. Sect. II. Part II.* And the Eye being infinitely extended from the Plane of the Projection, the Rays will become FA and GB, or infinitely near to them; and the Lines FA and GB being perpendicular to the Plane, the Rays will be so too, or infinitely near to Perpendiculars QED.

DEFINITION II.

That Plane, into which the Sphere together with its Circles, is to be transferred, is called the Plane of the Projection; and that Circle, in which the Representation falls, is called the Primitive.



DEFINITION. III.

A Point in the Surface of the Sphere, is projected into a Point, where; a Ray perpendicular to the Plane of the Projection passing through the Eye, and that Point, till it meet with the Plane of the Projection.

PROP. II.

A Right-line, perpendicular to the Plane of the Projection, is projected into a Point, where that Right-line cuts the Plane of the Projection.

DEMONSTRATION

THIS Proposition will appear plain from *Defin. I.* because a Line that is perpendicular to any Place, is but a Point in respect to that Plane.

PROP. III.

A right Line, as AB parallel (or as CD oblique) to the Plane ELZ, is projected into a right Line, as EZ and RS, and are always comprehended between their extream Perpendiculars, AE, BZ, and CR, DS.

[Pl. I. Fig. 2. Place the Moving Plane perpendicular to the other.]

DEMONSTRATION

THE Lines AE BZ being supposed equal, and perpendicular to the Line EZ, the Line AB is parallel also to EZ, per 6 Euclid II. and it is plain, that the Eye seeing the Points

Points A and B, by perpendicular Rays, the Rays will be A E and B Z, which are, by Supposition, perpendicular, and will, of Consequence, inclose the Space E Z, which must therefore be a right Line, because parallel to A B.

Again, Supposing the Eye to see the Line C D (obliquely posited to the Plane E L Z) by the Rays C R and D S, 'tis likewise evident, that the Distance any where between the Rays, as R S will be a right Line on the Plane of the Projection.

C O R O L. I.

A right Line that is parallel to any Plane, is projected into another right Line that is equal to it, as being between the same Perpendiculars; and on the contrary, a right Line oblique to the Plain, is projected into a Line shorter than its self.

For, draw N D parallel to the Plane E L Z, then is $N D = R S$ by the first Part of this Cor. But the Triangle N C D being a right-angled Triangle, because C R is perpendicular to E Z, by Supposition, and therefore the Hypothenuse C D must be longer than N D or R S, its equal. *Per Prop. XIV. Sect. II. Part I.*

C O R O L. II.

The Plane Superfices A B Z E, when it is placed perpendicular to the Plane E L Z, is projected into the Line E Z; for the Eye placed perpendicular to the Plane of the Projection, must be perpendicular to the Edge of the Plane A B Z E, and therefore no other Part than the Line A B can be seen by the Eye; *Ergo*, no

other Part is to be projected, which before has been proved to be projected into EZ .

Note, What has been here said, may be likewise understood of a Circular Superfices, for it is very easy to conceive, that the Circle EnZ will be projected into the Diameter EZ .

C O R O L. III.

By this we may project any Part of a Circle, as the Arch En , for if we let fall a Perpendicular, from the End of the Arch to the Plane ELZ , 'tis plain, from what has been already said, that Em will be the Representation of the Arch En , which is the versed Sine of the same Arch.

P R O P. IV.

A Circle, parallel to the Plane of the Projection, is projected into a Circle equal to its self.

THIS Proposition will appear evident from a due Consideration of the last, and its Corollary.

P R O P. V.

A Circle, placed oblique to the Plane of the Projection, is projected into an Ellipsis.

N. B. *In order to the Demonstration of this, and some of the following, I shall premise these few following Lemmas, which are Mr. Ward's Properties of the Ellipsis.*

IF a Cone is cut by a Plane oblique to its Bases, that Section will be a Plane, which is called an Ellipsis; in which are two noted Lines, and in one of them two noted Points; the Lines are called Diameters, Transverse, and Conjugate; the longest T S (*Fig. 38.*) is called Transverse, and the shorter n N is called the Conjugate, both being drawn through the Middle of the Ellipsis, dividing it into two equal Parts, and at right Angles one to another.

The two Points are called *Nodes*, *Focus's*, or burning Points, and are equally distant from the Middle of the Conjugate Diameter both ways, and lie in the Transverse Diameter.

All right Lines drawn within the Ellipsis, and parallel to one another, and can be divided into two equal Parts by a Diameter, are called *Ordinates*, with respect to the Diameter that divides them, and if they are parallel to the Conjugate, they are called *Ordinates rightly applied*, and those two that pass thro' the *Focus's*, are called *Latus Rectum*.

N. B. *If*

N. B. If the Transverse Diameter of an Ellipsis, as TS , be intersected or divided into any two Parts, by an Ordinate rightly applied, as at the Points C and a , then are the Parts TC and CS ; Ta and aS called Abscissas, and by the Rectangle of any two Abscissas, is meant the Rectangle of any two Parts, that being added together, will make the Transverse Diameter, as $TC + CS = TS$, and $Ta + aS = TS$, &c.

L E M M A I.

*As the Rectangle of any two Abscissas,
Is to the Square of half the Ordinate that divides
them;*

*So is the Rectangle of any other two Abscissas,
To Square of $\frac{1}{2}$ the Ordinate that divides them.*

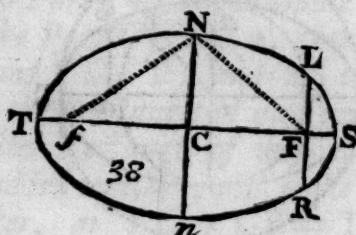
That is $Ta \times Sa : ba^2 :: TC \times SC : NC^2$.

D E M O N S T R A T I O N.

Let the Ellipsis, be circumscribed and inscribed with Circles, then from any Point, as B in circumscribed Circles Periphery, draw the right Line Ba parallel to the Semiconjugate NC , then will ba be a Semiordinate rightly applied to the Transverse Diameter TS ; from the Point b , in the Periphery of the Ellipsis, draw db parallel to the Transverse TS , and draw the Radius BC , then will $\triangle BCa$ be similar to $\triangle cfd$.

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third Term, to find a fourth Proportion, which will be LR the *Latus Rectum*.



Thus	1	$TC \times SC : NC^q :: TS : LR$
But	2	$TC = SC$ and $NC = Cn$
Therefore	3	$TC \times SC = \frac{1}{4} TS^q$
And	4	$NC^q = \frac{1}{4} Nn^q$
1 3 4	5	$\frac{1}{4} TS^q : \frac{1}{4} Nn^q :: TS : LR$
5 ∴	6	$\frac{1}{4} TS^q \times LR = \frac{1}{4} Nn^q \times TS$
6 × 4	7	$TS^q \times LR = Nn^q \times TS$
7 ÷ ST	8	$TS \times LR = Nn^q$
8 into Ana.	9	$TS : Nn :: Nn : LR$

Hence it is evident the LR thus found is an Ordinate, by which any other Ordinates, whose Distance from F or C are given, are regulated and found.

LEM.

L E M M A III.

From the Square of $\frac{1}{2}$ the Transverse, subtract the Square of $\frac{1}{2}$ the Conjugate, and extract the Square Root of their Difference, and that Root is the Distance of the Focus, from the common Center of the Ellipsis.

	1	$TS \times LR = Nn^q$ per 8th of the Last
	2	$TS : LR :: TF \times SF : LF^q$. By Lem. 2.
That is	3	$TS : LR :: TC + CF \times TC - CF : (\frac{1}{4} LR^q = LF^q)$
3 $\therefore$	4	$TS \times \frac{1}{4} LR^q = LR \times TC^q - CF^q$
4 $\div LR$	5	$TS \times \frac{1}{4} LR = TC^q - CF^q$
1 $\div \frac{1}{4}$	6	$TS \times \frac{1}{4} LR = \frac{1}{4} Nn^q = NC^q$
5 = 6	7	$NC^q = TC^q - CF^q$
7 —	8	$CF^q = TC^q - NC^q$
8 $\sqrt{\quad}$	9	$CF = \sqrt{TC^q - NC^q}$. Q. E. D.

L E M M A IV.

To describe an Ellipsis, having the Transverse and Conjugate Diameters given, viz. 12 and 20.

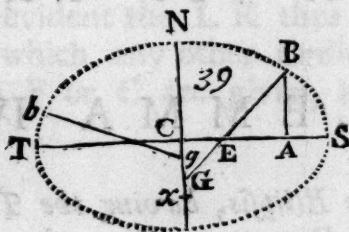
Having found the *Latus Rectum*, per Lem. II. and the Distance between the Conjugate and Focus, by the third we may then proceed to find other Ordinates; first the *Latus Rectum* LR is found = 7.2 and $CF = 8$.

Then, if it were to find any Semiordinate between C and F , 'tis only assigning the Distance, and then by the first Lemma, as $TC \times CS : NC^q ::$ so
is

is the Distance from T to the Part between C and F, multiplied by the remaining Part of the Transverse, to the Square of the Semiordinate, at that Division of the Transverse. It is evident, that if many of these Ordinates be found, and the Ends of them joined by a Curve, it is from what has been said, that that Curve will be an Ellipsis. Or,

G E O M E T R I C A L L Y,

From the End of the Conjugate Diameter N, set off half the Transverse TC from N to x , then take any Point in Cx at Pleasure; suppose G, set the Distance Cx from G to E, draw GE, and prolong it to B, so that GB may be equal to TC, or $EB = NC$, and where-ever the Point G was taken between C and X, the Point B will touch (or fall in) the Periphery of the Ellipsis.



D E M O N.

DEMONSTRATION.

Draw B A perpendicular to T S, then the $\triangle G C E$ and $\triangle B A E$ are similar.

Consequently	1	$CE:AE::EG:EB$ per 18 Geo.
And	2	$CE+AE:AE::EG+EB:EB$
But	3	$CE+AE=CA$, and $EG+EB=TC$, and $EB=NC$
Therefore	4	$CA:AE::TC:NC$
4 in Squares	5	$CA^q:AE^q::TC^q:NC^q$
5 $\therefore$	6	$\frac{CA^q \times NC^q}{TC^q} = AE^q$
Per 14 Geo.	7	$NC^q - AB^q = AE^q$ That is ($EB^q - AB^q = AE^q$)
6 = 7	8	$\frac{CA^q \times NC^q}{TC^q} = NC^q - AB^q$
8 $\times TC^q$	9	$CA^q \times NC^q = NC^q \times TC^q - AB^q$ ($\times TC^q$)
9 +	10	$NC^q \times TC^q - CA^q \times NC^q =$ ($AB^q \times TC^q$)
10 Analogy	11	$TC^q:NC^q::TC^q-CA^q:AB^q$
That is	12	$TC \times CS:NC::TC+CA \times$ ($TC-CA:AB^q$)

Which is *Lem. I.* therefore the Point B must be truly found.

Hence it follows, that if a convenient Number of such Points be found with a Line drawn through them by an even Hand, will give the Periphery of the Ellipsis required.

These *Lemmas* being premised, I shall go through with the Demonstration of the Proposition, viz. that a Circle, oblique to the Plane of the Projection, will be projected into an Ellipsis.

DEMON-

[Plate I. Fig. 3. Place the Moving Plane so that V and V, R and R may come together.]

D E M O N S T R A T I O N.

If the Diameter of the Circle $TRVS$, pass through the Diameter of the Circle upon the Plain of the Projection, cross the Diameter TC , at right Angles with the Diameters LC and CR , draw NR and bV perpendicular to the Plain TLS (from the Points R and V in the Plain $TRVS$ to be projected) which therefore must be parallels, because Perpendiculars to the same Plane.

Because NR is perpendicular to the Plane of the Projection, the thin Plane $NR C$ will be so to *per* 18. *Euclid* XI. and TC is supposed perpendicular to RC and NC , therefore it is so to ba and Va , because supposed parallel to NC and RC , and because NR and bV are perpendicular to the Plane of the Projection, they are Perpendiculars to NC and ba Lines in that Plane, and of Consequence the Angle $RNC = Vab =$ a right Angle. Lastly, since Va and RC are parallel, and in the same Plane they will have the same Inclination, therefore that $\angle RCN = Vab$, and of Consequence the Triangles $NR C$, and Vab are similar; therefore, *per* 18 *Sett.* II. *Part* I. $NC : ba :: RC : Va$, which in Squares, $NC^q : ba^q :: RC^q : Va^q$, and $RC^q = TC \times CS$ being Radius's of the same Circle, and $Va^q = Ta \times Sa$ *per* 22. *Sett.* II. *Part* I. therefore it will be $NC^q : ba^q :: TC \times CS : Ta \times Sa$, which is the Property of the Ellipsis (*per* *Lem.* 1.) And suppose NC a Diameter to a Circle, and ba another Line, which cuts TS at right Angles, then $TC \times CS = NC^q$ and $Ta \times Sa = ba^q$ *per* 22. *Sett.* II. *Part*

Part I. which being multiplied together, is $TC \times CS \times ba = Ta \times Sa \times NCq$, which being turned into an Analogy $NC : ba^q :: TC \times SC : Ta \times Sa$, which then will be the Property of a Circle; so that the Periphery being limited with the Lines NC and ab , must be either an Ellipsis, or a Circle; and if a Circle, then $LC = C$, which is impossible: Or otherwise thus, The Diameter RC of a Circle being placed oblique to the Plain, must be represented by a Line shorter than itself, *per Cor. I. Prob. III.* therefore the Line NC , its Representation, is not the Diameter of a Circle, but of an Ellipsis. Ergo, $TN \ b S$ is a Semi-Ellipsis. Q. E. D.

PROP. VI.

The Length of the Semi-conjugate Diameter from the Center of the Plane of the Projection, is equal to the Cosine of the Angle of Elevation of the given oblique Circle above the Plane of the Projection, set off upon that Diameter that is perpendicular to the Transverse Diameter of the Ellipsis, which is always the Diameter of the Primitive.

N. B. *The Primitive is the Circle GRQN on the Plane of the Projection.*

DEMONSTRATION.

Supposing QAG , the oblique Circle, to be represented, it is evident RA is the Measure of the Angle RCA , which is the Angle of the Elevation of the Circle to be projected, above the Plane of the Projection, and AS being parallel to ZC , $AB = BC$; but AS is the Sine of the
B b
Arch.

Arch $R B$, $A B$ its Cosine, and SC being $= R B$, is the Cosine likewise, and the Sine AS being perpendicular to the Plain of the Projection, and let fall from the Periphery of the oblique Circle, limits the Periphery; therefore the Cosine of the Elevation, is the true Distance from the Center:

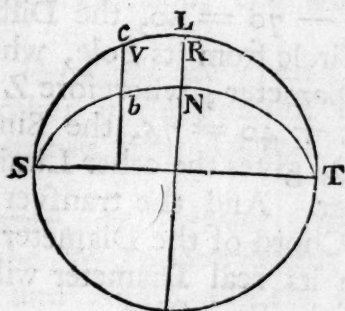
So that we have here an expeditious Way of finding the conjugate Diameter of the Ellipsis, it being only to cross transverse Diameter of the Ellipsis, at right Angles in the Middle, with a right Line, and set the Cosine of the given oblique Circle's Elevation from C to S , and it's done; and it is very plain, that the Diameter of the Circle will be the transverse Diameter of the Ellipsis, because it will be the common Section of the two Planes; and having the transverse and conjugate Diameter, the Ellipsis itself will be easily described by *Lem. IV.* and from a due Consideration of these Propositions, the two following Problems will appear to be easily deduceable from them.

P R O P. I.

To divide the Representation of a given oblique Circle into its proper Number of Degrees.

IT is evident, by consulting the Figure to *Prop. IV.* that $TN\hat{b}$ is the Representation of the Semicircle $TRVS$; therefore $N\hat{b}$ must likewise be the Representation of the Arch VR ; so that if we were to lay off the Representation of the Arch RV , upon the Ellipsis, it is only letting the Perpendicular fall upon the Plane of the Representation from the Points R and V ; or supposing the Semicircle $TRVS$, moved upon the common Section of the two Planes TS , 'till it

it fall on the Plane of the Projection, and the Arch $R V$ fall upon $L c$ (*Fig. 5.*) therefore 'tis only making a Circle about the given Representation, whose Diameter is equal to the transverse Diameter of the Ellipsis, and that Circle shall be equal to the given oblique Circle; therefore, if I would put off the Arches $S V$ and $R V$, upon the



Representation, it is only letting fall Perpendiculars from the Points R and V , or c and L , to the Diameter $S T$, which will cut the Representation in N and b ; so shall $b S$ represent $S V$, and $N b$ represent $R V$; and so of any other Arch or Number of Degrees.

P R O P. II.

To describe a lesser oblique Circle, and parallel to a great Oblique one; and at any given Distance from it, as suppose 70 Degrees, from GSQ , Fig. 4. suppose PI the Diameter of such a Circle.

LET GAQ be the great oblique Circle, whose Elevation is 55° , and Representation GSQ ; $AP = 70^\circ$, the Distance of the lesser oblique Circle from the great one: Then draw PI parallel to AC , so shall PI be the Diameter of the lesser oblique Circle; and, because oblique, will be projected.

jected into a Line less than itself, and will be the conjugate Diameter of the Ellipsis. In order to project which, $AZ = 35^\circ$, consequently $ZP = 70 - 35 = 35$; wherefore the Sine of 35 set from C, the Center of the Primitive, to L, will give the Representation of P, one Limit of the conjugate Diameter, and the Representation of I may be thus found, AP being $= 70^\circ$, PI must be 40; for $90 - 70 = 20$, the Distance of the lesser oblique Circle from its Pole, which doubled, is 40° for its Diameter; wherefore $ZP + PI = ZI$, that is $35 + 40 = 75$, the Sine of which, set from C to O, gives the other Limit of the conjugate Diameter. And the transverse Diameter is equal to the Chord of the Diameter of the lesser Circle, because its real Diameter will be parallel to the Plane of the Projection.

C O R O L.

By what has been said, it is plain, that the Radius of a Circle, parallel to the Primitive, is equal to the Cosine of its Distance from the Plane of the Projection.

P R O P. III.

To project the Sphere Orthographically upon the Plane of any great Circle.

BEFORE the Sphere can be projected, it is necessary to understand the following Astronomical Definitions.

I. The *Axis* is an imaginary Line, about which the Heavens seem to revolve, the Ends of which are called *Poles*; the uppermost of which is called the
North

North Pole, and the undermost is called the *South Pole*.

II. The *Zenith* and *Nadir*, are two Points in the Heavens diametrically opposite one to the other; the *Zenith* is that Point in the Heavens that is directly over our Heads, and the *Nadir* is that Point directly under us, and are both equally distant from the Horizon, viz. 90 Degrees.

III. The *Meridians* are great Circles intersecting one another in the Poles of the World, and cut the Equator at right Angles.

IV. *Azimuth*, or *Vertical* Circles, are great Circles of the Sphere intersecting one another in the Zenith and Nadir, and cut the Horizon at right Angles; and that Azimuth Circle that passeth thro' the East and West Points of the Horizon, is called the *Prime Vertical*.

V. The *Horizon* is a great Circle that cuts all the vertical Circles at right Angles, and divides the Sphere into two equal Parts called *Hemispheres*, the uppermost, or that which comes within the Bounds of our Eye, is called the *Northern Hemisphere*, and the lowermost, or that which is hid from us, is called the *Southern Hemisphere*.

VI. The *Equator* is a great Circle of the Sphere, whose Plane is perpendicular to the *Axis* of the World, and its Periphery is every where equally distant from the Poles of the World, viz. 90 Degrees; and upon this Circle Time is measured, and every 15 Degrees of it make an Hour.

VI. The *Ecliptick* is a great Circle cutting the Equator into two equal Parts, at two opposite
B b 3
Points,

Points, which are called the Points of *Aries* and *Libra*; and the Angle at the Interfection of these two Circles, is $23^{\circ} 29'$: According to the Observations of the late Reverend Mr. *Flamsteed*, the *Ecliptick* is divided into 12 equal Parts called Sines, each of which contains 30 Degrees, and to distinguish them one from another, it has been thought necessary to distinguish them by the Marks and Names, viz.

<i>Aries</i>	♈	<i>Libra</i>	♎
<i>Taurus</i>	♉	<i>Scorpio</i>	♏
<i>Gemini</i>	♊	<i>Sagittary</i>	♐
<i>Cancer</i>	♋	<i>Capricorn</i>	♑
<i>Leo</i>	♌	<i>Aquarius</i>	♒
<i>Virgo</i>	♍	<i>Pisces</i>	♓

The six last of which, are called Northern, and the six last Southern, Signs; and where *Aries* and *Libra* are placed, the Equinoctial and Ecliptick cut each other.

VIII. The *Colures* are two Meridians, dividing the Equinoctial into four equal Parts, one of which Meridians passing thro' the Points of ♈ and ♎, is called the *Equinoctial Colure*, the other passeth by ♊ and ♋, and is called the *Solstitial Colure*.

IX. The *Tropicks* are two small Circles, parallel to the Equinoctial, and distant from it $23^{\circ} 29'$, limiting the Sun's greatest Declination; and that which is on the North Side of the Equator is called the *Tropick of Cancer*, and that on the South Side is called the *Tropick of Capricorn*.

X. *Almicantbers*, or *Parallels of Altitude*, are small Circles parallel to the Horizon, growing less and less, 'till they terminate in the Zenith.

XI. *Parallels of Latitude*, are Circles that are parallel to the Equator, and grow less and less, 'till they terminate in the Poles.

XII. *Latitude of a Place*, is the Elevation of the Pole above the Horizon of the Place; or it is the Distance of the Zenith from the Equator.

These Definitions being known, it will be easy to project any (or all) of these Circles, upon the Plane of any one of them, as will appear from the following Examples.

EXAMPLE I.

To project the Sphere upon the Plane of the solstitial Colure, for the Latitude of $50^{\circ} 54'$.

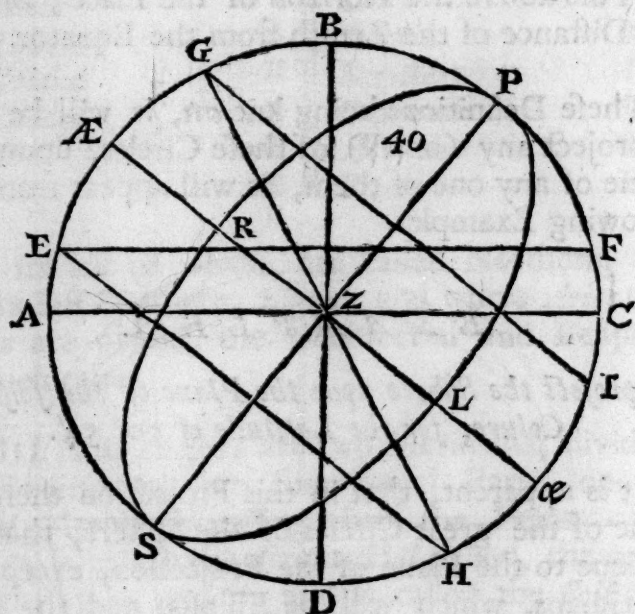
It is apparent, that in this Projection there are none of the great Circles of the Sphere, that are oblique to the Plane of the Projection, except the Meridians, which will be Ellipsis, and the other Circles, right Lines, by *Cor. II. Prop. III.*

Suppose A B C D to represent the solstitial Colure, and to be the Plane of the Projection, then any Diameter, as A C, may represent the Horizon; $50^{\circ} 48'$ set from C to P, gives the North Pole of the World, and from A to S, gives the South; and 90° set from P to $\mathcal{A}$ and $\mathfrak{a}$, which limits the Equinoctial, by *Def. VI.* of this *Prob.* so that $\mathcal{A} z \mathfrak{a}$, will be the Representation of the Equinoctial, by *Def. V.* Then $23^{\circ} 29'$ set each Way from $\mathcal{A}$ and $\mathfrak{a}$, to G E and I H,

B b 4

will

will be the Limits of the two Tropicks, and the right Lines G I and E H, will be the Representation of them, and the Line G H, the Representation of the Ecliptick, by *Def. VII.* and E F may represent any Almicanther or Parallel of Altitude; so that now we have finished the Projection, unless it be putting in the Representation of a Meridian, at any given Elevation, above the Plane of the Projection, which may be thus done.



Suppose the Elevation of the given oblique Circle to be 60.

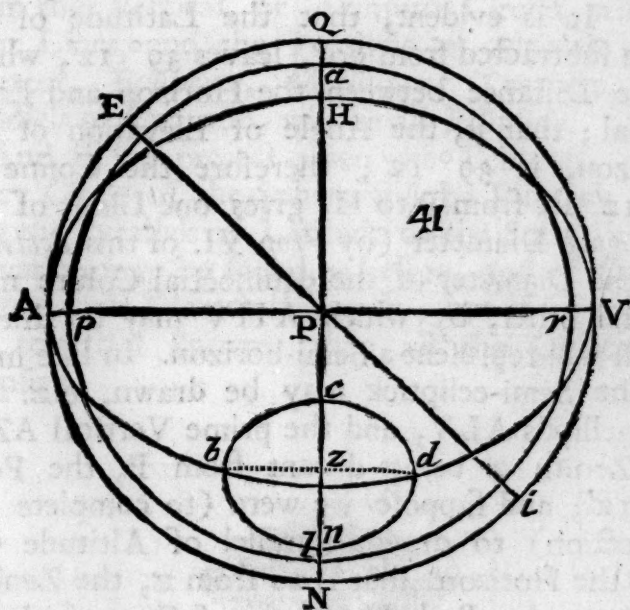
By *Def. III.* of this *Prob.* every Meridian passeth thro' the Poles, therefore the Axis *SP* must be the Diameter of every Meridian, and, of consequence, one Diameter to the Ellipsis, which being crossed at right Angles, by the Equinoctial, the conjugate Diameter must lie some-where in it, the Limits of which is the Cosine of the Complement

ment of Elevation, viz. the Cosine of 30, set off each Way from the Center of the Primitive, to R and L, so that now we have the transverse and conjugate Diameter of an Ellipsis given, to draw the Ellipsis itself, which may be done by *Lem. IV. Prop. V.*

EXAMPLE II.

So project upon the Plane of the Equinoctial for the same Latitude as before.

UPON a due Consideration of the Circles of the Sphere, and the Position of the Plane of the Projection to them, we shall find all the



Meridians to be right Lines, because they must pass thro' the Poles, which, in this Case, will be projected into the Center of the Primitive.

By

By *Prop. IV.* of this *Section*, all Parallels of Latitude are projected into Circles; and by *Prop. V.* all Circles not perpendicular, nor parallel to the Plane of the Projection, that is, to the Equinoctial, are projected into Ellipsis's, such as the Horizon, Ecliptick, and Parallels of Altitude and Azimuths.

Let AEQVIN be the Equinoctial upon whose Plane the Projection is made, draw any two Diameters at right Angles, on to the other, which will be the Representation of the two Colures, NPQ is the solstitial Colure, and APV the equinoctial Colure. Then, by the *Corol.* to *Prob. II.* of this *Section*, draw *arlp*, and distant from the Primitive $23^{\circ} 29'$, and that Circle will represent the Tropick of *Cancer*, and of *Capricorn* also. It is evident, that the Latitude of the Place subtracted from 90° , leaves $39^{\circ} 12'$, which is the Distance between the Horizon and Equinoctial; that is, the Angle of Elevation of the Horizon, is $39^{\circ} 12'$; therefore the Cosine of $39^{\circ} 12'$ set from P to H, gives one Limit of the conjugate Diameter (by *Prop. VI.* of this *Section*) and the Diameter of the equinoctial Colure must be the other, by which AHV may be drawn, which will represent a Semi-horizon. In like manner the Semi-ecliptick may be drawn, viz. the Semi-ellipsis ALV, and the prime Vertical AZV, the Zenith z being distant from P, the Pole, $39^{\circ} 12'$; and suppose we were (to complete the Projection) to draw a Parallel of Altitude 70° from the Horizon, that is 20 from z, the Zenith, we may, by *Prob. II.* of this *Section*, find the Points n and c, each 20° from the Zenith; and by *Prob. I.* we may measure 20° each Way from z, upon the prime Vertical from b to d, which will give two Diameters to an Ellipsis, which being finished,

finished, will be the Parallel required, and 70° Distance from the Horizon.

N. B. I have here drawn only the Semi prime Vertical, Semi-horizon, and Semi-ecliptick, because the Scheme should not be too much confus'd with Lines; it being my Design only to shew how any Circle may be projected, rather than to project all the Circles in the Sphere; which, had it been done, the Scheme would have been so perplex'd, that it would have been very difficult to distinguish one Line from another, and so of consequence it would have been useless.

If we project upon the Plane of the Horizon, then the Vertical, or Azimuth Circles, will be right Lines, and the Parallels of Altitude; the Equator, Ecliptick, Meridians, Tropicks, and Parallels of Latitude, will be all Ellipsis's.

And if we project upon the Ecliptick, then every Circle of the Sphere will be Ellipsis's (except the Parallels of Latitude of the Stars) whose Elevation may be found as before, and of Consequence their Diameters known; but if we project the terrestrial Sphere, then all the Circles are Ellipsis's.





THE
ELEMENTS
OF
SPHERICAL TRIGONOMETRY.

SECT. II.

*Of the Sterographical Projection of the
SPHERE.*



IN the *Sterographical Projection of the Sphere*, the Eye is supposed to be upon the Surface of the Sphere, and to see the Sphere, with all the Circles thereon, by a Cone of Rays passing directly from the Eye, to the Periphery of each particular Circle; that is the Eye, is the Vertex of the Cone of Rays, and the Circle seen, is the Base, from whence will be easy to conceive, that there will be two Sorts of Cones (*viz.* Right and Oblique) to be considered, and are so made by the Position of the Circles, which is the Cone's Base.

DEFI-

DEFINITIONS.

I. The Pole of any great Circle, is a Point upon the Surface of the Sphere, from whence all right Lines drawn to the Periphery of that Circle, are equal: Or the Pole of a Circle is a Point that is every-where 90° distant from it.

II. The Plane of the Projection, is the Plane of a great Circle infinitely produced every Way, and perpendicularly opposed to the Eye placed on the Surface of the Sphere.

[Fig. 5. Place the two moving Planes so that A and A may become one Point.]

III. That great Circle whose Plane is extended every Way, and in which the Projection is made, is called the Primitive Circle. Let the Eye be placed at A on the Surface of the Sphere, from which, thro' B, the Center of the Sphere, draw the Radius AB, to which suppose a Plane to be infinitely extended; and at right Angles to it, cutting the Sphere in the Circle OESC, an infinite Plane so drawn, is the Plane of the Projection, and the Circle OESC is called the primitive Circle.

Hence it is evident, that A is the Pole of the primitive Circle, it being equally distant from it every Way: And the Arches AS, AE, AO, AC, are all Quadrants, and AB the Radius to them: And, secondly, that all the great Circles which pass thro' the Eye at A, as EAC, and OAS, are at right Angles to the Primitive; for all of them pass thro' the Radius AB, which is supposed at right Angles to it, and consequently, per 18. Eucl. 11. they will be so to.

IV. A

IV. A Point is there projected into a Point in the Plane of the Projection, where a Ray passing from the Eye thro' that Point, meets the Plane of the Projection.

V. A Line, whether strait or crooked, is there represented in the Plane of the Projection, where Rays passing from the Eye thro' all the Points of that Line to be projected, meet the Plane of the Projection.

From hence it is evident, first, That if a strait Line passeth thro' the Eye, its Representation will be a Point in the Plane of the Projection; for one and the same Ray, will pass thro' all the Points of the same Lines, C S E O (*Fig. 5.*) being the primitive Circle, and the Eye at A, the Representation of the Line A B, passing thro' it, will be the Point B, *viz.* that Point where it cuts the Primitive, or in case it falls without the Primitive, where it cuts the Plane of the Projection.

And, secondly, That the Projection of a Circle passing thro' the Eye, will be an infinite right Line; for in the Projection of the Semicircle, that is opposed to the Eye, and is the other Half of E A C, will be projected into E C, and the Projection of the Semi-circle E A C, in which the Eye is placed, will fall in the Line E C, produced infinitely both Ways from E and C.

VI. A Circle passing thro' the Eye, is called a right Circle.

VII. A Circle not right to the Eye, but perpendicular to the Eye, or parallel to the Plane, of the Projection, is called a direct Circle.

VIII. A

VIII. A Circle neither at right Angles, nor parallel to the Plane of the Projection, but inclined to it, is called an oblique Circle.

In order that the following Propositions may be rendered plain and easy, I shall premise the four following *Lemmas*, concerning the *Sphere* and *Cone*.

LEMMA I.

Let ABCDL represent a Sphere; let ABCD be a Circle in that Sphere: If a Line be drawn from r, the Center of that Sphere, perpendicular to the Plane of the Circle, that Perpendicular will, if produced, fall on the Poles of that Circle.

[Fig. 7. Place the great moving Planes so that L and L may come together.]

DEMONSTRATION.

Draw the Diameters BD and AC, and then draw the Lines LA, LB, LC, and LD, the Triangles DLR, CLR, ARL, and BRL, have the Sides AR, BR, RC, and DR, equal, being the Radius's of the same Circle; and the Side LR is common to all the Triangles; and LR being supposed perpendicular to the Plane, the Angles LRA and LRB, &c. must be right Angles; therefore the four Triangles are equal in all Respects, and the Lines AL, BL, CL, &c. must be equal; then the Pole A, where they meet, must be the Pole of the Circle (*per Def. I.*) and the Line LR passing thro' that Point, must needs pass thro' the Pole. The like may be done for the under Pole.

LEMMA

I

L E M M A II.

If DLB, a great Circle of the Sphere, cut any other Circle at right Angles, it bisects that Circle, and passeth thro' its Poles.

[Fig. 7. Place the Planes as before.]

D E M O N S T R A T I O N.

From the Center r , of the great Circle DLB which is likewise the Center of the Sphere, let fall a Perpendicular to the Circle ABCD, and that Perpendicular, which is part of the Diameter of the great Circle DLB, must needs pass thro' the Center r , of the Circle; for supposing the Circle placed perpendicular any where else, supposing on mn , then the Circle DLB will be out of the Sphere, and therefore cannot be said to be perpendicular to any Part of it, because it does not touch it all, therefore no other Line, but the Diameter of the Circle LRD, can be the common Section of the two Planes, which divides it in the Middle. And by *Lemma I.* the Line LR being supposed perpendicular to ABCD, must needs pass thro' the Poles of it.

LEMMA

L E M M A III.

If a right Cone be cut parallel to its Base by a Plane, that Section will be a Circle.

N. B. A right Cone is when all the Rays from the Periphery of its Base, to the Vertex, are equal.

[Fig. 7. Place the great moving Plane so that L and L may become one Point, and the small moving Plane be placed parallel to ABCD.]

D E M O N S T R A T I O N.

Suppose the Cone LABCD to be cut down its Axis LR, that Section will be the Triangle ALC, whose Base AC is the Diameter of the Circle, that is the Cone's Base. Again, suppose the Cone to be cut by a Plane, as $abcd$, parallel to its Base: I say, that the Figure $abcd$ is a Circle; for, take any Triangles, as ALR, the Line ar will be parallel to AB, because it is in a Plane which is supposed parallel to the Plane ABCD, and of consequence, to any Line in that Plane, and the Axis LR is perpendicular to that Plane; therefore Lr , apart of that Line, is perpendicular to $abcd$, and of consequence, the Angles LRA and Lra , are right Angles, and the Angle RLA is common to both, therefore they are similar: And, per 18. Sect. II. Part I. as $LR : RA :: Lr : ra$, in like Manner we may prove, that $LR : RB :: Lr : rb$; and as $LR : RA :: Lr : ra$, &c. and the Sides RA, RB, RC, RD, being equal, therefore each, or any of them, may be made the second Term to all, or any of the foregoing Proportions, and, of consequence, the Lines ra, rb, rc, rd ,

C c

are .

are all equal, because they may be made the fourth Term, to one and the same second Term, and these Lines being equal, must be the Radii of one and the same Figure, which can be no other but a Circle.

LEMMA IV.

The Section of a scalenous Cone cut sub-contrary to its Base, will be a Circle.

N.B. *A scalenous Cone hath a Circle for its Base, but the Rays or Lines from its Vertex, to the Periphery of its Base, are unequal.*

[Fig. 6. Place the triangular Plane perpendicular to the Circle, and the other two Planes perpendicular to the Triangular.

Let GRFZ be the Base of a Cone, and A its Vertex, and GA, and AF, Lines drawn from the Vertex to the Periphery of its Base; and if that Cone be cut thro' its Axis, the Section is the Triangle AGF, in which draw IH, in such a manner, that the Angle AIH may be equal to the Angle AFG; then will the Triangles AIH, and AFG, be similar, and are said to be subcontrarily posited. Imagine a Plane to cut the Cone thro' the Line IH that is perpendicular to the Plane of the Triangle AGF: I say, that the Section IBH will be a Circle.

DEMONSTRATION.

Take any Point, as P, any where in the Plane IBH, and draw PB perpendicular to IH, then thro' P draw DK parallel to the Diameter GF, and imagine a Plane to cut the Cone thro' DPK, perpendicular to AGF, and parallel to GZFR, that

that Section DBK will be a Circle, and DK its Diameter, *Lem. III.* the Plane DBK being perpendicular to the Triangle AGF, as well as the Plane IBH, and one Plane cutting the other in the Point P, PB is the common Section of the two Planes, and perpendicular to both the Lines DK and IH. And $DP \times PK = BP^2$ *per 22. of Sect. II. Part I.* and the Triangles IBD and KPH, are similar (for the $\angle IPD = \angle KPH$, *per 2. Sect. II.* and the $\angle AIH = \angle AKD = AFG$, by the sub-contrary Position of the Line IH, consequently their Complements DIP and PKH, are equal) therefore $DP : PI :: PH : PK$, *per 18. Sect. II.* Ergo $DP \times PK = PI \times PH = PB^2$; therefore the Point P must be in the Periphery of a Circle, whose Diameter is IH, because the Parts of the Diameter IH multiply'd together, is $= PB^2$, which is the Property of a Circle, consequently IBH is a Circle.

P R O P. I.

A right Circle is projected upon the Plane of the Projection, into a Line of $\frac{1}{2}$ Tangents.

D E M O N S T R A T I O N

IT is evident that the Semi-circle passing thro' the Eye, will be projected into the Diameter AC (*per. Def. V.*) which is the Diameter of the given Circle, and of the Primitive also, which cross with another Diameter; and at right Angles to it, as BD, draw BS any where to the Periphery of the given right Circle: I say, that En is the Semi-tangent of the Arch DS, being its Representation; for En is the Tangent of the Arch Ea, and the Radius BE is equal to the Radius ES, and the Angle DES is double the

C c 2

Angle

P R O P. III.

The Representation of a Circle of the Sphere placed oblique to the Eye, will be a Circle in the Plane of the Projection.

[Fig. 6. Place the Plane AGF perpendicular to FGZ and IBH at right Angles to AGF, then will IBH be the Plane of the Projection.]

D E M O N S T R A T I O N.

TAKE any Circle inclined to the Plane of the Projection, whether it be a greater or a lesser, for the same Demonstration will serve for all: Let the Cone of Rays, under which the Eye sees the Circle GZF cutting the Plane of the Projection in IBH, which will be the Representation of the given oblique Circle. It is evident, that if AGZFA be the Cone of Rays by which the Eye sees the Circle FZG, and that the Plane IBH be parallel to the given Circle FZG, the Base of the Cone of these Rays, then IBH, its Representation, is a Circle, *per Lem. III.* but if the Plane of the Projection be direct to the Eye, as it always is, and of consequence, oblique to the Base, then that Plane cuts the Cone of Rays, subcontrary to its Base, and the Section IBH, the Representation of the given oblique Circle, is a Circle, *per Lem. IV. Q. E. D.*

Before I proceed any farther, it will be necessary to lay down another Definition, which it was not proper to do before.

Def. IX. The Line IH prolong'd which is the common Section of the Plane of a great Circle passing thro' the Eye at A , and of the Plane of the Projection, is called the *Line of Measures*; if that Plane which passeth thro' the Eye at A , is perpendicular to the given oblique Circle. Or (*Fig. 11.*) the Line of Measures is that infinite right Line Bf , which is perpendicular to the common Section of the Plane of the Projection, and Plane of the Circle to be projected.

From hence it is evident, that one Line of Measures will not serve all Circles, but only such as are perpendicular to it; that is, the Line of Measures is that right Line which passeth thro' the Center of the Representation of an oblique Circle, and is perpendicular to one Diameter of it; so that it is plain there may be many Lines of Measures in one and the same Projection, where many Circles are concerned, as will plainly appear in the following Propositions.

N. B. The Eye is always supposed to be in one of the Poles of the primitive Circle.

PROP.

P R O P. IV.

If a great Circle is to be projected upon the Plane of another great Circle, its Center shall be in the Line of Measures, and distant from the Center of the Primitive, the Tangent of the Elevation of the given Circle, above the Plane of the Projection.

[Fig. 11. Place the moving Planes so that E on one, may coincide with E on the other.]

LET the given Circle to be projected be AECF, oblique to ABCD, the Plane of the Projection, then is AC the common Section of the two Planes, and Bf the Line of Measures, per last Definition: And it is likewise plain, that the Eye at P, the under Pole of the Primitive, projects the Point E, one End of the Diameter of the oblique Circle to be projected in *e*, and it projects the other Point F of the Diameter of the oblique Circle into *f*, both in the Line of Measures, which being bisected in N, gives the Center of the Circle *eCfA*, the Representation of the Circle AECF; and, therefore, what is to be prov'd is, that ON is the Tangent of the Arch BE, the Elevation of the oblique Circle AECF above the Plane of the Projection. Make the Arch *Be* = BE, draw *eO*, *eC*, *em*, *Cf*, *CN*, and the Arch *O m*.

D E M O N S T R A T I O N.

The Angle $eCA = \frac{1}{2} eOA$, per 15. Sect. II.
 and Angle $eCA = CfO$, per 20. Sect. II. the
 Angle $ONC = NfC + NCf$, per 4. Sect. II.
 = eOA , consequently the Angle OCH, which
 C c 4 Is

is the Complement of the Angle ONH , is equal to the Angle eOB , which is the Complement of eOA , which is the Elevation of the given oblique Circle above the Plane of the Projection; but ON is the Tangent of the Angle OCN , *Ergo* ON is the Tangent of the Elevation. *Q. E. D.*

P R O P. V.

If a lesser Circle, whose Poles lie in the Plane of the Projection, was to be projected, the Center of its Representation shall be in the Line of Measures, distant from the Center of the Primitive, by the Secant of that lesser Circle's Distance from its Pole, and its Radius shall be equal to the Tangent of that Distance.

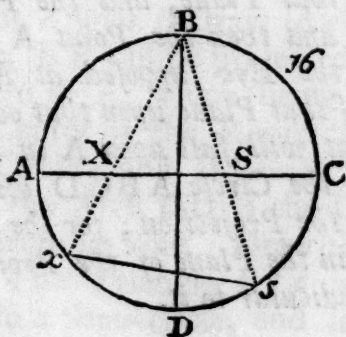
D E M O N S T R A T I O N.

SUPPOSE BD the Line of Measures, and EF the Diameter of the lesser Circle (EeF to be projected) and common Section of the two Planes; the Poles of the lesser Circle EeF , will be B and D , by *Lem. II.* the Circle $ABCD$ cutting it at right Angles. Draw AeE and AfF , the Diameter, EF will be projected into ef , supposing the Eye at A , which being bisected, will give O the Center of the Representation of the given lesser Circle; and D being the Pole of the lesser Circle, the Arch ED is the Distance of the lesser Circle from its Pole. Draw the Lines QE , EO , Ef . The Triangle Efe is right-angled at E , being in a Semi-circle, and is divided into two similar ones, *per 19. Sect. II.* therefore $\angle eEN = \angle EfN$, which is equal to $\angle fEO$, *per 4. Sect. II.* because $EO = Of$: But the $\angle eEN$ is equal to $\angle EAQ = \angle QEA$, because they stand

PROP. VI.

All lesser Circles, whose Poles lie not in the Plane of the Projection, are projected by laying off from the Center of the Primitive, both Ways, upon the Line of Measures, the Tangent of half the Distance of each Extremity of the Diameter of the given lesser Circle, from that Pole of the Primitive which is opposite to the Eye, and the Middle of these two projected Extremities, will be the Center of the Representation of the given lesser Circle.

IN order to the right understanding that Proposition, let us imagine a Plane erected perpendicular to $ABCD$, which will (supposing the Eye at B) be the Plane of the Projection, and the Line sx , the Diameter of the lesser Circle, to be projected, it is evident, that the Pole of this Circle cannot lie in the Plane of the Projection, and that D is the opposite Pole of the Primitive.



DEMONSTRATION.

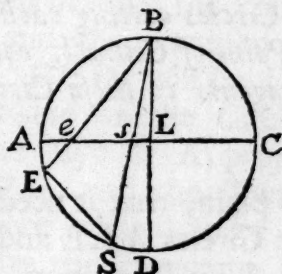
It is evident, that XS is the Representation $\propto s$, and dS of Ds , and dX of $\propto D$; but Xd is the half Tangent of $\propto D$, and dS the half Tangent of Ds , per Prop. I. of this Sect. Q.E.D.

PROP. VII.

If a lesser Circle to be projected, fall intirely on one Side of the opposite Pole of the Primitive, and do not encompass it as in the last Proposition, then will its Diameter be equal to the Difference of the half Tangent of its greatest and nearest Distance from the Pole of the Primitive, set off from the Center of the Primitive, one and the same Way, in the Line of Measures.

DEMONSTRATION.

LET us suppose the Plane of the Projection and Primitive to be a Circle perpendicular to the Plane of the Circle $ABCD$, and on the Diameter ALC , then is B the Plane passing thro' the Eye,



and D the opposite Pole. Let ES be the Diameter of a lesser oblique Circle. It is evident, that the Eye at B sees the Point S by the Ray BS , and con-

consequently s is the Projection of S into the Line of Measures ; and in like manner, e is the Representation of E , into the same Line of Measures : But Ls is the half Tangent DS , the nearest Distance of the lesser oblique Circle from the remotest Pole of the Primitive (*per Prop. I. of this Sect.*) and Le is the half Tangent of DE , the greatest Distance of the lesser oblique Circle, from the Pole of the Primitive, by the same ; and it is likewise evident, that ES is the Difference of the lesser Circle's greatest and nearest Distance from D , *Ergo*, es , the Difference of the half Tangents of the lesser Circle's greatest and least Distance from its remotest Pole. $\mathcal{Q}. E. D.$

P R O P. VIII.

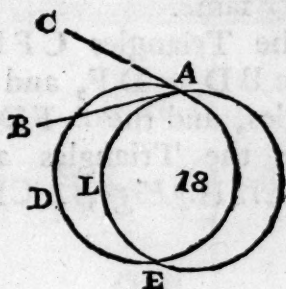
In the Sterographick Projection, the Angles made by any Circles on the Surface of the Sphere, are equal to the Angles made by their Representation on the Plane of the Projection.

IN order to demonstrate this Proposition, I premise the following *Lemma*, viz.

That any two Circles cutting each other, do form an Angle at the Point of Contact, equal to the Angle formed by the Tangents to these Circles, meeting in the same Point.

And this is so plain, that it needs little Demonstration ; for the Circles ALE and ADE , meeting at the Point A by Inspection, forms the same Angle with the Tangents CA and BA , because the infinitely small Portions of the Circles ALE , ADE , do coincide with the Tangents CA and BA , consequently have the same Direction ; there-

therefore the curvilinear Angle $DAE =$ to the right-lined Angle CAB formed by the Tangents; and that the Proposition is true, I thus demonstrate.



DEMONSTRATION.

[Fig. 19. Place the two triangular Planes perpendicular to the Plane $AEBS$, and the circular Plane to lay on the Edge of them.]

Suppose the Eye at A , projecting the Angle RBS upon the Plane $EFSDC$, draw BD a Tangent to the Circle BS , and BC a Tangent to the Circle RS , to the common Point B ; then is the Angle $CBD = \angle RBS$, per Lemma to this Prop. The Plane $EFSDC$ being the Plane of the Projection, is perpendicular to the Circle BS , passing thro' the Eye, as is likewise the Plane BCD made by Supposition, and the common Section of the Planes $EFSDC$, BCD , will be likewise perpendicular to the Plane of the Circle BS , per 19. *Euc. II.* and the Eye at A projects the Tangent BD into ED , and the Tangent BC into FC ; and therefore what is to be proved is, that $\angle CBD$ made by the Tangents, is $= \angle CFD$, made by the projected Tangents.

Draw

398 *The* ELEMENTS *of, &c.*

Draw BQ parallel to ES, and join AQ; then the $\angle DBA = AQB$, *per last of Sect. II.* $= \angle ABQ$, *per 4. Sect. II.* $= \angle BED$, *per 6. Sect. II.* $= \angle DBF$, *per 4. Sect. II.* therefore $BD = DF$, *per same.*

Then, in the Triangles CFD, and CBD, there is the Side $BD = DF$, and CD is common to both Triangles, and the $\angle FDC = BDC = 90^\circ$; therefore the Triangles are similar and equal, *per 3. Sect. II. Ergo, $\angle CFD = \angle CBD$.*
Q. E. D.



THE



THE
ELEMENTS
OF
SPHERICAL TRIGONOMETRY.

SECT. III.

Of STEROGRAPHIC PROBLEMS.

THE Degrees of any oblique Circle upon the Sphere, are equal to one another; but the Representation of these Degrees upon the Plane of the Projection, are very unequal, as is evident from *Fig. II.* where *AeCf* is the Representation of an oblique Circle upon the Sphere, one Half of which is projected into *AeC*, and the other into *CfA*; but these two are very unequal to one another, nor are the Degrees in each of these equal among themselves; so that, in the first Place, I shall shew how to divide the Representation of any oblique Circle, into its proper Number of Degrees. In order to do which, I premise the following *Lemma*.

LEMMA

L E M M A.

If thro' the remotest Poles of two equal Circles of the Sphere, any other Circle be drawn, it will cut off equal Portions from the Periphery of the two equal Circles.

N. B. These Circles on the Sphere that are equal, are either great Circles (which are all equal) or lesser ones, and the same Demonstration will serve for both.

[Fig. 14. Place the moving Planes so that they may all be perpendicular to the Plane ACOL, and put the two thin Pieces through the Slits at N and B.]

Let L be the upper Pole of the great Circle CES, and A the lower Pole of the Circle OHS, and thro' these two remotest Poles of the two great Circles, let any other Circle be drawn, as AEHL: I say, it will cut off equal Portions of the Circles CES and OHS; that is, $CE = OH$. Draw the Lines AL, LC, OA, CA, OL, and where the Circle AEHL cuts the two Circles in E and H, let fall Perpendiculars to LA, and then join the Lines CB, CH, ON, HO.

D E M O N S T R A T I O N.

1st. The Arch $LOC = ACO = 90^\circ$, per Definition of a Pole, and their Chords LC, and AO, are equal, per 21. Sect. II. Then the Arch CO is common to both the Arches LOC, ACO, Ergo, the Arch CA = Arch OL, and of consequence, CA and OL, their Chords, are equal.

2dly. In the Triangles LCA, AOL, the Side $OL = CA$, and $OA = CL$, and the Side AL is

is common to both Triangles, *Ergo*, they are equal in all respects, *Ergo*, $\angle CLA = \angle OLA$.

3dly. The Arch $HEA = EHL = 90^\circ$, being the Distance of each Circle from its Pole, and the Arch HE common to both, therefore AE and HL are equal; and EB and NH being the Sines of these Arches, are therefore equal. Now in the Triangle ABE , LNH , there is proved two Sides in one, equal to two Sides in the other, and the $\angle s$, B , and N , are right Angles by Supposition; therefore the Triangles are equal in all respects, *Ergo*, $AB = NL$.

4thly. In the Triangle CAB and OLN , it has been proved, that $CA = OL$, and $NL = AB$, and the $\angle A = \angle L$, therefore the two Triangles are similar and equal, *Ergo*, $CB = ON$.

Lastly, In the Triangles CBE and ONH , it has been proved, that $CB = ON$, and $EB = HN$, and the Angles B and N are right Angles, therefore the two Triangles are equal in all respects: *Ergo*, the Chords CE and OH , are equal, and of consequence, the Arches that belong to them are equal. *Q. E. D.*

Or this *Lemma* may be demonstrated after a much shorter Manner, by the following Method.

It was proved by the preceding Demonstration, *Art. 3.* that EB and HN are equal, and, *per 20. Sect. II. Part I.* equal Chords, and, of consequence, equal Sines, subtend equal Arches in equal Circles; but the Lines EB and HN , are Sines to the Arches CE and HO : *Ergo*, the Arch $CE =$ Arch HO . *Q. E. D.*

As to lesser Circles, which are equal, the Demonstration will be much the same, because they will be equally distant from the Center of the Sphere, and consequently equally distant from their remotest Poles, for the Center of a great Circle, is the middle Point between the Poles of that Circle; wherefore all the right Lines that can be drawn from the Poles of these Circles to their Periphery, will be equal, as in *Part I.* *LC* and *LE*, are equal to *AO* and *AH*, would be still so, tho' the Circles *OHS* and *CES*, were lesser Circles, in case they were equal; and since the Angle *CLE* and *OA H*, in lesser, as well as greater Circles, are equal, the Bases *CE* and *OH* will be equal, and consequently the Arches which they subtend, would be equal also.

P R O B. I.

To find the Pole of any Circle in the Plane of the Projection.

THE Pole of a great Circle direct to the Eye, will be its Center; for if it be direct, the Eye must be in its Axis, and consequently the two Poles must be projected into the Center.

2dly. The Pole of a right Circle may be found by setting off 90° from one of its extrem Points. Suppose we were to find the Poles of the right Circle *AC*, take 90° from the Line of Chords, and set from *C* to *D*, will give *D* its Pole; for *AC* being a right Circle, *ABDC* will be perpendicular to it, consequently (*per Lem. II. Sect. II.*) will pass thro' its Poles, and the opposite Point *S*, will be the other Pole.

3dly. The

Circle, and 90 Degrees set from e on the Primitive, and a Ruler laid on C and that Point, it would have cut the Line of Measures in H , which is the above Method.

4thly. The projected Pole of a lesser Circle direct to the Eye, is the same with that of a great Circle, viz. its Center.

5thly. The Pole of a lesser Circle that is right to the Eye, is the same with the Pole of a great Circle, to which it is parallel, and consequently will be projected into the same Point. Or it may be done thus,

Suppose BO the Diameter of a lesser Circle right to the Eye at B . Erect KEI at right Angles to the Diameter BO , and those two Points, K and I , will be the Poles; then join DI , and prolong KD , and those two Points where they cut the Line of Measures AC , will be the projected Poles.

6thly. The projected Pole of a lesser oblique Circle, is the same with that of a greater oblique Circle, to which it is parallel.

P R O B. II.

To divide the Representation of a great oblique Circle into its proper Number of Degrees.

[Fig. 11. Place the moving Plane BER perpendicular to the Primitive, and the other so that E and E may come together.]

LET $ABCD$ be the Primitive, and $AfCe$ the Representation of an oblique Circle, which is to be divided into any Number of Degrees required, viz. into the Number of Degrees that is contain'd in KT in the Primitive.

By

By *Prob. I.* of this *Section*, find *H*, the Pole of the given oblique Circle, and then a Ruler laid on *H*, and the Points *K* and *T* will cut the oblique Circle in *k* and *t*; so will *kt* be the Representation of the Number of Degrees that is contained in *KT*, and so of any other Arch or Number of Degrees.

D E M O N S T R A T I O N.

The Eye at *P* sees the Circle *ABCD* at right Angles, and the Circle *AECF* obliquely, and *P* is the under Pole of one, and *S* the upper Pole of the other; therefore, by the last *Lemma*, any Circle that passeth thro' *S* and *P*, whose Diameter is *SP*, will cut off equal Portions from each Circle. If therefore all these Circles are projected upon the Plane of the Primitive, with respect to the Eye at *R*, the same thing will happen in the Projection, which happened in the Circles before they were projected; and it is evident, by *Prop. IV.* of the last *Section*, that the oblique Circle *AECF* is projected into the Circle *Aecf*, and the *CERD* is projected into the Diameter *BDf*, which is the Line of Measures infinitely extended, *per Def. V.* of this *Section*, and the Poles *S* and *P* are projected into the Point *H* in the Line of Measures, *per Def. V.* as being one right Line; therefore any Line that passeth thro' the Point *H* (which is the Representation of the Poles of the primitive and oblique Circles) and cuts the Circle *Aecf*, and *ABCD*, will cut off equal Portions from them both; that is, the Arches *AK* and *KT* are equal to the Arches *Ak* and *kt*; and so of any other Circle. Q. E. D.

C O R O L.

Hence we may easily divide the Representation of any oblique Circle into the Number of Degrees that A T represents.

Suppose it was required to find the Number of Degrees that A *t* represents; Find H, the Pole of the given oblique Circle, by *Prob. I.* and then a Ruler laid on H and *t*, will cut the Primitive in T; and the Distance A T apply'd to the Line of Chords, gives the Number of Degrees that A *t* contains.

P R O B. III.

To divide the Representation of a lesser oblique Circle into its proper Number of Degrees.

[Fig. 20. Place the moving Plane perpendicular to the other.]

LET the primitive Circle be MBND, and EO*e* the Representation of an oblique Circle, whose real Diameter is ST, with respect to the Eye at A, it is evident from what has been already said, that P, the true Pole of the oblique Circle, is projected into H in the Line of Measures BD. Set off the Distance GL (the Distance of the Center of the Primitive, from the Center of the oblique Circle) from L (the Center of the Primitive) to *a*, thro' *a*, draw *b c*, parallel to the Line of Measures; then is *b c*, the Diameter of a Circle equal to the oblique Circle ST, because they are equally remote from the Center of the Primitive; and the Center of the Circle ABCD, in which they both are, and the Circle whose Diameter is *b, a, d* is projected into QXVYZ upon the Plane of the Projection. Now if the Circle

Circle QXY be divided into Degrees, as suppose XVY , and the Lines, as XH , VH , YH , be drawn, they will cut the Representation of the oblique Circle into the same Number of Degrees; that is, $do = XY$, and $oe = VY$, &c.

DEMONSTRATION.

The Circle, whose Diameter is c, a, b , is direct to the Eye at A , and the Circle whose Diameter is S, T oblique to it, and A is the upper Pole of one, and P the under Pole of the other; and therefore any Circle passing thro' A and P , will cut off equal Portions of the Circle whose Diameter is c, a, b , and the Circle whose Diameter is S, T , by the *Lemma*; and the same in their Projection on the Plane of the Primitive; and the Circle whose Diameter is S, T , is projected into EOK , and c, a, b into $QXVZ$, and the Poles P and A are projected into the Point H , therefore H is the Representation of the two Poles, and of consequence right Lines (which are the Representation of Circles) drawn from H to any Part of QVZ , will cut off the same Parts from EOK ; *Ergo*, $do = XV$, and $oe = VY$, which was to be proved.

COROLL.

Hence we have every practical Method of dividing any lesser oblique Circle proposed. Suppose the lesser Circle be 40° distant from its Pole, we have nothing to do but to find a Circle that shall be equal to it, and parallel to the Primitive, which may be done by taking the Semi-tangent of its Distance from its remotest Pole, *viz.* 140° on the Tangent of 70 , and therewith, from the Center of the Primitive, describe a Circle upon

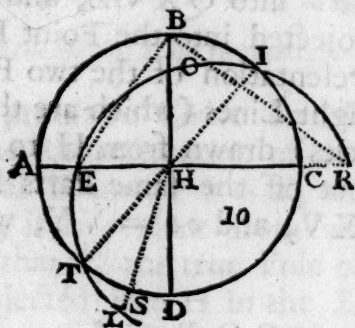
the Plane of the Primitive, and this will be the Representation of a Circle, equal to the Circle to be divided; and consequently Lines drawn from the Pole H, to any Number of Degrees in the Circle QVZ, will in like manner divide the Circle EOK.

P R O B. IV.

To draw a great Circle thro' any two Points given within the Periphery of the Primitive.

SUPPOSE we were to draw a Circle thro' the Points E and o, ABCD being the Primitive.

Draw a Diameter, as AC, thro' either of the given Points, as thro' E, and cross it with another Diameter at right Angles, as BD. Then



from the Point E, to either of the Points B or D, draw a Line, as EB, on the Point B, erect a Perpendicular to EB, and continue that Perpendicular until it intersect the Diameter AC (continued, if need be) in R, and then thro' the Points EOR draw a Circle, which will be the great Circle required.

DEMONSTRATION.

The great Circle $TEOR$ intersects the Primitive in T and I , draw a right Line, as TI ; now we have nothing else to prove, but that the Line TI passeth thro' the Center of the Primitive; for then it is a Diameter of a great Circle. Let us suppose HI a Semidiameter, and continued, it must of necessity fall upon the Point T ; for it is impossible it should do otherwise. Let it fall on S , and continue it 'till it meet the great Circle in L ; the $EH \times HR = BH^q$, *per* 22. *Sect. II. Part I.* and $LH \times HI = EH \times HR = BH^q = HI \times HT = HI \times HS$.

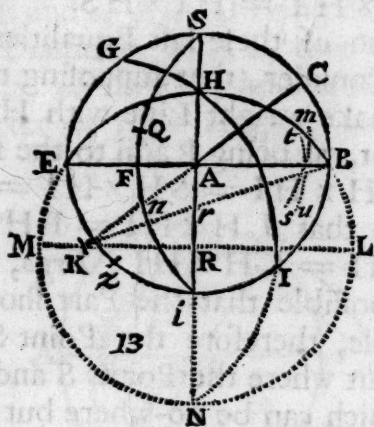
The Reason of these last Equalities will appear plain, if we consider, that supposing neither HT , or SH , to make a right Line with HI , yet they are equal to it, as being Radii to the same Circle; therefore $TH \times HI = SH \times HI = BH^q$, and it is proved, that $LH \times HI = BH^q$; and likewise $SH \times HI = LH \times HI$; *Ergo*, $LH = SH$, which is impossible that the Part should be equal to the Whole, therefore the Point S cannot fall any where but where the Points S and L must fall together, which can be no-where but at the Point T , where the great Circle meets the Periphery of the Primitive.

PROB.

P R O B. V.

To describe a spherical Angle of any Number of Degrees.

A *Spherical Angle* is the Inclination of two great Circles upon the Sphere meeting in a Point, and whose Measure is the Arch of a great Circle intercepted between the two Arches, which constitute the Angle, and at 90° Distance from the angular Point.



CASE I.

When the angular Point is in the Center of the Primitive, as at A, and to make an Angle of 37° , draw the Diameter EAB, and take 37° in your Compasses from the Line of Chords, and set it from B to C; then draw AC, and the Angle CAB is the Angle required; for the Radii AC and BC, are the Representations of part of two great Circles, and are Quadrants, and consequently the Measure of the Angle is the Distance BC upon the Primitive.

CASE

CASE II.

If the angular Point be at S in the Periphery of the Primitive, and it is required to make an Angle of 45° .

In this Case, upon due Consideration, we shall find, that it is no other than to draw the Representation of an oblique Circle, whose Elevation above the Plane of the Projection, is equal to the given Angle; so that if, from the Point S, we draw a Diameter, and perpendicular to it another EB, then is EB the Line of Measures, where the Tangent of 45° set from A, the Center of the Primitive, to B, gives B the Center of the Representation of the oblique Circle required; and the Distance SB, the Radius (*per Prop. IV. Sect. II.*) and the Circle SFi being drawn, will constitute an Angle at S equal to 45° , as was required: Or by the same *Prop.* you may take the Secant of the given Angle, and set from S to B on the Line of Measures, and that Secant is the Radius of the given Circle.

N. B. If your Angle was above 90° , proceed with the Supplement to 180 , and it is evident that the Angle FSB is that Angle required.

CASE III.

If the angular Point be any where within the Periphery, but not in the Center, as at H, and it was required to make an Angle of 35° ,

Proceed as before in drawing a Diameter thro' the given Point, as SHR, and continue it longer, if need be; cross it at right Angles with another Diameter, as EB, then thro' the Points
E, H, B

E, H, B draw a great Circle, as HLN'M, and thro' the Center of that Circle R, draw another Diameter ML perpendicular to SR; then, supposing HLN'M as a primitive Circle, and we were to make an Angle at H equal to the given one 35° , we may find its Center K in the Line of Measures, and draw GHIN, then is the Angle $\text{IHB} = \angle \text{GHE} = 35^\circ$, as was required; and the Reason of the Thing appears obvious to any one, that it is impossible that it should be otherwise, and there remains nothing to prove, but that the Circle GHIN is a great Circle in the Primitive ESB*i*, as well as in the Circle HLN'M.

D E M O N S T R A T I O N.

The great Circle, whose Representation is HLN'M, is equal to the Primitive ESB*i*, because all great Circles of the Sphere are equal; and the great Circle whose Representation is GHIN, is supposed equal to LMNH, therefore it must be equal to ESB*i*. Q. E. D.

Or we may make an Angle at H, without making it of two oblique Circles; that is, by the Intersection of a right Circle and an Oblique, and then the Construction will be exactly the same as before, only we must set the Tangent of the Complement of the given Angle, from R to K, and with the Radius KH draw GHI, then will $\angle \text{RHI}$ be equal to 55° , as was required.

PROB.

P R O B. VI.

To draw the Representation of one great Circle, perpendicular to the Representation of another great Circle.

IT is evident from *Lem. II. Sect. II.* that when any two Circles pass through one another's Poles, they are perpendicular one to another, therefore if we have a great Circle given, it is only finding its Poles and drawing a Circle thro' them, and that Circle so drawn, will be the Perpendicular required.

C A S E I.

The Representation of a Circle, perpendicular to the Primitive, is any Diameter; for the Center of the Primitive, is the Representation of both its Poles.

C A S E II.

The Representation of a Circle, perpendicular to a right Circle, may be drawn by drawing a Diameter thro' its Poles, as *BD*, is perpendicular to the right Circle *AC*, it passing thro' its Poles *B* and *D*.

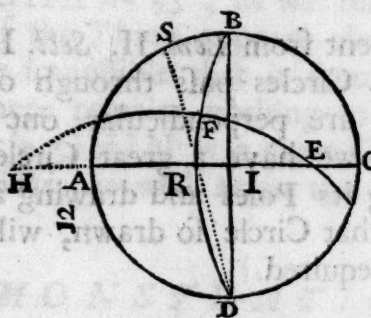
C A S E III.

If we were to draw the Representation of an oblique Circle, perpendicular to a right one, as *AC*, 'tis done by drawing any oblique Circle that shall pass thro' both its Poles *B* and *D*, and of consequence the Center of such a Circle must lie in the right Circle, continued, if need be, to which it is required to draw a Perpendicular to.

C A S E

CASE IV.

To draw the Representation of one oblique Circle, perpendicular to another, as perpendicular to BFD, and thro' the Point F. Find E and H



the Poles of the given oblique Circle (by *Prob. I.* of this *Section*) and then thro' the Points H, F, and E, draw a Circle, as the Circle HFE, which is the Perpendicular required.

P R O B. VII.

To measure the Quantity of any Part of a great Circle in the Projection.

CASE I.

THE Arch of (*See Fig. to Prob. VI.*) the Primitive, as SB, is measured by taking the Distance SB in your Compasses, and applying it to the Line of Chords, will give the Number of Degrees it contains.

CASE

CASE II.

The Arch of a right Circle RI , may be measured by taking the Distance IR in your Compasses, and applying it to the Line of half Tangents for the Radius of the Primitive, and that will shew the Number of Degrees; or it may be done by laying a Ruler on its Pole D , and the Points R and I , the Extremities of the given Arch, and that will cut the Primitive in S and B , and the Distance SB applied to the Chords, will shew the Number of Degrees that the Arch RI contains.

CASE III.

The Arch of any oblique Circle is measured by Cor. to Prob. II. of this Section.

PROB. VIII.

To measure any Spherical Angle. (See Fig. to Prob. V. of this Section.)

1. IF we were to measure the Angle CAB where the angular Point is in the Center of the Primitive, it is only applying the Distance CB to the Line of Chords, and it will give you the Measure of the Angle required to be measured.

2. If we were to measure an Angle in the Primitive, as the Angle ESF , it is done by applying FA to the Scale of half Tangents, which subtracted from 90 , leaves the Angle ESF ; or it may be done by finding the Center of SQF , as B , and the Distance AB applied to the Line of Tangents, gives the same as before.

3. To

3. To measure an Angle within the Primitive, but not in the Center, as the Angle IHB . Find the Poles of both the Circles that constitute the Angle, as r and n , and a Ruler laid on the angular Point H , and the Poles r and n , will cut the Primitive in z and i , the Distance $z i$ applied to the Line of Chords, will give the Measure of the Angle required; or it may be done by finding the Centers of the two oblique Circles R and K , and that Distance applied to the Scale of Tangents (for the Radius of that of the two Circles, which is to be esteemed the Primitive to the other) will give the Measure of the required Angle. The Reason of these Cases will appear plain from *Prob. V. of this Section.*

P R O B. IX.

To draw the Representation of one Circle, parallel to the Representation of another.

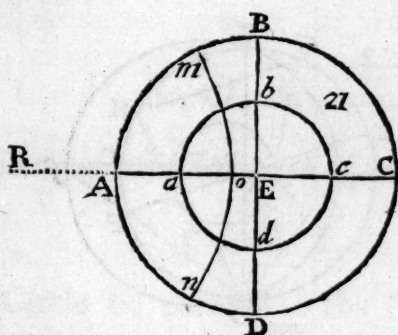
C A S E I.

TO draw a Circle parallel to the Primitive, and at any given Distance from it, as suppose 30° ; take half the Tangent of the Complement of its Distance from the Primitive, and therewith, from the Center, describe a Circle, as $a b c d$, and that is the Representation of the Circle required.

C A S E II.

To draw a Circle parallel to a right Circle, and any given Distance from it, as 30° , from the right Circle BED , and parallel to it, from B to m , and from D to n , set the Chord of the given Distance; and

and with the Tangent of the Complement of the given Distance, and setting one Foot in the Points m , or n , the other will fall on the Line of Measures prolonged in R; and then with the same Distance, and one Foot in R, draw the Circle $m o n$, which will be parallel to B D, and distant from it 30° , per Prop. V. Sect. II. Part II.



CASE III.

To draw a Circle parallel to an oblique Circle, and distant from it any Number of Degrees, as 50, from the oblique Circle B e D, it may be thus done, Find p , the Pole of the given oblique Circle, and by the Scale of half Tangents, measure the Distance between that Pole and the Center of the Primitive, as $p x$, and suppose it to measure 32° , then $32 + 40$, the Complement of the Distance, $= 72^\circ$; and the half Tangent of 72° set from x to b , is the Limits of the Circle one Way, and $42^\circ - 32 = 8$; and the half Tangent of 8 set from x to g , gives the Limits of the Circle the other Way, and the Distance $b b$ bisected in o , gives the Center o , and $o b$ the Radius of the Parallel required.

E e

But

AE; then $AB - AE - EG = 90 - 32 - 50 = 8 = GB$, and $FC = AE$, $EG = HF$; therefore $BC + FC - FH = 90 + 32 - 50 = 72 = BH$; then, by *Prop. VI.* of the last *Section*, make $gx =$ half Tangent of GB , and $xb =$ half Tangent BH , and you have the Diameter as before, but with more Certainty, and less Trouble.

PROB. X.

To measure any projected Arch of a parallel Circle.

CASE I.

IF the Circle be parallel to the Primitive, then a Ruler laid thro' the Center, and the extream Points of the Arch to be measured, will cut the Primitive in the proper Number of Degrees that was required to be known in the parallel Circle.

CASE II.

If a Circle be parallel to a right one, as *m, o, n* (See *Fig. to Prop. IX.*) any Part of that Circle is measured, or, indeed, divided into its proper Number of Degrees, by the *Corol. to Prob. III.* of this *Section*.

CASE III.

If a Circle be parallel to an oblique Circle, any Arch of it may be measured by the same *Corol.*

P R O B. XI.

To draw a lesser Circle perpendicular to a great Circle.

THIS Problem (tho' indeed it may properly be said to be Part of the VIth) may now easily be perform'd; for if any great Circle whatever, either direct, right, or oblique, were given, and it was required to draw a lesser Circle perpendicular to it, 'tis but drawing a great Circle perpendicular to the given great Circle, and then drawing a Circle parallel to that which is likewise said to be perpendicular to the given Circle too.

N.B. It is necessary to take Notice, that tho' this Circle so drawn is called a Perpendicular to the first Circle, it is not so in Fact; for no lesser Circle can be perpendicular to a great Circle; for if perpendicular, the Intersection of the two Circles should make four right Angles, as it does, but the Angles on each Side the Perpendicular are not equal, as will appear from the Sight of any Sphere or Globe, for none but great Circles of the Sphere can be at right Angles one to another.

P R O B. XII.

To make an Angle at the primitive Circle of any Number of Degrees that shall pass thro' any Point within the Primitive, as of 40° , to pass thro' the Point Q. (See the Fig. to Prop. V.)

WITH the Tangent of the given Angle, and one Foot in the Center of the Primitive, draw an Arch tu , then will the Center of an

an oblique Circle, whose Elevation is $= 40^\circ$, be somewhere in tu , then with the Secant of the same, and one Foot in the Point Q, draw nm , and where these two intersect, must be the Center of the oblique Circle SQF, to make the Angle required, *per Prop. IV. and its Corol. Sect. II.*

If the Propositions in *Sect. II.* and the Problems in the last, be understood, it will be no great Difficulty to project the Sphere upon the Plane of any great Circle, and an Example or two will make it very intelligible.

P R O B. XIII.

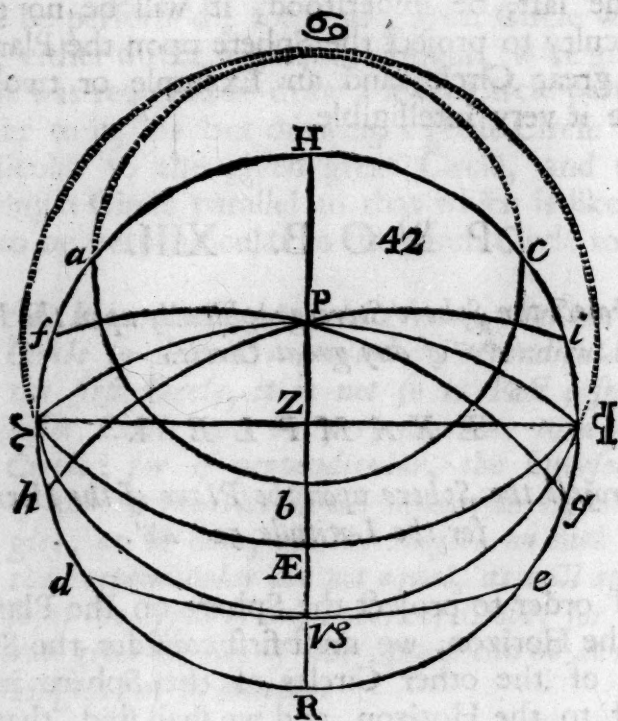
To project the Sphere Stereographically upon the Plane of any great Circle.

E X A M P L E I.

To project the Sphere upon the Plane of the Horizon, for the Latitude $50^\circ 48'$.

IN order to project the Sphere on the Plane of the Horizon, we must first consider the Situation of the other Circles of the Sphere in respect to the Horizon, and we shall find, that the solstitial Colure passing thro' the Eye in the Nadir, will be projected into right Lines, *per Def. V. Sect. II.* and the Equinoctial will be projected into an oblique Circle, whose Elevation above the Plane of the Projection, is equal to the Distance of the Equinoctial from the Horizon, *viz.* the Complement of Latitudes of the Place, and the Ecliptick will be projected into an oblique Circle, whose Elevation will be $23^\circ 29'$ less than the Equinoctial, &c.

These Things being premised, draw any Circle, as $\vee H \cong R$, which let represent the Horizon and Plane of the Projection. Draw any Diameter, as HR , which may represent the solstitial Colure, it is evident, by *Def. III. Sect. II.* that z is the Projection of the Zenith and Nadir, because they are the Poles of the Horizon.



The Elevation of the Pole of the World above the Horizon, is equal to the Latitude of the Place, may be set off upon the Diameter HR , from H to P , or its Complement $39^{\circ} 12'$, from z to P ; and the Equinoctial $\vee \text{Æ} \cong$, whose Elevation above the Horizon, is $39^{\circ} 12'$, may be drawn by *Prob. V. Sect. III.*

The

The two Tropicks abc and dve , may be drawn parallel to the Equinoctial, and distant from it $23^{\circ} 29'$, by *Prob. IX.* of this Section: Then if we draw the great Circle $\gamma P \simeq$ thro' the equinoctial Points of *Aries* and *Libra*, and P the Pole, then will that great Circle represent the equinoctial Colure, being at right Angles to HR , the solstitial Colure: And as the Ecliptick makes an Angle of $23^{\circ} 29'$, with the Equinoctial in the Points γ and $\simeq$, it will also make an Angle with the Horizon of $16^{\circ} 31'$; that is, the Representation of the Ecliptick will be elevated above the Horizon $16^{\circ} 31'$; and of consequence, if we draw such a Circle $\gamma, v, \simeq$, it will truly represent the Ecliptick.

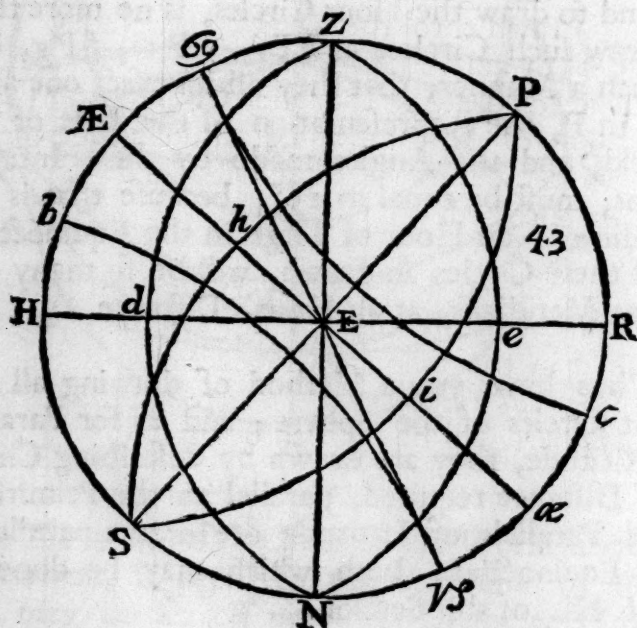
And to draw the Hour Circles, is no more than to draw such Circles, as bPi , $\gamma P \simeq$, fPg , &c. in such a Manner, that they all intersect one another in P , the Representation of the Pole of the World, and the Angles made by these Intersections, must be equal to 15° , because that is the Measure of an Hour of Time in the Equinoctial: And these Circles so drawn, will be so many different Meridians, at an Hour's Distance.

Thus have you a Method of drawing all the great Circles of the Sphere; and as for Parallels of Altitude, they are drawn by describing Circles at a Distance required, parallel to the Primitive: And Parallels of Latitude are drawn parallel to the Equinoctial; both which may be done by *Prob. IX.* of this Section.

EXAMPLE II.

To project the Circles of the Sphere upon the Plane of a Meridian for the Latitude of $50^{\circ} 48'$.

LET HZRN be any Meridian. It is evident that the Poles of the World will be somewhere in that Meridian, because all Meridians pass thro' the Poles, which Poles let be P the North, and S the South Poles, and join them with the Diameter SP, which will represent another Meridian, and will be the Axis of the World, and SbP, and SiP, are two others.



Set from the Line of Chords, the Latitude of the Place from P to R, and from S to H, and draw the Diameter RH, then is HR the Representation of the Horizon; and if we set 90° from R to

R to Z and N, then will Z and N represent the Zenith and Nadir, and R will be the North Point, and S the South Point of the Representation of the Horizon, and the East and West Points will be projected into the Center of the Primitive, because the Eye is supposed to be in one of them.

If we cross the Diameter S P at right Angles with another Diameter, as $\mathcal{A}E\mathcal{E}\mathcal{a}$, it will represent the Equinoctial being every Way 90° distant from the Poles of the World.

Again set off $23^\circ 29'$ from $\mathcal{A}$ to $\mathcal{E}$, and from $\mathcal{a}$ to $\mathcal{v}$, and draw $\mathcal{E}\mathcal{v}$, which will then represent the Ecliptick, because it makes an Angle $\mathcal{A}E\mathcal{E} = 23^\circ 29'$, with the Equinoctial.

And any great Circles, as $N\mathcal{a}Z$ and $N\mathcal{e}Z$, being drawn thro' the Zenith and Nadir, will be vertical or azimuth Circles.

And for drawing the lesser Circles, as Tropicks, Parallels of Altitude and Latitude, they may be drawn parallel to the Equinoctial and Horizon, as before directed.

I have been designedly very short in these Projections, because the Thing will be enough explain'd in the following Projections of the Cases of *Spherical Trigonometry*, where all the Circles of the Sphere will be differently apply'd.





THE
ELEMENTS
OF
SPHERICAL TRIGONOMETRY.

SECT. IV.

Of SPHERICAL TRIGONOMETRY.

I.  *S Plain Trigonometry* teacheth us to measure the Sides and Angles of a right-lined Triangle, so *Spherical Trigonometry* teacheth us to measure the Sides and Angles of Triangles, whose Sides are Parts of Circles.

2. A spherical Angle is made by the meeting of two Circles.

3. A spherical Triangle is a Portion of the Surface of the Sphere, contained between the Intersections of three Circles.

4. The

4. The Quantity of an Angle is measured by the Arch of a Circle every Way 90° distant from the angular Point, and intercepted between the two Circles, that from the Angle.

Properties of Spherical Triangles.

1. When one great Circle crosseth another great Circle, the Sum of the contiguous Angles are equal to two right Angles, because their Measure is a Semicircle.

2. When one great Circle crosseth another, the opposite Angles are equal, as in the crossing of right Lines, and may be demonstrated by the same Method.

3. The greater Angle is opposite to the greater Side, as in plain Triangles.

5. Two Triangles mutually equilateral are also equiangular.

6. If two Sides, and the included Angle of one Triangle, are equal to two Sides and the included Angle of another Triangle, the two Triangles are equal in all respects.

7. Two Sides of a spherical Triangle, are always greater than a third.

P R O P. I.

All great Circles of the Sphere cut one another into two equal Parts.

THIS has been already demonstrated in Lem. II. Sect. II.

C O R O L.

From hence it follows, that every Side of a Triangle is less than a Semicircle.

P R O P. II.

The opposite Angles at the Sections of two great Circles, are equal; that is, the $\angle ABS = \angle ACS$.

[Fig. 5. Place the moving Planes that A and A may come together.]

D E M O N S T R A T I O N.

BY Def. III. the Arch of a great Circle intercepted between A and S, is the Measure of the Angle A E S, as it is likewise the Measure of the Angle A C S, being 90° distant from each, consequently the Angles are equal, because the same Arch is the Measure of them both. Q. E. D.

P R O P.

P R O P. III.

In any spherical Triangle, as EDS, if the Sum of the Legs including any Angle, be greater than a Semicircle; that is, if $ED + ES$ be greater than the Semicircle EAC, then the internal Angle at the Base E, is greater than the outward opposite Angle DSC, and the two internal Angles DES and ESD, are greater than two right Angles.

[Fig. 5. Place the moving Planes so that A and D may come together.]

D E M O N S T R A T I O N.

DE + ES being greater than the Semicircle EDC, and SD is greater than DC, consequently the Angle DCS = DES, is greater than DSC, per Prop. the third, which was to be proved.

Again, $\angle DES + \angle ESD$, is greater than $\angle ESD + \angle DSC$, but $\angle ESD + \angle DSE = 180^\circ$; therefore $\angle DES + \angle ESD$ is greater than 180° .
Q. E. D.

P R O P.

P R O P. IV.

If $EA + AS$ is equal to the Semicircle EAC , then the internal Angle at E is equal to the outward opposite Angle ASC , and the two internal Angles at the Base $E + S = 180^\circ$.

[Fig. 5. Place the moving Planes so that A and A may come together.]

D E M O N S T R A T I O N.

$EA + AS$, being equal to the Semicircle EAC , the Side $AS = AC$, and consequently the $\angle ACS = \angle EES$ (per Prop. II.) = ASC (per Property the second); and $AES + ESA = ASE + ASC$, but the last Side of the Equation is equal to two right Angles, consequently the other is so too. *Q. E. D.*

P R O P. V.

If $dS + SE$ be less than a Semicircle, then the internal Angle at the Base E , is less than the outward Opposite $\angle DSC$.

[Fig. 5. Place the moving Planes so that A and d , may come together.]

THE Demonstration of this will be just in the same Manner with the two last Propositions.

C O R O L.

Hence it is plain, that in an Ifofceles Triangle, if one of the equal Legs be greater, equal, or less than

than a Quadrant, the Angle at the Base is accordingly, equal, or less than a right Angle.

P R O P. VI.

The Sum of the three Sides of a spherical Triangle, are less than four right Angles.

[Fig. 5. Place A and A together.]

D E M O N S T R A T I O N.

IN the Triangle CAS, the Side AS is less than $AC + CS$ (by Property 7.) and adding $EA + ES$ to both, than $AS + EA + ES$, less than $AC + CS + EA + ES$, which are equal to four right Angles being two Semicircles.
Q. E. D.

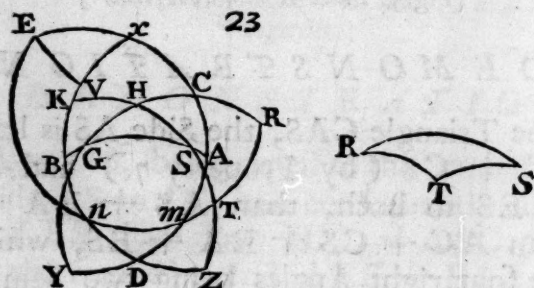
P R O P. VII.

The Poles of the Sides of any Triangle GHS, constitute another Triangle nxm, which we may call supplemental to the Triangle GHS; for the Supplements of the Angles of the Triangle nxm, are equal to the Sides of the Triangle GHS, and the Supplements of the Sides of the Triangle nxm, are equal to the Angles of the Triangle GHS.

D E M O N S T R A T I O N.

FROM the angular Points of the Triangle G, H, S, as Poles, describe on the Globe three great Circles $\propto AY$, $RTmn$, and $\propto BnZ$, and then finding the Centers of the given Sides, continue them to Y and Z, and to Z and K, and to B and D; m being the Pole of GH, mY equal to a Quadrant

Quadrant (because it is the Distance of the Pole of that great Circle from its Periphery) equal Ax , x or D being the Poles of the Circle GS , therefore adding m on both Sides mx , $mx = AYn$, being the Pole of HS , $Zn = Bx$ equal to a Quadrant, and adding Bn to both Sides, then $xn = Bz$ equal to Supplement of BK , which is the Measure of the Angle HSg .



Again, $mn = TR$, *i. e.* the Distance between the Poles of any two great Circles, is equal to the Inclination of those Circles: But n being the Pole of HS , and m the Pole of GH , mn will be the Distance between the two Poles. And the Angle THR is the Inclination of the two Circles, whose Measure is TR , wherefore $mn = TR$, and adding mT to both, then $mR = Tn$; but m being the Pole of GH : mR equal to a Quadrant, and of consequence Tn is a Quadrant also: nm being equal to TR , nm will be equal to the Measure of the Supplement of the Angle GHS , so that the latter Part of the Proposition is proved, *viz.* that the Supplements of the Sides of the Triangle nxm , are equal to the Angles of the Triangle GHS .

And that the first Part is true, may be thus demonstrated.

The Triangle nDm constituted between the three next Poles, hath its Sides equal to the Angles,

SPHERICAL TRIGONOMETRY. 433

Angles, and its three Angles equal to the Sides of the Triangle GHS , save the greatest Side mn , is the Supplement of the $\angle H$, and the $\angle D$, of the Side GS . *Q. E. D.*

PROP. VIII.

Any Angle of a Triangle, with the Difference of the other two, is less than two right Angles; that is, $\angle G + \angle H - \angle S$, is less than two right Angles. (See the last Figure.)

DEMONSTRATION.

nx is less than $xm + mn$ (because two Sides of a Triangle is greater than the the third);

$$\text{But } \left\{ \begin{array}{l} mn = 2 \angle s - \angle H \\ xm = 2 \angle s - \angle G \\ xn = 2 \angle s - \angle S \end{array} \right\} \text{ By the last.}$$

Therefore, by Restitution, $2 \angle - \angle S$, less than $2 \angle s - \angle G + 2 \angle s - \angle H$. The last Step $+ \angle G + \angle H$, is then $2 \angle s - \angle S + \angle G + \angle H = 4 \angle s$: Then each Side $- 2 \angle s$, it will be $C + H - S$, less than $2 \angle s$. *Q. E. D.*

PROP. IX.

If two Triangles are mutually equiangular, they are mutually equilateral.

DEMONSTRATION.

SINCE the two Triangles are equiangular by Supposition, their supplemental Triangles will be equilateral by the 7th. Since the Supplements
F f of

of the Sides of their supplemental Triangles will be respectively equal to the Angles of two equal Triangles, consequently the Sides of the supplemental Triangles will be respectively equal to one another; but the supplemental Triangles being equilateral, will be equiangular, by the 5th Property; and being equiangular, the Sides of the two equiangular Triangles given, will be respectively equal, as being equal to the Supplements of the two equilateral supplemental Triangles, by Prop. VII. of this Section.

P R O P. X.

The three Angles of every spherical Triangle, are greater than two right ones, and less than six right ones.

D E M O N S T R A T I O N.

BY Prop. VI. foregoing, the Sides $nx + xm + mn$, are less than four right Angles (See Fig. to Prop. VII.) But $nx + xm + mn = 6 \text{ Ls} - \angle H - \angle G - S$, per Prop. VIII. being the Sum of the three first Steps in that Proposition, consequently $6 \text{ Ls} - H - G - S$, is less than 4 Ls ; and the last Step divided by four right Angles, and then $+ H + S + G$, it will be 2 Ls less than $\angle H + \angle G + \angle S$, which is the first Part of the Proposition; that is, $\angle H + \angle G + \angle S$, greater then two right Angles.

The Sum of the internal Angles of any Triangle, are less than the Sum of the internal and external Angles taken together; but they are equal to six right Angles (because they are the Angles formed by the Interfection of two great Circles taken three times, and the Angles formed by the meeting of two great Circles, are equal to two right

right

right Angles (*per Prop. I.*) therefore the three times such Intersections, which will be at the three Angles, is equal to six right Angles) *Ergo*, the Internal are less than six right Angles.

P R O P. XI.

Of several Arches of Circles falling from the same Point of the Sphere's Surface, on another Circle, the greatest is that which passeth thro' the Pole of that Circle.

[Fig. 24. Place the moving Planes so that A and A may come together.]

D E M O N S T R A T I O N.

LET P be the Pole of the Circle DWC, and W the Pole of the Circle DPC, then will AD (= AP + DP) be greater than AB, greater than AE, or than AC, and the Arch BWC greater than BP or BD. *Q.E.D.*

P R O P. XII.

In an oblique angled Triangle, if the Angles at the Base are alike, that is, both acute, or both obtuse, the Perpendicular let fall from the opposite Angle, will fall within the Triangle, and the quadrantal Arch without; but if they are unlike, then the Perpendicular falls within the Triangle, and the Quadrant without.

D E M O N S T R A T I O N.

FOR in the Triangle AEF acute-angled at E and F, a Perpendicular let fall upon EF from the opposite Angle A, as AC falls within the Triangle,

Triangle, and the Quadrant AW without (supposing a Circle passing thro' A and W).

Also in the Triangle BAG (supposing a Circle passing thro' B and A) obtuse angled at B and C , a Perpendicular AD to the Base BG , falls within the Triangle, and the Quadrant AW without.

But in the Triangle BAE , whose Angle B is acute, and BEA obtuse, the Perpendicular AC to BE continued, falls without the Triangle, but the Quadrant from A , as AW , falls within the Triangle.

These Propositions being understood, it will be very easy from them to draw the following Theorems.

THEOREM I.

In the spherical Triangles BAC and BHE right-angled at A and H . If the Angle B , at the Base HBA , and B be acute, then the Sine of the Hypotenuse's shall be proportional to the Sines of the perpendicular Arches.

[Fig. 25. Place the two triangular Planes perpendicular to the Plane to which they are fix'd, and the circular moving Plane, so that C and C , and E and E , may come together.]

DEMONSTRATION.

THE right Lines CD and EF being perpendicular to the same Plane, are parallel (*per 6 Euc. II.*) Also FR and DP are perpendicular to the Radius OB , and are likewise parallel; wherefore the Planes of the Triangles EFR and CDP , and CP and ER , (the common Section of the Planes, with the Plane passing thro' RE and CO are) parallel, by 16 *Euc. II.* Therefore the Triangles CDP and EFR are similar, and

and of consequence their Sines proportional. But CP is the Sine of the Hypothenuſe BC, ER is the Sine of the Hypothenuſe BR, and CD is the Sine of the perpendicular Arch CA, and EF is the Sine of the perpendicular Arch EH; therefore, by Proportion, $CP : CD :: ER : EF$; that is,

As the Sine of one Hypothenuſe,
Is to the Sine of the Perpendicular;
So is the Sine of the other Hypothenuſe,
To the Sine of its Perpendicular.

N. B. S	}	Stands for	{	Sine.
T				Cofine, or Sine of Comp.
R				Tangent.
Tc				Radius
				Cotangent, or Tangent of Complement.

THEOREM II.

The ſame Things ſuppoſed as before, QA and HK, the Sines of the Baſes, are proportional to LA and GH, the Tangents of the perpendicular Arches.

DEMONSTRATION.

AFTER the ſame Manner as in the laſt Theorem (*See the ſame Figure*) we can demonſtrate, that the Triangles QAL and KHG, are ſimilar; wherefore QA (the Sine of the Baſe AB) : AL (the Tangent of the Perpendicular CA) :: HK (the Sine of the Baſe BH) : HG (the Tangent of the Perpendicular EH). Q.E.D.

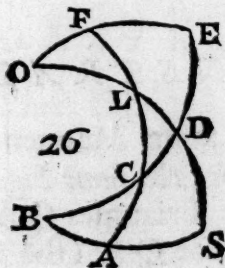
THEOREM III.

In the spherical Triangles ABC, right angled at A, it will be,

*As the Cosine of the $\angle B$ (at the Base AB),
Is to the Sine of the $\angle ACB$ (at the Perpendicular AC);
So is the Cosine of the Perpendicular AC,
To the Radius.*

DEMONSTRATION.

PRODUCE the Sides of the Triangle ABC until they become Quadrants; that is $BS = BD = AL = 90^\circ$; then from B and C, as Poles, describe the Circles OLDS and OFE; then $DO = CE = CF = OE = 90^\circ$, and of consequence $DE = BC$, and $CA = LF$, and the right Angles are A, S, D, E, and AS the Complement of BA, and CD the Complement of BC, and LC the Complement of CA, and DS is the Measure of the Angle B, as is EF, that of the Angle $FCE = \angle BCA$, and $DS = OL$.



In the Triangles FCE and LCD, right-angled at D and E, having both the same acute Angle C, it will be, *per Theorem I.* As S, LD : S, FE

$S, FE :: S, CL : S, CF$; that is, As $\Sigma, \angle B$
 $: S, \angle C :: \Sigma, CA : R.$ Q. E. D.

From these three Theorems, and what may be drawn from the preceding Figure, all the Cases of right-angled spherical Trigonometry may be solved.

THEOREM IV.

*As the Cosine of the Base,
 Is to the Cosine of the Hypothenuse;
 So is the Radius,
 To the Cosine of the Perpendicular.*

DEMONSTRATION.

IN the Triangles ASL and CDL (*Fig. to Theorem III.*) right-angled at S and D, having the same acute Angle, therefore, by *Theorem I.* As $S, AS : S, CD :: S, LA : S, LC$; that is, As $\Sigma, BA : \Sigma, BC :: R : \Sigma, CA.$ Q. E. D.

THEOREM V.

*As the Sine of the Base,
 Is to the Radius;
 So is the Tangent of the Perpendicular,
 To the Tangent of the Angle at the Base.*

DEMONSTRATION.

IN the Triangles ABC and SBD (*See the last Figure*) having the same acute Angle B, and right-angled at S and A, it will be (*per Theorem II.*) $S, BA : S, BS :: T, AC : T, DS$; that is, As $S, BA : R :: T, AC : T, \angle B.$ Q. E. D.

THEOREM VI.

*The Cosine of the vertical Angle C,
Is to the Radius ;
As the Tangent of the Perpendicular,
Is to the Tangent of the Hypothenuse.*

DEMONSTRATION.

IN the Triangles OED and OFL, right-angled at F and E, having the same acute $\angle O$, it will be (*per Theorem II.*) As S, OF : S, OE :: T, FL : T, DE; that is, As $\angle C$: R :: T, CA : T, BC.

THEOREM VII.

*The Sine of the Hypothenuse,
Is to the Radius ;
As the Sine of the Perpendicular,
Is to the Sine of the Angle at the Base.*

DEMONSTRATION.

IN the last Triangles mentioned, it will be (*per Theorem I.*) As S, ED : S, OD :: S, FL : S, OL; that is, As S, BC : R :: S, CA : S, $\angle B$.

THEOREM

THEOREM VIII.

*The Radius,
Is in Proportion to the Sine of the Hypothenuſe;
As the Tangent of the Angle at the Perpendicular,
Is to the Tangent Complement of the Angle at
the Baſe.*

DEMONSTRATION.

IN the Triangles FEC and DLC, right-angled at D and E, having the ſame acute Angle C, it will be, by *Theorem II.* As S, CE : S, CD :: T, FE : T, LD; that is, As R : Z, BC :: T, LC : Tc, LB.

It is here proper to obſerve, that the Angles B and C are ſuppoſed acute, and each Side leſs than a Quadrant.

From theſe eight Theorems (or, indeed, from the two firſt, for the ſix laſt are only Conſequents from them) are all the Proportions in the following Table taken. In which Table there is ſhewn, at one View, what is given, and what is required, in all the Caſes of right-angled ſpherical Trigonometry, with the Theorem for the Solution, and from what Theorem that Proportion was taken.

Caſes.

Cases.	Things given	Required.		What Theor.
PROPORTIONS.				
1	BA, CA	BC	$R : \Sigma, CA :: \Sigma BA : \Sigma BC$	4
2	BA, CA	$\angle B$	$S, BA : R :: T, CA : T, \angle B$	5
3	BC, $\angle C$	$\angle B$	$R : \Sigma, BC :: T, \angle C : T, \angle B$	8
4	BC, $\angle C$	CA	$R : \Sigma, \angle C :: T, BC : T, CA$	6
5	BC, $\angle C$	BA	$R : S, BC :: S, \angle C : S, BA$	7
6	CA, $\angle C$	BC	$\Sigma, \angle C : R :: T, CA : T, BC$	6
7	CA, $\angle C$	$\angle B$	$R : \Sigma CA :: S, \angle C : \Sigma \angle B$	3
8	BA, $\angle B$	CA	$B : S, BA :: T, \angle B : T, CA$	5
9	CA, $\angle B$	BA	$T, \angle B : T, AC :: R : S, BA$	5
10	CA, $\angle B$	BC	$S, \angle B : S, CA :: R : S, BC$	7
11	CA, $\angle B$	$\angle C$	$\Sigma, CA : R :: \Sigma, B : S, C$	3
12	CA, BC	$\angle B$	$S, BC : R :: S, CA : S, \angle B$	7
13	CA, BC	$\angle C$	$T, BC : T, CA :: R : \Sigma, \angle C$	6
14	CA, BC	BA	$\Sigma, CA : R :: \Sigma, BC : \Sigma, BA$	4
15	$\angle B, \angle C$	BC	$T, C : T, \angle B :: R : \Sigma, BC$	8
16	$\angle B, \angle C$	CA	$S, \angle C : \Sigma, \angle B :: R : \Sigma, CA$	3

The Cases in the foregoing Table, are all that can happen in *Right-angled spherical Trigonometry*; and as for the Solution of *Oblique-angled Triangles*, we must have some more Theorems before we can raise the Proportions for them, and most of these will arise upon the Consideration, that the perpendicular Arch AC let fall from the $\angle C$, upon the Base DB continued, if need be, make two right-angled Triangles BAC and DAC, out of the given Triangle.

THEOREM

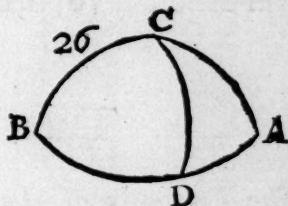
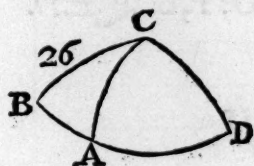
THEOREM IX.

The Cosine of the Angles B and D at the Base BD, are proportional to the Sines of the vertical Angles BCA and DCA.

DEMONSTRATION.

For $\left\{ \begin{array}{l} \overline{M}, B : \overline{S}, BCA :: \overline{M}, CA : R \\ \overline{N}, D : \overline{S}, DCA :: \overline{N}, CA : R \end{array} \right\}$ per Theo. III.

Ergo, $\overline{N}, B : \overline{S}, BCA :: \overline{N}, D : \overline{S}, DCA$. Q.E.D.



THEOREM X.

The Cosines of the Sides BC and DC, are proportional to the Cosines of the Bases BA, DA.

For $\left\{ \begin{array}{l} \overline{M}, BA : \overline{M}, BC :: R : \overline{M}, CA \\ \overline{N}, DA : \overline{N}, DC :: R : \overline{N}, CA \end{array} \right\}$ per Theo. IV.

Ergo, $\overline{N}, BC : \overline{N}, DA :: \overline{N}, CD : \overline{N}, DA$. Q.E.D.

THEOREM

THEOREM XI.

The Sines of the Bases BA and DA, are in a reciprocal Proportion of the Tangents of the Angles B and D, at the Base BD.

DEMONSTRATION.

	1	$S, BA : R :: T, AC : T, B$	$\} T^{theor.}$
	2	$S, DA : R :: T, AC : T, D$	$\} V.$
	3	$S, BA \times T, B = R \times T, AC$	$\} 2. \& 3. in$
	4	$S, DA \times T, D = R \times T, AC$	$\} Equat.$
Consequently	5	$S, BA \times T, B = S, DA \times T, D.$	
5 in Anol.	6	$S, BA : S, DA :: T, D : T, B.$	$\} Q.E.D.$

THEOREM XII.

The Tangents of the Sides BC and CD, are in a reciprocal Proportion to the Sines of the vertical Angles BCA and DCA.

For $\left\{ \begin{array}{l} T, BC : R :: T, CA : \sum, BCA \\ T, DC : R :: T, CA : \sum, DCA \end{array} \right\}$ Theorem VI.

Whence, as in the last, $T, BC \times \sum, BCA = T, DC$
 $\times \sum, DCA.$

Which in Anol. is $T, BC : T, DC :: \sum, DCA : \sum, BCA.$ Q.E.D.

THEOREM

THEOREM XIII.

The Sines of the Sides BC and DC, are reciprocally proportional to the Sines of their opposite Angles B and D.

DEMONSTRATION.

For $\left\{ \begin{array}{l} S, BC : R :: S, CA : S, \angle B \\ S, DC : R :: S, CA : S, \angle D \end{array} \right\}$ per Theor. VI.

Whence, as in the two last $S, BC \times S, \angle B = S, DC$
 $\times S, \angle D.$

Which in Anol. is $S, BC : S, DC :: S, \angle D : S, \angle B.$
 $\mathcal{Q}.E.D.$

THEOREM XIV.

In any spherical Triangle ABC, the Product of CF $\times$ AE (or of FM $\times$ AE) contained under the Sines of the Leg BC and BA, is to the Square of the Radius, as IL (or IA — LA, the Difference between the versed Sine of the Base CA, and the versed Sine of AM, the Difference of the Legs) to GN, the versed Sine of the Angle B.

[Fig. 27. Place the Plane PNG perpendicular to PON, and the other Planes so that P and P, and C and C, may come together.]

DEMONSTRATION.

THE Sine FC = FM, is the Sine of one Leg, and AE is the Sine of the other Leg, and BM is equal BC, consequently AM is the Difference of the Legs, and LA is the versed Sine of the Difference of the Legs, and IA is the versed

versed Sine of the Base CA, consequently the Part IL is the Difference between the versed Sine of the Base, and versed Sine of the Difference of the Legs; and it is likewise plain, that if from B, as a Pole, we describe the Circle PN, then is PN the Measure of the Angle ABC contained between the Legs, and GN is the versed Sine.

It is plain that BP and BN are Quadrants, if from B, as a Pole, we describe a lesser Circle, (whose Plane is CFM) thro' C; the Planes of the Circles CFM and PGN, shall be parallel (*per Lem. III. Sect. II.*) and because the Plane GNP is perpendicular to the Plane BON, the Plane FCM will be so too; and supposing Lines, as PC and CH, to be drawn perpendicular to the Plane of the Circle BON, they will fall in the common Section of the Planes with BON, as suppose in G and H; again draw HI perpendicular to AO, and then the Plane of HI, as CHI, shall be perpendicular to AOB, because CH is perpendicular to it, whence AI is perpendicular to CI, by 18 *Euc. II.*

The Isosceles Triangles CFM and PON, are similar, because MF, NO, and CF, PO, are parallel, and the Angles CFM and PON are equal, because the Lines that constitute them are in the same Planes; and the Perpendiculars from the equal Angles P and C, divide the Triangles into similar ones; therefore it is, As FM : ON :: MH : GN; and because the Triangles AOE, DIH, and DLM, are similar, it will be, as AE : AO :: DL : DM; and as AE : AO :: ID : DH; and by Composition of Proportion, AE : AO :: DL + DI : DH + DM; that is, as AE : AO :: IL : MH; but it has been proved, that FM : ON :: MH : GN; and by Multiplication, AE × FM : AO × ON :: IL × MH : GN × MH; and by taking away MH from

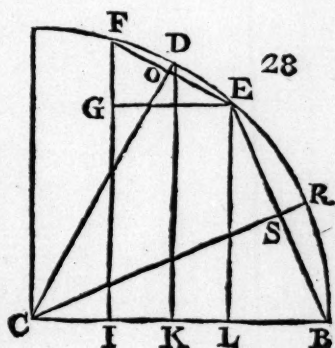
from the two last Terms, and squaring OA , or ON , because they are equal, and Radius's, it is as $AE \times FM : AO^2 :: IL : GN$, which was to be proved.

THEOREM XV.

The Difference of the versed Sines of two Arches, multiply'd by half the Radius, is equal to the Product of half the Sum of the Arches, by the Sine of half the Difference of the Arches.

DEMONSTRATION.

LET the two Arches be BE and BF , whose Difference is FE , which bisect in D , and draw the Radius CD , and the Chord EF ; from the Point E draw GE parallel to CB , and from the Points E , D , and F , draw Perpendiculars to CB , as FI , DK , and EL ; then shall $GE = IL =$ Difference of the versed Sines of the two



Arches, and DK the Sine of half the Sum of the Arches, and FO the Sine of half the Difference of the Arches, and the Triangles CDK and FGE are

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are similar; therefore $DK : GE :: (DC : FE)$
 $:: \frac{1}{2} DC : \frac{1}{2} FE$; whence, by turning it into an
Equation, it will be $DK \times \frac{1}{2} FE = GE \times \frac{1}{2} DC$
 $= IL \times \frac{1}{2} DC$. Q.E.D.

THEOREM XVI.

*The versed Sine of any Arch, multiply'd by half the
Radius, is equal to the Square of the Sine of half
the said Arch.*

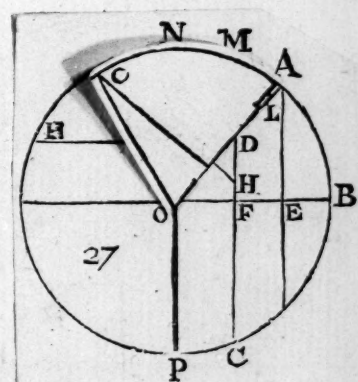
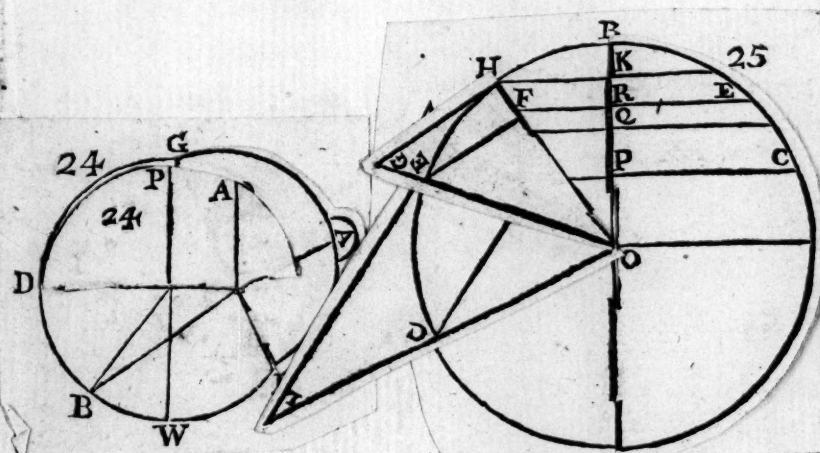
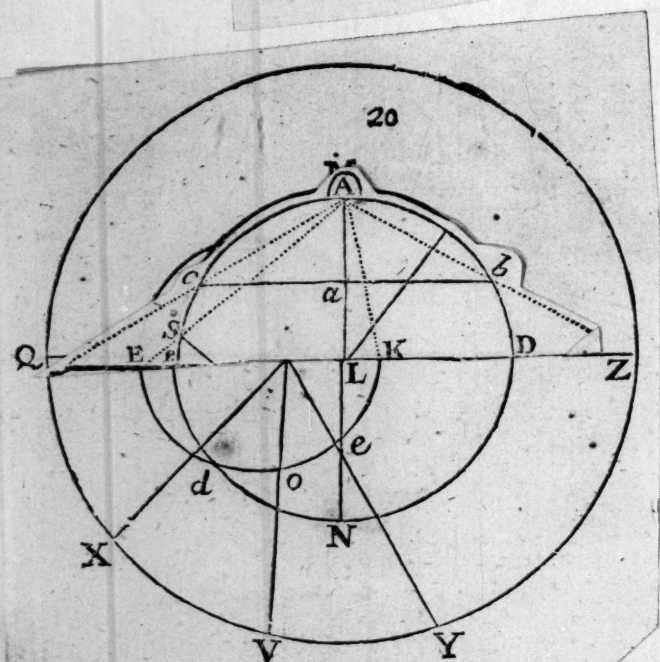
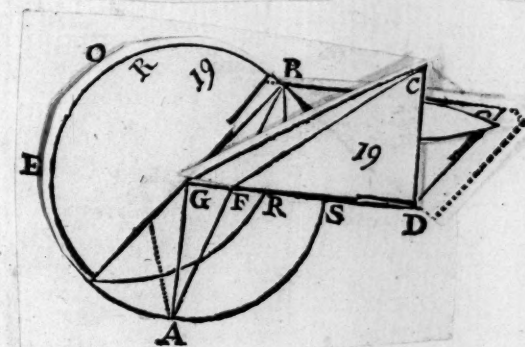
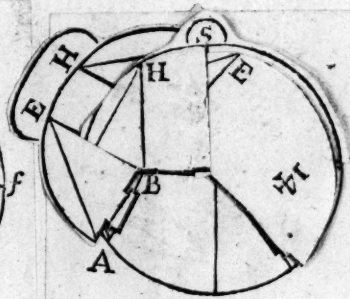
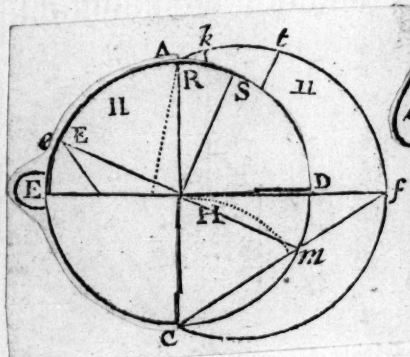
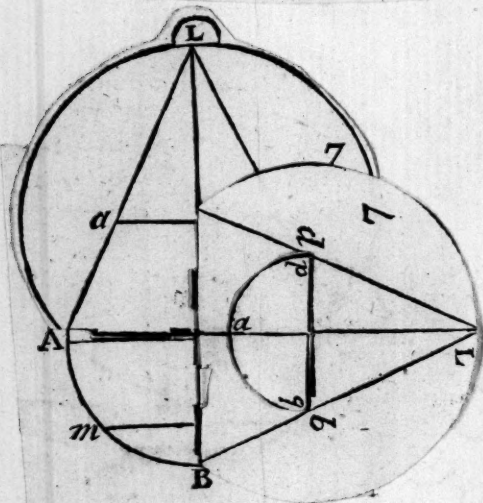
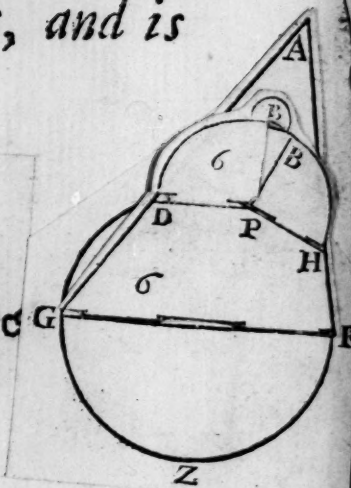
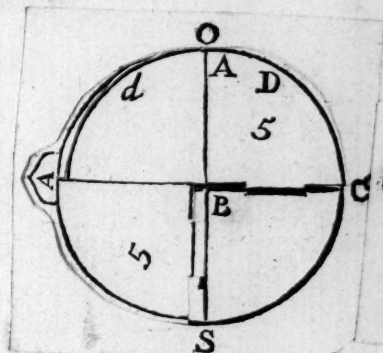
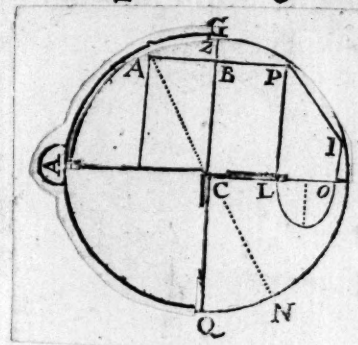
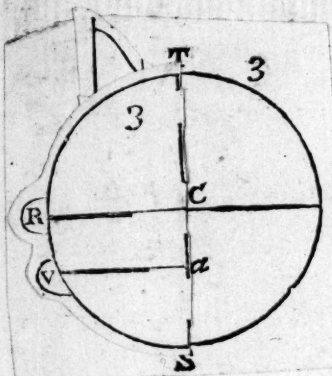
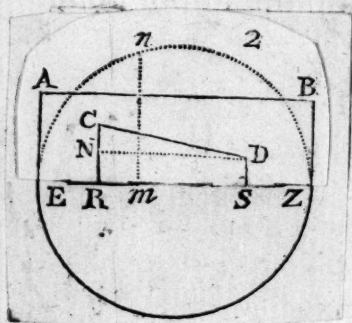
DEMONSTRATION.

THE Triangle CBS (See Fig. to the last Prop.)
and ELB, are similar, because the Angles
L and S are right Angles, and the Angle B com-
mon to both; therefore, as $LB : BE :: BS :$
 CB , which turned into an Equation, is $LB \times$
 $CB = BE \times BS$, and $EB \times \frac{1}{2} CB = BS \times \frac{1}{2}$
 $BE = BS^2$.



THEOREM

*This PLATE contains all the Moveable Cuts, and is
to be placed facing Page 448.*



THEOREM XVII.

In any spherical Triangle, as ABC , whose Legs containing the Angle B , are BC and AB , and the Base subtending that Angle, is AC . If the Arch AM be taken equal to the Difference of the Legs, then shall the Product of the Sines of the Legs, be to the Square of the Radius; as the Sine of the Arch $\frac{AC + AM}{2} \times \frac{AC - AM}{2}$, is to the Square of one Half of the Sine of the Angle B . (See Fig. to Theorem XIV.)

DEMONSTRATION.

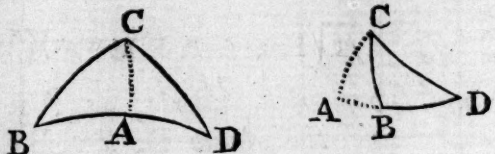
As $S, BA \times S, BC : R^q :: IL : \sqrt{d} S, \angle B$; or as $IL \times \frac{1}{2} R : \frac{1}{2} R \times \sqrt{d} S, \angle B$, per Theor. XIV.

And $IL \times \frac{1}{2} R = S, \frac{AC + AM}{2} \times S, \frac{AC - AM}{2}$ per Theorem XV.

And $\sqrt{d} S, \angle B \times \frac{1}{2} R = S, \frac{1}{2} \angle B^q$, per Theo. XVI.

Ergo, $S, BA \times S, BC : R^q :: S, \frac{AC + AM}{2} \times S, \frac{AC - AM}{2} : \frac{1}{2} S, \angle B^q$. Q.E.D.

By the Help of these last Theorems, and the foregoing eight, may all the Cases of oblique-angled spherical Triangles be solved. And, as before, in right-angled spherical Triangles, so here I shall give a Table to shew what is given, and what is required, and the Proportions for solving them, and then shall proceed to the Practice, and give the geometrical Constructions, and logarithmetical Calculations to all the different Cases of right and oblique-angled spherical Triangles.

CASE.	GIVEN.	REQUIRED.	
			PROPORTION.
1	BC CD B	D	$S, CD : S, B :: S, BC : S, D.$ This Case is ambiguous, because the Angle B may be either greater or less than a Quadrant, or its Supplement to 180° , both having the same Sine, the Proportion from <i>Theorem XIII.</i>
2	BC CD B	BD	$R : \Sigma, B :: T, BC : T, BA,$ per <i>Theorem VI.</i> $\Sigma, BC : \Sigma, BA :: \Sigma, DC : \Sigma, DA,$ per <i>Theor. X.</i> The Sum or Difference of BA and DA, according as the Perpendicular falls within or without the Triangle, is equal to BD, which cannot be known, unless the Species of the Angle D be first known.
3	BC CD B	C	$\Sigma, BC : R :: Tc, B : T, BCA,$ per <i>Theorem VIII.</i> and $T, DC : T, BC :: \Sigma, BCA : \Sigma, DCA,$ per <i>Theorem XII.</i> The Sum and Difference of these Angles BCA and DCA, according as the Perpendicular CD falls within or without the Triangle, is the Measure of the Angle BCD.

CASE.	GIVEN.	REQUIRED.	PROPORTION.
4	BC B D	CD	$S, D : S, BC :: S, B : S, CB,$ which Side is ambiguous ; the Proportion taken from <i>Theor.</i> XIII.
5	BC B D	C	$\Sigma, BC : R :: T, B : T, BCA,$ <i>per Theor.</i> VIII. and $\Sigma, B : S, BCA :: \Sigma, D : S, DCA,$ <i>per Theor.</i> IX. Wherefore the Sum or Difference of the two said Angles, is the Angle C required, according as the Perpendicular falls within or without, which may be known by <i>Prop.</i> XII. of this Section.
6	BC B D	BD	$R : \Sigma, B :: T, BC : T, BA,$ <i>per Theor.</i> VI. $T, D : T, B :: S, BA : S, DA,$ <i>per Theor.</i> XI. The Sum or Difference of BA and DA, is equal to BD, according as the Perpendicular falls within or without, which may be known as before.
7	BC B D	D	$R : \Sigma, BC :: T, B : T, BCA,$ <i>per Theor.</i> VIII. $S, BCA : S, DCA :: \Sigma, B : \Sigma, D,$ <i>per Theorem</i> IX. And whether the Angle D shall be of the same Affection with B, may be known as before.

CASE.	GIVEN.	REQUIRED.	PROPORTION.
8	BC B C	DC	$\Sigma, BC : R :: T, B : T, BCA,$ <i>per Theorem VIII.</i> $\Sigma, DCA :$ $\Sigma, BCA :: T, BC : T, DC,$ <i>per</i> <i>Theorem XII.</i> And whether DC shall be greater or less than a Qua- drant, may be known from the Solutions of Ambiguities of right- angled spherical Triangles follow- ing.
9	DB BC B	DC	$R : \Sigma, B :: T, BC : T, BA,$ <i>per</i> <i>Theor. VI.</i> $\Sigma, BA : \Sigma, BC ::$ $\Sigma, DA : \Sigma, DC,$ <i>per Theorem X.</i> And whether DC shall be greater or less than a Quadrant, may be known from the above mentioned Solutions.
10	BC BD B	D	$R : \Sigma, B :: T, BC : T, DA,$ <i>per Theorem VI.</i> $S, DA : S, BA$ $:: T, B : T, D,$ <i>per Theorem XI.</i> and if $\{ BA \}$ ^{greater} $\{ BD \}$ _{less} then $\{ \text{like} \}$ D $\{ \text{unlike} \}$ B.

CASE.	GIVEN.	REQUIRED.	PROPORTION.
11	BC BD CD	C	$S, BC \times S, CD : R^q :: S, \frac{DB + CM}{2} S, \frac{DB - CM}{2}$ $: S, \frac{1}{2} C^q, \text{ per Theor. XVII. Here } DC - CB = CM.$
12	C B D	CD	Since, by <i>Prop. VII.</i> of this Section, the Poles of the Circles BC, CD, BD, do constitute another Triangle, whose Sides shall be the Supplements to the Angles B, D, C, and the Angle's Supplements to the Sides BC, CD, DB; therefore 'tis only changing the Angles into Sides, and the Sides into Angles, and then the Proportion will be as in the last Case.



THE
ELEMENTS
OF
SPHERICAL TRIGONOMETRY.

SECT. V.

*To Project and Calculate all the CASES of
Right-angled spherical Trigonometry.*



MOST, if not all, the Cases of right and oblique-angled spherical Triangles, may be made ambiguous ; that is, they may have two Answers, tho' one will only answer the Tenor of the Question. Therefore I shall, before I come to the Projection of right-angled spherical Triangles, say something in order to determine which shall be the true Solution ; and I shall do the like before I come to project the Cases of oblique spherical Triangles.

1. The Legs of the right Angle, are of the same kind with their opposite Angles: So in the Triangle BDA (*See Fig. to Prop. XI. of the last Section*) right-angled at D, because that DA is
greater

greater than a Quadrant, the Angle DBA is so too (*per Property 3. Sect. IV.*) and in the Triangle BCA, because AC is less than a Quadrant, the Angle CBA is so too; that is, it is less than the right Angle CBB.

2. If the Legs, and consequently the Angles, are of the same, or different Kinds, the Hypothenuse is accordingly lesser or greater than a Quadrant: So in the Triangles DAE and EAC, the Legs DA and DE are greater, and CA and CE are lesser than a Quadrant (which is what is meant by being of the same kind) the Hypothenuse AE is less than a Quadrant; but in the Triangle BDA, whose Legs BD and DA, are not of the same kind, but one greater, and the other less, than a Quadrant, the Hypothenuse BA is greater than a Quadrant, as BP.

3. On the contrary, if the Hypothenuse be less than a Quadrant, than are the Legs and the Angles opposite to them, less than a Quadrant; but if the Hypothenuse is greater than a Quadrant, then are the Legs that constitute the right Angle of different kinds, *viz.* one greater, and the other less than a Quadrant.

4. If the Hypothenuse be greater or lesser than a Quadrant, each Leg shall be like, or unlike its adjacent Angle; for if the Hypothenuse be less than 90° , the Legs are similar, by *Solution 2.* as also the Angles by the 1st. wherefore each Leg will be similar to its adjacent Angle; but if the Hypothenuse is greater than 90° , the Legs are unlike, by *Solution 2.* and the Angles being of the same kind with the Legs, by the 1st. each Leg will be dissimilar to its adjacent Angle and *vice versa*.

Before I come to project or calculate spherical Triangles (as my Design is to apply the Cases to some astronomical Problems) I shall add some few more astronomical Definitions, which begin where those in Page 375 end, and are absolutely necessary for the understanding the following Cases.

Def. I. The Declination of the Sun is an Arch of a Meridian intercepted between the Sun and the Equinoctial.

II. Ascension is the Rising of the Sun or Star, or any Part of the Equinoctial, above the Horizon.

III. Right Ascension is an Arch of the Equinoctial intercepted between the Beginning of *Aries*, and any Meridian, and counted according to the Order and Succession of the Signs: Or it's that Degree and Minute of the Equinoctial, which come to the Meridian with the Sun.

IV. Oblique Ascension is an Arch of the Equinoctial between the Beginning of *Aries*, and that Part of the Equinoctial which riseth with the Center of the Sun, *è Contra*, for Descension.

V. Ascensional Difference is the Difference between the right Ascension and oblique Ascension: Or it is an Arch of the Equinoctial contained between the Axis of the World, and that Meridian which passeth by the Sun.

VI. Amplitude is an Arch of the Horizon contained between the Sun's Rising or Setting, and the East and West Points, and sheweth how far the Sun or Star Riseth or Setteth from the East or West Points of the Horizon.

VII. Azi-

VII. Azimuth of the Sun is an Arch of the Horizon contained between the Meridian of a Place, and an Azimuth Circle passing thro' the Sun.

VIII. Altitude is that Arch of an Azimuth Circle (passing thro' the Sun or Star) intercepted between the Sun or Star, and the Horizon.

IX. Latitude of a Place is the Distance of the Zenith of that Place from the Equinoctial: Or it is the Elevation of the Pole above the Horizon of that Place.

C A S E I.

Given $\begin{cases} AB = 60^{\circ} 48' \\ BC = 50^{\circ} 48' \end{cases}$ Required AC.

Or given the Latitude of the Place, and Sun's Amplitude; to find the Sun's Declination.

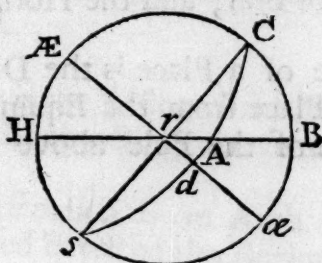
Geometrical Construction.

LET HCS be the Primitive; draw any right Circle, as KB, then make $rA =$ Complement of $AB = 33$ (per Prob. II. Sect. III.) and $CB = 50^{\circ} 48'$, per fame; then thro' the Points C, A, and S, draw the great Circle CAS, which finishes the Triangle ABC; and AC measured (per Prob. VII. Sect. III.) is $71^{\circ} 15'$, what is required, draw $\text{Æ}r\text{æ}$ Perpendicular SC.

In this Projection let the Primitive HCS represent the solstitial Colure, then will the Eye be in either *Aries* or *Libra*, which are the East and West Points, and will be projected into the Center (per Def. IV. Sect. II.) and let HB represent the

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the Horizon, and CB will be the Latitude of the Place, and C the Pole, r A the Amplitude North, and SC will be the Axis of the World, and $\text{Æ}r\alpha$ the Equinoctial, and $A d$ the Sun's Declination; therefore AC subtracted from $A d$, leaves $18^{\circ} 45'$ for the Sun's Declination.



By Calculation.

As Radius 90	-	-	-	10,000000
Is to the $\sphericalangle$, BA = 60	-	-	-	9,698970
So is the $\sphericalangle$, CB = 50 48	-	-	-	9,808067
To the $\sphericalangle$, AC = 71 15	-	-	-	9,507037

Which subtracted from 90, leaves $18^{\circ} 45'$ for the Sun's Declination.

CASE

CASE II.

Given $\begin{cases} AB = 60^\circ 00' \\ CB = 50^\circ 48' \end{cases}$ Required the Angle C.

Or there is the Amplitude and Latitude of the Place given; to find the ascensional Difference.

Geometrical Construction,

IS the same as in the foregoing Case, and the Angle C will be the Complement of the ascensional Difference; for rd is the ascensional Difference (per Def. V. to this Section) and $dæ$ is the Complement to rd ; but $dæ$ is the Measure of the Angle C (per Def. III. Sect. IV.) therefore it is the Complement of the ascensional Difference.

By Calculation,

As $S, CB = 50^\circ 48'$	-	-	9,889271
Is to Radius 90	-	-	10,000000
So is $T, AB = 60$	-	-	10,238561
To the $T, \angle C = 65^\circ 54'$	-	-	10,349290

Which subtracted from 90 ($= ræ$) $= 24^\circ 26' =$ Sun's ascensional Difference.

CASE

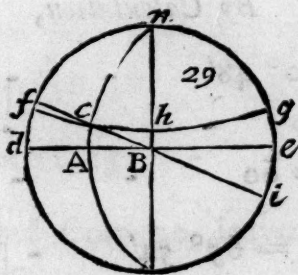
C A S E III.

Given $\left\{ \begin{array}{l} CB = 54^{\circ} 00' \\ \angle B = 23 \quad 29 \end{array} \right\}$ Required the Angle C.

Or there is given the Sun's Place in the Ecliptick, and Sun's greatest Declination; to find the Angle form'd by the Meridian and Ecliptick.

Geometrical Construction.

DR A W *de* any right Circle to the Primitive *dne*, and make the Angle $dBf = 23^{\circ} 29'$ (per Prob. Sect. III.) then cross the right Circle *de* at right Angles with another, as *nB*, then set 54° from B to C (per Prob. II. Sect. III.) and draw the great Circle *nCA*, which compleats the Triangle ABC.



This Projection is made upon the Plane of the solstitial Colure, and the Primitive *dne*, the solstitial Colure, and the Eye, being at *Aries*, or *Libra* (which are projected into the Point B) will project the Equinoctial and Ecliptick into the two right Circles *de* and *fi*, making the Angle at B equal to $23^{\circ} 29'$, being the Angle between the Ecliptick and Equinoctial; and *n* being the Pole *ACn* will be the Meridian passing thro' the given Point

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Point of the Ecliptick, consequently the Angle $ACB = 75^\circ 40'$, is the Angle between the Ecliptick and Meridian required.

By Calculation.

As Radius 90	-	-	-	10,000000
Is to Σ , BC = 54	-	-	-	9,769219
So is T , $\angle B = 23^\circ 29'$	-	-	-	9,637956
To T_c , $\angle C = 75^\circ 40'$	-	-	-	9,407175

C A S E IV.

Given $\left\{ \begin{array}{l} CB = 54^\circ 00' \\ \angle B = 23^\circ 29' \end{array} \right\}$ Required the Side CA.

Or there is the Sun's greatest Declination, and Place in the Ecliptick given; to find its present Declination and right Ascension.

Geometrical Projection.

THE Projection is the same exactly as in the last Case, and it is as evident, that fCg is a Parallel of Declination for the Time, and CA the Distance of the Sun from the Equinoctial.

By Calculation.

As Radius 90	-	-	-	10,000000
Is to S , Hypothenufe BC 54	-	-	-	9,907958
So S , $\angle B = 23^\circ 39'$	-	-	-	9,600409
To the S , CA = $18^\circ 48'$	-	-	-	9,508367

C A S E

CASE V.

Given $\left\{ \begin{array}{l} CB = 54^{\circ} 00' \\ \angle B = 23 \quad 09 \end{array} \right\}$ Required the Side AB.

Or there is the Sun's greatest Declination, and Place in the Ecliptick given; to find its right Ascension.

THE Geometrical Projection is the same as in the two foregoing Cases, and AB will be the Sun's right Ascension (*per* Def. II. of this Section) and may be measured (*per* Prob. VII. Sect. III.

By Calculation.

As Radius, or 90°	- - -	10,000000
Is to Σ , $\angle B = 23^{\circ} 29'$	- - -	9,962453
So is $\mathcal{T}$, $BC = 54^{\circ}$	- - -	10,138739
To $\mathcal{T}$, $AB = 51^{\circ} 37'$	- - -	10,101192

Which is equal to the Sun's right Ascension required.

CASE

CASE VI.

Given $\left\{ \begin{array}{l} AB = 51^{\circ} 37' \\ \angle B = 23^{\circ} 29' \end{array} \right\}$ Required CB.

Or there is given the Sun's greatest Declination, and right Ascension; to find the Sun's Longitude from Aries.

Geometrical Projection.

HAVING drawn the Primitive (See the last Fig.) which let be the solstitial Colure, make the Angle $fBd = 23^{\circ} 29'$, as in the last three Problems; then upon the Equinoctial de , set off $51^{\circ} 37'$ from B to A (per Prob. II. Sect. III.) and then thro' the Points A and n , draw the great Circle ACn , which finisheth the Projection, and gives the Hypothenuse $CB = 54^{\circ}$, the Sun's Longitude from Aries, as was required.

In the two Projections already done, in one the angular Point given is in the Primitive, in the other it is in the Center of the Primitive: I shall now do this last, so that the angular Point shall be any-where within the Primitive, but not in the Center, as in B.

At the Point B make the Angle $ABC = 23^{\circ} 29'$, with two oblique Circles (by the last Case of Prob. V. Sect. III.) and, by Prob. II. lay off $51^{\circ} 37'$ from B to A, at the Point A draw the oblique Circle CAS perpendicular to iAn (by Prob. VI. Sect. III.) which finisheth the Triangle.

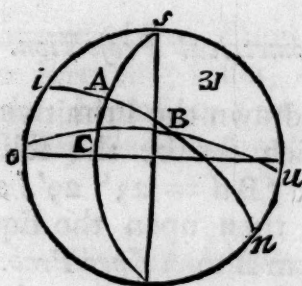
By

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By Calculation.

As Σ , B = $23^{\circ} 29'$	-	-	9,962453
Is to Radius 90°	-	-	10,000000
So is $\mathcal{T}$, AB = $51^{\circ} 37'$	-	-	10,101211
To $\mathcal{T}$, BC = 54°	-	-	10,138758

Which is equal the Sun's Longitude from *Aries*.



C A S E VII.

Given $\left\{ \begin{array}{l} AB = 51^{\circ} 37' \\ \angle B = 23^{\circ} 21' \end{array} \right\}$ Required the Angle C.

Or there is the Sun's greatest Declination, and right Ascension given; to find the Angle form'd between the Ecliptick and Meridian.

THE Projection is the same as in the last Case.

By Calculation.

As Radius 90°	-	-	10,000000
Is to Σ , AB = $51^{\circ} 37'$	-	-	9,793035
So $\mathcal{S}$, $\angle B = 23^{\circ} 39'$	-	-	9,600409
To Σ , $\angle C = 75^{\circ} 40'$	-	-	9,393444

Which is equal the Angle between the Ecliptick and Meridian.

C A S E

C A S E VIII.

Given $\left\{ \begin{array}{l} AB = 51^{\circ} 37' \\ \angle B = 23^{\circ} 21' \end{array} \right\}$ Required AC.

THE Geometrical Construction, and the Parts, are the same with the two last Cases, and AC will be the Sun's Declination, which may be measured (by *Prob. VII. Sect. III.*)

By Calculation.

As Radius 90°	-	-	-	-	10,000000
Is to S, $AB = 51^{\circ} 37'$	-	-	-	-	9,894246
So is $\mathcal{T}$, $\angle B = 23^{\circ} 29'$	-	-	-	-	9,637956
To the $\mathcal{T}$, $BA = 18^{\circ} 48'$	-	-	-	-	9,532202

Which is equal the Sun's Declination.

H h

CASE

C A S E IX.

Given $\begin{cases} \angle A C = 18^{\circ} 48' \\ \angle B = 23^{\circ} 22' \end{cases}$ Required $A B$.

Or there is given the Sun's greatest and present Declination ; to find his right Ascension.

Geometrical Projection.

DR A W *dne* a Primitive (See Fig. to Case III.) which let be the solstitial Colure, and make the Angle $f B d = 23^{\circ} 29'$, which will be the Sun's greatest Declination, then (by *Prob. IX. Sect. III.*) draw $f C g$ parallel to $d e$, the Equinoctial, and thro' the Point C , where that Parallel cuts the Equinoctial, draw the Circle $A C n$, which finisheth the Triangle, the Parts of which are the same with all the foregoing Cases, because the Projection is upon the same Plane.

By Calculation.

As $\angle B = 23^{\circ} 29'$	- - -	9,637956
Is to the $\angle A C = 18^{\circ} 48'$	-	9,532205
So is the Radius 90°	- -	10,000000

To the right Ascension $A B = 51^{\circ} 37'$ 9,894249

CASE

CASE X.

Given $\left\{ \begin{array}{l} AC = 18^\circ 48' \\ \angle B = 23^\circ 29' \end{array} \right\}$ Required BC.

THE Geometrical Projection is the same with last, and in this Case it is plain, that BC will be the Sun's Longitude from Aries.

By Calculation.

As S, $\angle B = 23^\circ 29'$	-	-	-	9,600400
Is to S, $CA = 18^\circ 48'$	-	-	-	9,508214
So is the Radius 90°	-	-	-	10,000000
To S, $BC = 54^\circ$	-	-	-	9,907803

CASE XI.

Given $\left\{ \begin{array}{l} \angle B = 18^\circ 48' \\ AC = 23^\circ 29' \end{array} \right\}$ Required the Angle C.

THE Geometrical Projection is the same with the two last Cases; and the Angle required, is the Angle form'd between the Meridian and Ecliptick.

By Calculation.

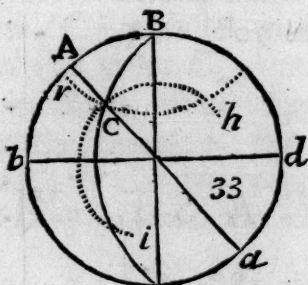
As the Σ , $AC = 18^\circ 48'$	-	-	-	9,976189
Is to Radius 90°	-	-	-	10,000000
So is the Σ , $\angle B = 23^\circ 29'$	-	-	-	9,962453
To S, $\angle C = 75^\circ 40'$	-	-	-	9,986264

C A S E XII.

Given $\left\{ \begin{array}{l} \text{Hypotenuse } BC = 52^\circ \\ \text{and Perpend. } AC = 30 \end{array} \right\}$ Requir. $\angle B$.

Geometrical Construction.

CROSS the Primitive with two right Circles perpendicular one to another, as bd & Bi , then draw the Circle iCb parallel to the Primitive, and distant from it $30^\circ 6'$ (by *Prob. IX. Sect. III.*)

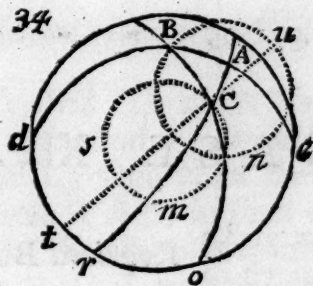


and by the same draw rc parallel to bd , and distant from it the Complement of 52° ; and where these two intersect one another, draw the great Circle CB , which finisheth the Triangle.

Or supposing the angular Point not in the Center, nor in the Primitive, as in the Point C , thro' the Point C draw any oblique Circle, as ACr , from C to A ; lay off 38° (by *Prob. II. Sect. III.*) and then thro' the Point A draw dAe perpendicular to ACr ; thro' the Point C , and the Center, draw tu ; then from C , (by *Prob. II.*) lay off each way the 52° equal to the Hypotenuse, and that Distance bisected in the Middle, gives

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gives the Center of the Circle nBn , which will be every way Distant from C 52° , consequently a great Circle drawn thro' B and C (by *Prob. IV. Sect. III.*) will compleat the Triangle, and give BC , the Hypothenuſe given, and the Angle CBA required.



By Calculation.

As the S , BC the Hypothenuſe 52°	9,896532
Is to the Radius 90° - - -	10,000000
So is the Sine of the Leg AC 30° -	9,798970
To the Sine of the Angle $B = 39^\circ 23'$	9,902438

C A S E XIII.

Given $\begin{cases} BC = 52^\circ \\ AC = 30 \end{cases}$ Required $\angle C$.

THE Geometrical Projection is the ſame as before, as is the Aſtronomical Part, which, in this, and the following Caſe, will be needleſs to repeat.

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By Calculation.

As $\mathcal{T}$, $BC = 52^\circ$	-	-	-	19,107190
Is to $\mathcal{T}$, $CA = 30^\circ$	-	-	-	9,937531
So is the Radius 90°	-	-	-	10,000000
To $\mathcal{N}$, $\angle C = 47^\circ 25'$	-	-	-	9,830341

C A S E XIV.

Given $\begin{cases} CA = 30^\circ \\ BC = 52 \end{cases}$ *Required* BA.

By Calculation.

As $\mathcal{N}$, $CA = 30^\circ$	-	-	-	9,937531
Is to Radius 90°	-	-	-	10,000000
So is $\mathcal{N}$, $BC = 52^\circ$	-	-	-	9,789342
To the $\mathcal{N}$, $BA = 44^\circ$	-	-	-	9,851811

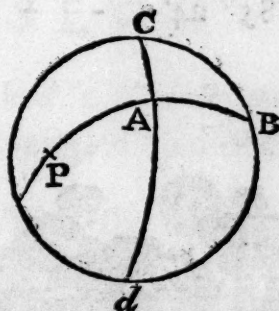
CASE

CASE XV.

Given $\left\{ \begin{array}{l} \angle C = 70^\circ \\ \angle B = 40^\circ \end{array} \right\}$ Required CB.

Geometrical Projection.

BY *Prob. V. Sect. III.* make the Angle C = 70° , and then find P, the Pole of the Circle CA d (by *Prob. I.*) then thro' the Pole P draw the oblique Circle PAB, in such a manner, that the Angle B at the Primitive, may be equal to 40° (by *Prob. XIII.*) which compleats the Triangle.



By Calculation.

As $\mathcal{T}$, C = 70°	-	-	-	-	10,438954
Is to $\mathcal{T}$, B = 40°	-	-	-	-	10,076186
So is Radius 90°	-	-	-	-	10,000000
To the $\mathcal{Z}$, BC = $69^\circ 51'$	-	-	-	-	9,637232

C A S E XVI.

Given $\left\{ \begin{array}{l} \angle C = 70^\circ \\ \angle B = 40^\circ \end{array} \right\}$ Required CA.

THE Geometrical Projection is the same as in the last Case, and then,

By Calculation.

As the S , $C = 70^\circ$	-	-	-	9,972986
Is to M , $B = 40^\circ$	-	-	-	9,884254
So is Radius 90°	-	-	-	10,000000
To M , $CA = 85^\circ 24'$	-	-	-	9,911268





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SECT. VI.

*To Project and Calculate all the CASES of
Oblique-angled spherical Triangles.*



MOST of the Cases of oblique-angled spherical Triangles, are sometimes ambiguous, and sometimes not; therefore it is necessary to determine, before the Operation, whether they are ambiguous or not; to do which, the following Rules are to be observed.

First, When the Sides BC, CD (See Fig. to Theorem IX.) and the Angle B opposite to one of them, are given to determine whether the Things sought are ambiguous or not.

If

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If the Side BC is $\begin{cases} \text{greater} \\ \text{less} \end{cases}$ than a Quadrant,
 and the Angle B $\begin{cases} \text{obtuse} \\ \text{acute} \end{cases}$ then is $\angle ACB$
 $\begin{cases} \text{acute} \\ \text{obtuse} \end{cases}$ per Solution III. of *ambiguous right-*
angled spherical Triangles; and of consequence the
 Side BA $\begin{cases} \text{less} \\ \text{greater} \end{cases}$ (per 1st of the same) than
 a Quadrant, and then the Thing sought is
 $\begin{cases} \text{ambiguous} \\ \text{not ambiguous} \end{cases}$; for if BA be less than a
 Quadrant, BD may be greater, or less than a
 Quadrant, because DA may be greater or less:
 But if BA be greater than a Quadrant, than DA
 must be less than a Quadrant, it is no matter whe-
 ther the Perpendicular falls within or without the
 Triangle; for if it falls $\begin{cases} \text{within} \\ \text{without} \end{cases}$ then $\begin{cases} AB + \\ AB - \end{cases}$
 $\begin{matrix} AD \\ AD \end{matrix} \} = BD.$

And if BA is $\begin{cases} \text{ambiguous} \\ \text{not ambiguous} \end{cases}$ then the
 other Things sought are $\begin{cases} \text{ambiguous} \\ \text{not ambiguous} \end{cases}$ whence
 this,

R U L E I.

If the Legs are of $\begin{cases} \text{like kind} \\ \text{unlike kind} \end{cases}$ then the
 Things sought are $\begin{cases} \text{ambiguous} \\ \text{not ambiguous} \end{cases}$ by this
 Rule are solved the 1st, 3d, and 4th Cases.

Secondly,

Secondly, When there are two Angles, B and D, and the Leg BC, opposite to one of them, given; to determine the Ambiguity.

It may be determined (*per Prop. XII. Sect. IV.*) whether AC falls within, or without the Triangle.

If the Angles BCA and B are of the $\begin{cases} \text{same} \\ \text{unlike} \end{cases}$ kind } then BA $\begin{cases} \text{less} \\ \text{greater} \end{cases}$ than a Quadrant, *per Solutions to Ambiguities of right-angled Triangles*: And if BA $\begin{cases} \text{less} \\ \text{greater} \end{cases}$ than a Quadrant, then AD $\begin{cases} \text{ambiguous} \\ \text{not ambiguous} \end{cases}$; for by the last it may be greater or less than a Quadrant, if BA be less than a Quadrant; but if BA is greater than a Quadrant, BD must be greater likewise, and therefore not ambiguous; and if BD is ambiguous, the rest of the Things sought will be so too, otherwise not. This Solution will do when the Perpendicular falls within the Triangle; whence this Rule.

R U L E II.

If the Side opposite to one of the Angles $\begin{cases} \text{greater} \\ \text{less} \end{cases}$ than a Quadrant, then the Things sought are $\begin{cases} \text{not ambiguous} \\ \text{ambiguous} \end{cases}$: By this Rule are solved the 2d, 5th, and 6th Cases.

Thirdly, Tho' the Theorem given in the last Case is what is given by most Writers of *Trigonometry*, yet it will only do when two Angles, and two Sides, are Quadrants (and consequently when it happens to be given otherwise, we must have Recourse

Recourse to some other Solution) which the late Mr. *Samuel Cunn* thus demonstrates:

For, if possible, let the Side of some Triangle RST (*See Fig. to Prop. VII. Sect. IV.*) be equal to the Measures of the Angles of the Triangle GHS ; that is, let RS , ST , RT , be equal to the Angles H , G , S , the Angles of the Triangle GHS ; and also, that the Measures of the Angles R , S , T , be equal to the Sides GH , GS , HS .

Produce mx and mn , 'till they meet in E , of which $\angle xE$ (*per Prop. VII. Sect. IV.*) the Measure of the Supplement of mx , which, by the same, was the Measure of the Angle G , therefore $\angle xE$ is the Measure of the Angle G . And in like manner the Side nE (the Supplement of mn , which, by the same, was the Supplement of the Angle H) is equal to the Measure of the Angle H itself; but the third Side xn is not the Measure of the Angle D , but it is its Supplement (*per VII. Sect. IV.*). Again, If the Angle Rxn , whose Supplement is $\angle xnm$, the Measure is GH (*per VII. Sect. IV.*) and of the Angle $\angle x n E$, whose Supplement is $\angle x n m$, the Measure is equal to HS ; but the third $nE x$, which is equal $\angle n m x$, the Measure is not equal to GS , but to its Supplement.

Then make $nV = RT = BK$, the Measure of the Angle S , and draw the great Circle EV ; and since RS is, by Supposition, equal to the Measure of the Angle GHS , which is equal to $\angle En$; and since the Measure of the Angle $SR T$, by Supposition is equal to GH , which is also equal to the Measure of the Angle $\angle x n E$. The Triangles RST , and nEV , are mutually equiangular (*per Prop. VI. Sect. IV.*) consequently $ST = EV =$ Measure of the Angle HGD ; to which Measure $\angle xE$ is also equal. Therefore $EV = Ex$, consequently the Angle $EVx = ExV$ (*per IV. Sect. IV.*) and the Angle ExV , whose Measure is
 $= GD,$

$= GD$, is $=$ Angle A , or nVE , since, by Supposition, the Measure of this is also equal to GD . Therefore the Angle $EVx = \angle EVn =$ a Quadrant, consequently ExV is a Quadrant also: Therefore (*per Lem. II. Sect. II.*) EV and Ex are both Quadrants.

But if EV is a Quadrant, and at right Angles to nx then E is the Pole of nx (*per Def. I. Sect. II.*) and so En is a Quadrant also, and the Enx a right one. Therefore if the Sides of a Triangle EnV , or its Equal RST , are equal to the Measures of the Angles of some other Triangle GHS , and the Measures of the Angles of the former, equal to the Sides of the latter. Two Sides of such a Triangle RST , or GHS , must be Quadrants, and two Angles are right ones.

Therefore if a Triangle RST be constructed, whose Sides are equal to the Measures of the Angles of some other Triangle GHS . The Measures of the Angles of the Triangle RST , shall not be equal to the Sides of the Triangle GHD , unless in the Case before mentioned, therefore it will not answer to change the Angles into Sides, as propos'd in *Case XII.*

But to find the Side GS of a spherical Triangle GHS , whose Angles are all given, produce mn , that Side of the supplemental Triangle, which is equal to the Supplement of the Measure GHS , the Angle opposite to the Side sought, and mx , either of the other Sides, 'till they meet in E , and there, as hath been shewn, the Sides Ex , En of the Triangle nEx , are exactly equal to the Measures of the Angle G and H , of the Triangle GHS , and of the Angles Enn , Enx of the Triangle Enn , are equal to GH and HS , but the Sides xn is equal to the Supplement of the Measure of the Angle GDH , and of the Angle xEn , the Measure

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Measure is equal to the Supplement of GH, from whence comes this certain Rule:

R U L E.

Charge one of the Angles adjacent to the Side fought, into its Supplement, and then work with the Measures of the Angles, as tho' they were Sides, and the Result shall be the Side fought.

C A S E I.

Given $\left\{ \begin{array}{l} BC = 39^{\circ} 12' \\ CD = 45 \quad 00 \\ B = 30 \quad 00 \end{array} \right\}$ Required the Angle D.

Or there is given Latitude of the Place, Hour of the Day, and Sun's Altitude; to find the Angle formed by the Meridian (or Hour Circle) and Azimuth Circle.

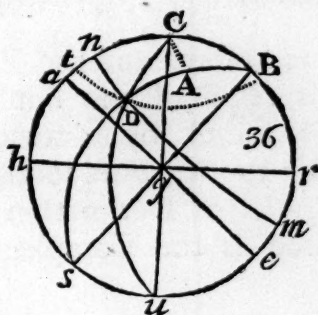
Geometrical Projection.

DR A W *br* in the Primitive, and cross it with another Diameter *Cu*, from C to B set off $39^{\circ} 12'$ (*per Prob. II. Sect. III.*) then at the Point B make an Angle of 30° (*per Prob. V.*) thro' B and the Center draw *SB*, and perpendicular to it *ae*, draw *t DB* parallel to *br*, and distance from it 45° , equal to the Sun's Altitude; and where that Parallel intersects the Meridian *SDB*, draw the great Circle *CDu*, which finisheth the Triangle *DCB*, and in order for the Calculation, let fall the Perpendicular *CA*.

In this Projection the Primitive and Plane of the Projection, is the Meridian at Noon, *br* the Horizon, *rB* the Latitude, B the Pole, C the Zenith, *SB* the Axis of the World, *ae* the Equinoctial

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noctial, and δDB a Parallel of Altitude, the $\angle ABC$, the Angle formed between the Meridian at Noon, and the Sun's Meridian two Hours from Noon, which is the Meridian BDS ; the Circle CDu is a vertical or azimuth Circle, and the Angle CDB , the Angle between it and the Meridian at a given Time, which was required.



By Calculation.

As $S, DC = 45$	-	-	-	-	9,849485
Is to $S, B = 30$	-	-	-	-	9,698970
So is $S, BC = 39\ 12$	-	-	-	-	9,800737
					19,499707
					9,849485
To the $S, \angle D\ 26\ 33$	-	-	-	-	9,650322

CASE

C A S E II.

$$\text{Given } \left\{ \begin{array}{l} BC = 39^{\circ} 12' \\ CD = 57 \quad 00 \\ B = 30 \quad 00 \end{array} \right\} \text{ Required BD.}$$

Geometrical Projection,

IS the same as before, and the Parts the same, only here is required the Sun's Declination, which may be found by drawing a Parallel (*n D m* thro' D) to the Equinoctial, which will represent a Parallel of Declination for the Time, and BD required is the Complement of Declination.

By Calculation.

As Radius 90	-	-	-	-	10,000000
Is Σ , B = 30	-	-	-	-	9,937531
So is $\mathcal{T}$, BC = 39 12	-	-	-	-	9,911467
To $\mathcal{T}$, BA 35 14	-	-	-	-	9,848998
As Σ , BC = 39 12	-	-	-	-	9,889271
Is to $\mathcal{M}$, BA = 35 14	-	-	-	-	9,912121
So is $\mathcal{N}$, DC = 45	-	-	-	-	9,849485
					19,761606
					9,889271
To the Σ , DA 41 49	-	-	-	-	9,872335

And the DA + BA = BD = 77 03.

CASE

CASE III.

$$\text{Given } \left\{ \begin{array}{l} BC = 39^\circ 12' \\ CD = 45^\circ 00' \\ B = 30^\circ 00' \end{array} \right\} \text{ Required } \angle C.$$

THE Projection is the same with the two foregoing Cases.

By Calculation.

$$\text{As } \Sigma, BC = 39^\circ 12' \quad - \quad - \quad - \quad 9,889,271$$

$$\text{Is to Radius} = 90^\circ \quad - \quad - \quad - \quad 10,000,000$$

$$\text{So is } \mathcal{T}, B = 30^\circ \quad - \quad - \quad - \quad 10,238,561$$

$$\text{To the } \mathcal{T}, BCA = 65^\circ 55' \quad - \quad - \quad 10,349,290$$

$$\text{As the } \mathcal{T}, DC = 45^\circ \quad - \quad - \quad - \quad 10,000,000$$

$$\text{Is to the } \mathcal{T}, BC = 39^\circ 12' \quad - \quad - \quad 9,911,467$$

$$\text{So is the } \Sigma, BCA = 65^\circ 55' \quad - \quad 9,610,729$$

$$\text{To the Cosine } DCA = 70^\circ 34' \quad 9,522,196$$

$$\text{And } \angle DCA + \angle BCA = \angle C = 136^\circ 29'.$$

C A S E IV.

$$\text{Given } \left\{ \begin{array}{l} \angle A = 30^\circ 00' \\ \angle C = 136^\circ 29' \\ BD = 77^\circ 03' \end{array} \right\} \text{ Required DC.}$$

Geometrical Projection.

DR A W BS (See Fig. to Prop. I.) and cross it at right Angles with *ae*; at the Point B make an Angle of 30° (by Prob. V. Sect. III.) and (by Prob. II.) make DB equal to $77^\circ 3'$; then (by Prob. XII.) make an Angle at the Primitive equal to $136^\circ 29'$, so that the great Circle that constitutes the Angle may pass through the Point D, the End of the given Side, which compleats the Triangle, and in order for the Calculation of the two following Cases, draw a Perpendicular from D, which suppose *Dn*, to fall without the Triangle.

In this Case there is given the Hour of the Day, the Sun's Azimuth and Declination, to find his Altitude; and in Case V. the same is given to find the Angle formed between the Meridian and Azimuth; and in Case VI. the same is given to find the Latitude.

In this Projection *beru* is the Meridian at Noon, and BS represents the Axis, *ae* the Equinoctial, B the Pole, and drawing *Cu* the prime Vertical, and *br* at right Angles to it, will represent the Horizon *nDm*, a Parallel of Declination, and *tDB* a Parallel of Altitude; and in the Triangle BDC, the Side CD is the Complement of Altitude, CB the Complement of Latitude, and DB the Complement of Declination; the Angle DCB the Azimuth; the Angle DBC, the Angle of the Time from Noon, equal to two Hours; and

BDC

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BDC the Angle formed by the Meridian and Azimuth.

By Calculation.

$$\begin{array}{rcl}
 \text{As } S, \angle C = 136 \ 29 = 43 \ 31 & & 9,837945 \\
 \text{Is to } S, DB = 77 \ 03 & - & 9,988811 \\
 \text{So is } S, \angle B = 30 & - & 9,698970 \\
 & & \hline
 & & 19,687781 \\
 & & 9,837945 \\
 & & \hline
 \end{array}$$

To the Sine DC = 45° the Altitude 9,849836

CASE V.

$$\text{Given } \left\{ \begin{array}{l} \angle B = 30^\circ \ 00' \\ \angle C = 136 \ 29 \\ DB = 77 \ 03 \end{array} \right\} \text{ Required the } \angle D.$$

THE Geometrical Projection, and Parts of it, are the same as in the last Case.

By Calculation.

$$\begin{array}{rcl}
 \text{As } \angle, BD = 77 \ 03 & - & 9,350443 \\
 \text{Is to Radius} = 90 & - & 10,000000 \\
 \text{So is } \angle, B = 30 & - & 10,238561 \\
 & & \hline
 \text{To } \angle, BDn = 82 \ 37 & - & 10,888118
 \end{array}$$

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As the Σ , $B = 30$ - - - - 9,937531

Is to S , $BDn = 82 \ 37$ - - - - 9,996384

So is Σ , $C = 136 \ 29$ - - - - 9,860442

19,856826

9,937531

To the S , $CDn = 56 \ 08$ - - - 9,919295

And $BDn - CDn = BDC = 26^\circ \ 29'$, the Angle required.

C A S E VI.

Given $\left\{ \begin{array}{l} \angle B = 30^\circ \ 00' \\ \angle C = 136 \ 29 \\ DB = 77 \ 03 \end{array} \right\}$ Required BC.

THE Geometrical Construction is the same with the last Case; and,

By Calculation,

As Radius = 90 - - - - 10,000000

Is to Σ , $\angle B = 30$ - - - - 9,937531

So is T , $BD = 77 \ 03$ - - - - 10,638368

To the T , $Bn = 75 \ 13$ - - - 10,575899

As

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As T , $\angle C = 136\ 29$	-	-	-	9,977503
Is to T , $\angle B = 30$	-	-	-	9,761439
So is S , $Bn = 75\ 13$	-	-	-	9,985380
				19,746813
				9,977503
To S , $Cn = 36\ 01$	-	-	-	9,769316

And $Bn - Cn = 39^\circ 12'$ equal to Complement of Latitude, whence the Latitude is $50^\circ 48'$.

C A S E VII.

$$\text{Given } \left\{ \begin{array}{l} BC = 39^\circ 12' \\ B = 30\ 00 \\ C = 136\ 29 \end{array} \right\} \text{ Required } D.$$

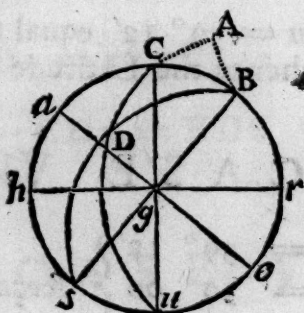
Geometrical Projection.

MAKE the Angle DBC equal 30 Degrees (by *Prob. V.* and by *Prob. II.*) set off from B to C $39^\circ 12'$, at the Point C make the Angle DCB equal $136^\circ 29'$, which compleats the Triangle; and for the Calculation let fall the Perpendicular BA upon DC prolong'd.

Here is given the Time from Noon two Hours equal to 30° , the Sun's Azimuth $136^\circ 29'$, and the Latitude of the Place, to find the Angle between the Meridian and Azimuth Circle. From C draw Cn , which is the prime Vertical, and perpendicular to it br the Horizon. From B draw BS , the Axis of the World, and perpendicular to it, ae the Equinoctial, &c.

By Calculation.

As Radius 90	-	-	-	10,000000
Is to Σ , BC = 39 12	-	-	-	9,889271
So is $\mathcal{T}$, C = 136 29	-	-	-	9,977503
To $\mathcal{T}$, CBA = 53 40	-	-	-	9,866774



And CBA + $\angle$ B equal DBA equal $83^{\circ} 40'$;
and then,

As $\mathcal{S}$, CBA = 53 40	-	-	-	9,906111
Is to $\mathcal{S}$, DBA = 83 40	-	-	-	9,997341
So is Σ , $\angle$ C = 136 29	-	-	-	9,860441
				19,857783
				9,906111
To the Σ , $\angle$ D = 26 33	-	-	-	9,951672

CASE

CASE VIII.

$$\text{Given } \left\{ \begin{array}{l} BC = 39^\circ 12' \\ B = 30^\circ 00' \\ C = 136^\circ 29' \end{array} \right\} \text{ Required DB.}$$

THE Geometrical Projection is the same as in the last Case, and here are the same Things given, to find the Declination of the Sun, the Complement of which is DB.

By Calculation.

$$\text{As Radius } 90 \quad - \quad - \quad - \quad 10,000,000$$

$$\text{Is } \Sigma, BC = 39^\circ 12' \quad - \quad - \quad - \quad 9,889,271$$

$$\text{So is } \mathcal{T}, C = 136^\circ 29' \quad - \quad - \quad - \quad 9,977,503$$

$$\text{To the } \mathcal{T}, CBA = 53^\circ 40' \quad - \quad - \quad 9,866,774$$

And CBA + B equal DBA equal $83^\circ 40'$.

$$\text{As the } \Sigma, DBA = 83^\circ 40' \quad - \quad - \quad 9,042,625$$

$$\text{Is to the } \Sigma, CBA = 53^\circ 40' \quad - \quad 9,772,675$$

$$\text{So is } \mathcal{T}, BC = 39^\circ 12' \quad - \quad - \quad 9,911,467$$

$$19,684,142$$

$$9,042,625$$

$$\text{To the } \mathcal{T}, DB = 77^\circ 09' \quad - \quad 10,641,517$$

C A S E IX.

$$\text{Given } \left\{ \begin{array}{l} DC = 45^\circ 00' \\ BC = 39 \quad 12 \\ C = 136 \quad 29 \end{array} \right\} \text{ Required DB.}$$

Geometrical Projection.

ANY where in the Primitive, as at C (*See the last Figure*) make the Angle DCB (by *Prob. V.*) equal to $136^\circ 29'$; and (by *Prob. II.*) set off from C to B $39^\circ 12'$; and from C to D 45° , draw the Diameter BS, and thro' the Points S, D and B, draw the Circle BDS, which compleats the Triangle; and, in order to calculate, let BA fall perpendicular to DC prolong'd.

Here is given the Azimuth $136^\circ 29'$, the Sun's Altitude 45° , and the Latitude of the Place $50^\circ 48'$, the Complement to which is $39^\circ 12'$; to find the Sun's Declination, as in the last Case, and the Parts of the Projection are the same.

By Calculation.

As Radius = 90	-	10,000000
Is to Σ , $\angle C = 136 \quad 29$	-	9,860442
So is $\mathcal{T}$, $BC = 39 \quad 12$	-	9,911467
To the $\mathcal{T}$, $CA = 30 \quad 36$	-	9,771909
And $CA + DC = DA = 75^\circ 36'$.		

As the Σ , CA = 30 36	-	9,934873
Is to the Σ , BC = 39 12	-	9,889271
So is the Σ , DA = 75 36	-	9,395658
		19,284929
		9,934873
To the Σ , DB = 77 03	-	9,350056

CASE X.

Given $\left\{ \begin{array}{l} DC = 45^\circ 00' \\ BC = 39 12 \\ C = 136 29 \end{array} \right\}$ Required $\angle D$.

As Radius = 90	-	10,000000
Is to the Σ , C = 136 29	-	9,860442
So is the $\mathcal{T}$, BC = 39 12	-	9,911467
To the $\mathcal{T}$, CA = 30 36	-	9,771909

And CA + DC equal DA equal $75^\circ 36'$.

As the $\mathcal{S}$, DA = 75 36	-	9,986137
Is to the $\mathcal{S}$, CA = 30 36	-	9,706753
So is the $\mathcal{T}$, C = 136 29	-	9,977503
		19,684256
		9,986137
To the $\mathcal{T}$, D = 26 33	-	9,698119

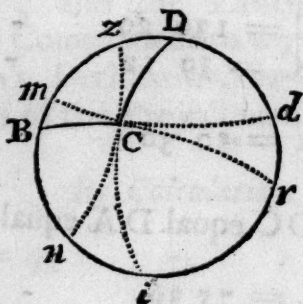
CASE

C A S E X I.

Given $\left\{ \begin{array}{l} BD = 85^\circ \\ BC = 59 \\ CD = 70 \end{array} \right\}$ Required C.

Geometrical Projection.

SET off 85° equal BD any where upon the Primitive, and draw a parallel Circle, which is every way distant from the Point D 70° (by *Prob. IX.*) as md , and by the same draw zn , and distant from B 59° , and where these two parallel Circles cut one another, draw the Circles iCD and BCr , which are two great Circles, and compleats the Triangle.



By

By Calculation.

$$\text{As } S, BC \times S, CD \left\{ \begin{array}{l} 59^\circ - 9,933066 \\ 70 - 9,972986 \end{array} \right\} 19,906052$$

$$\text{Is to the Square of the Radius} - 20,000000$$

$$\text{So is } S, \frac{BD + DC - BC}{2} \times \left. \begin{array}{l} \\ \\ \end{array} \right\} 19,650536$$

$$\frac{BD - DC - BC}{2} \left\{ \begin{array}{l} 9,871073 \\ 9,779463 \end{array} \right\}$$

$$39,650536$$

$$19,902842$$

$$\text{To the Square of } \frac{1}{2} S, \angle C - 19,747697$$

The Square Root of which is $9,873848 = 48^\circ 25'$, which doubled, gives $96^\circ 50' = \angle C$.

C A S E XII.

$$\text{Given } \left\{ \begin{array}{l} B = 64^\circ 00' \\ C = 96 \quad 50 \\ D = 56 \quad 00 \end{array} \right\} \text{ Required } DC.$$

BEFORE we can construct this Case, we must observe, that as it is now given, there can be no Projection according to the Theorem given in the Table; because, according to the Solution of oblique Ambiguities, it will not answer, unless there be two Angles Quadrants; but in this Case neither of them are so, therefore that Theorem will not answer either in the Projection or Calculation. But, by the third Rule of this Section, change the Angle opposite to the Side given, and one of the other Angles, into Sides, and

and the Supplement of the other Angles, into a third Side, and then proceed with these Sides to find the Angles, which will be the Measures of the two Sides, and a Supplement to the third.

Imagine the Side (*See the last Figure*) BD the Supplement of the Angle C, and the Side DC = $\angle B$, and the Side BC = $\angle D$; then of consequence $\angle C$ will equal the Supplement of BD, and DC equal B, and BC equal D, then have we the Sides to find the Angles, which must be the Measures of the Sides.

By Calculation.

$$\text{As } S, BD \times S, BC \quad - \quad - \quad - \quad 19,915478$$

$$\text{Is to the Square of the Radius} \quad 20,000000$$

$$\text{So is } S, \frac{DC + BD - BC}{2} \times S, \left. \begin{array}{l} \\ \\ \frac{DC - BD - BC}{2} \end{array} \right\} \begin{array}{l} \\ \\ 19,353446 \end{array}$$

$$\text{To the Square of } S, \text{ of } \frac{1}{2} \angle B \quad - \quad 19,415478$$

$$\text{Of which the Square Root is} \quad - \quad 9,718984$$

Which is equal to the Sine of $31^\circ 39'$, which doubled is $63^\circ 18' = \angle B = DC$.

In the Solution of these Cafes, I have followed the Theorems just as they are delivered in the Tables, and, I presume, have rendered them easy enough to be understood; and my not varying the Cafes to more astronomical Problems, was because it was not my Design so much to shew practical Astronomy, as to demonstrate both speculative and practical spherical Trigonometry; and in these Demonstrations I have made the Schemes to move or elevate, so as to represent the several

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several Solids and Circles treated of; because in the Practice of Teaching I have found Learners very much at a Loss to understand the Nature and Properties of Solids when represented on Planes; and I have always found this Method I have here used, the best Way of rendering these Demonstrations easy; and if in these Elements I have done any thing to the Advancement of the Science, or have made it more plain than what has already been done, I shall then think my Time well bestow'd, and my End in some measure answered.



THE



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